Lecture 1: Introduction to Dark Matter Direct Detection

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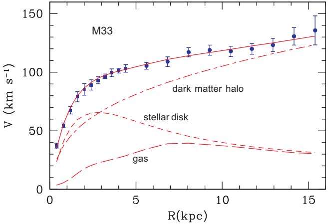
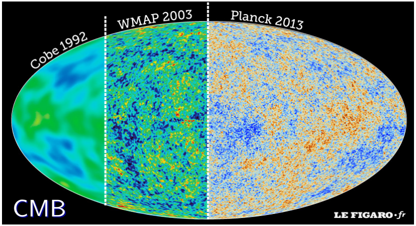
Frontiers in Dark Matter, Neutrinos, and Particle Physics
Theoretical Physics Summer School

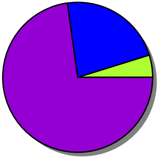
Sun Yat-Sen University, Guangzhou
July 27-28, 2017

Dark Matter in the Universe

Dark matter (DM) makes up most of the matter component in the Universe, as suggested by astrophysical and cosmological observations







Planck 2015
[1502.01589]

Cold DM (25.8%)
$\Omega_c h^2 = 0.1186 \pm 0.0020$

Baryons (4.8%)
$\Omega_b h^2 = 0.02226 \pm 0.00023$

Dark energy (69.3%)
$\Omega_\Lambda = 0.692 \pm 0.012$

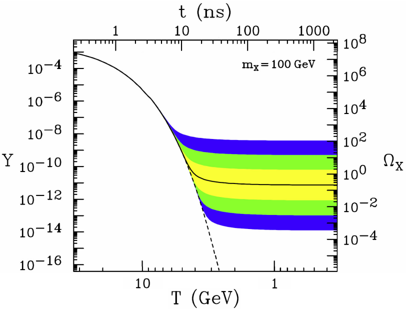
DM Relic Abundance

If DM particles (χ) were thermally produced in the early Universe, their **relic abundance** would be determined by the annihilation cross section $\langle \sigma_{\rm ann} v \rangle$:

$$\Omega_\chi h^2 \simeq \frac{3 \times 10^{-27}\ \mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle \sigma_{\rm ann} v \rangle}$$

Observation value $\Omega_\chi h^2 \simeq 0.1$

$$\Rightarrow \quad \langle \sigma_{\rm ann} v \rangle \simeq 3 \times 10^{-26}\ \mathrm{cm}^3\,\mathrm{s}^{-1}$$



[Feng, arXiv:1003.0904]

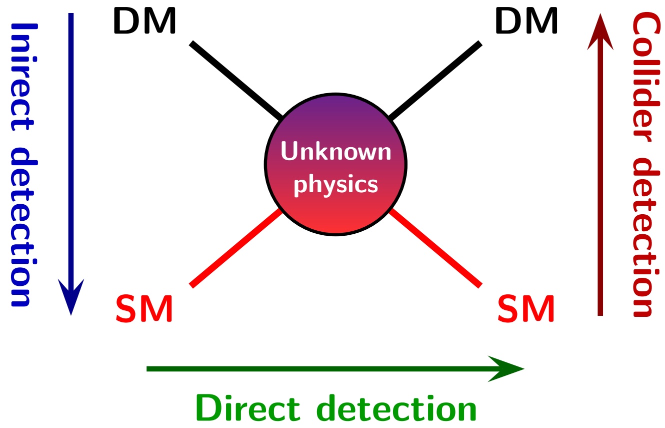
Assuming the annihilation process consists of two weak interaction vertices with the $\mathrm{SU(2)_L}$ gauge coupling $g \simeq 0.64$, for $m_\chi \sim \mathcal{O}(\mathrm{TeV})$ we have

$$\langle \sigma_{\rm ann} v \rangle \sim \frac{g^4}{16\pi^2 m_\chi^2} \sim \mathcal{O}(10^{-26})\ \mathrm{cm}^3\,\mathrm{s}^{-1}$$

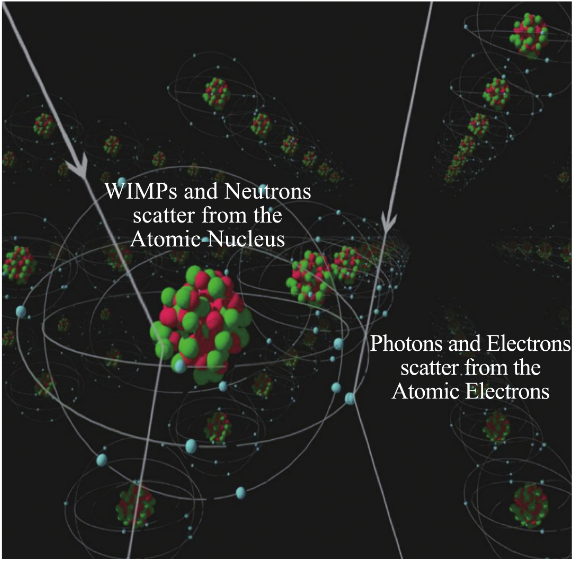
$\Rightarrow$ A very attractive class of DM candidates:

Weakly interacting massive particles (WIMPs)

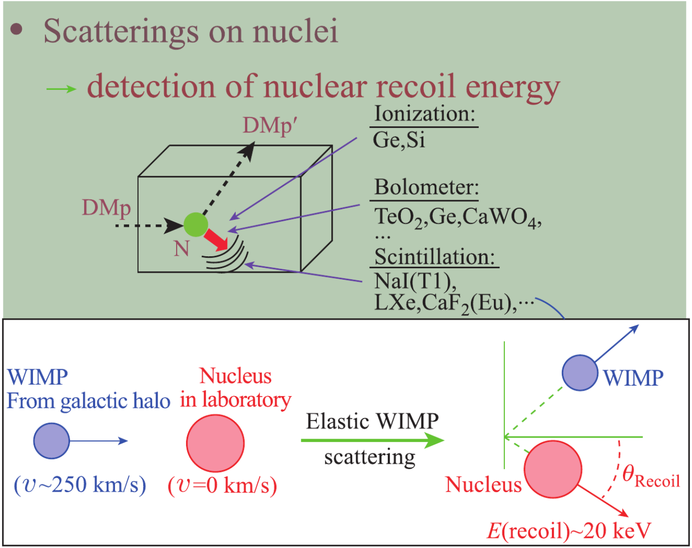
Experimental Approaches to WIMP Dark Matter



WIMP Scattering off Atomic Nuclei



Direct Detection



[Bing-Lin Young, Front. Phys. 12, 121201 (2017)]

WIMP Velocity Distribution

During the collapse process which formed the Galaxy, WIMP velocities were "thermalized" by fluctuations in the gravitational potential, and WIMPs have a **Maxwell-Boltzmann velocity distribution** in the **Galactic rest frame**:

$$\tilde{f}(\tilde{\mathbf{v}})d^3\tilde{v} = \left(\frac{m_\chi}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m_\chi \tilde{v}^2}{2k_B T}\right) d^3\tilde{v} = \frac{e^{-\tilde{v}^2/v_0^2}}{\pi^{3/2} v_0^3} d^3\tilde{v}, \quad v_0^2 \equiv \frac{2k_B T}{m_\chi}$$

$$\langle \tilde{\mathbf{v}} \rangle = \int \tilde{\mathbf{v}} \tilde{f}(\tilde{\mathbf{v}}) d^3\tilde{v} = \mathbf{0}, \quad \left\langle \tilde{v}^2 \right\rangle = \int \tilde{v}^2 \tilde{f}(\tilde{\mathbf{v}}) d^3\tilde{v} = \frac{3}{2} v_0^2$$

Speed distribution: $\tilde{f}(\tilde{v})d\tilde{v} = \dfrac{4\tilde{v}^2}{\sqrt{\pi} v_0^3} e^{-\tilde{v}^2/v_0^2} d\tilde{v}$

For an **isothermal** halo, the local value of v_0 equals to the **rotational speed of the Sun**:

$$v_0 = v_\odot \simeq 220 \mathrm{km/s}$$

[Binney & Tremaine, *Galactic Dynamics*, Chapter 4]


Galactic disk and dark halo

[Credit: ESO/L. Calçada]

Velocity dispersion: $\sqrt{\langle \tilde{v}^2 \rangle} = \sqrt{3/2}\, v_0 \simeq 270 \mathrm{km/s}$

Earth Rest Frame

The WIMP velocity distribution $f(\mathbf{v})$ seen by an observer on the Earth can be derived via **Galilean transformation**

$$\tilde{\mathbf{v}} = \mathbf{v} + \mathbf{v}_{\text{obs}}, \quad \mathbf{v}_{\text{obs}} = \mathbf{v}_{\odot} + \mathbf{v}_{\oplus}$$

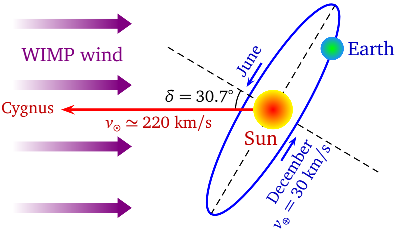
Velocity distribution: $f(\mathbf{v}) = \tilde{f}(\mathbf{v} + \mathbf{v}_{\text{obs}})$

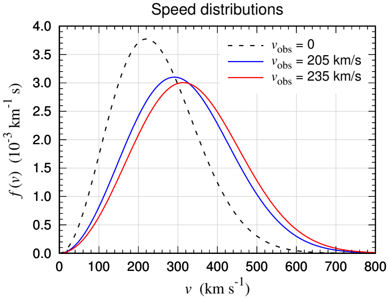
Speed distribution:

$$f(v)dv = \frac{4v^2}{\sqrt{\pi}v_0^3} \exp\left(-\frac{v^2 + v_{\text{obs}}^2}{v_0^2}\right) \times \frac{\tilde{v}_0^2}{2vv_{\text{obs}}} \sinh\left(\frac{2vv_{\text{obs}}}{v_0^2}\right) dv$$

Since $v_{\oplus} \ll v_{\odot}$, we have ($\omega = 2\pi/\text{year}$)

$$v_{\text{obs}}(t) \simeq v_{\odot} + v_{\oplus} \sin\delta \cos[\omega(t - t_0)]$$
$$\simeq 220\ \text{km/s} + 15\ \text{km/s} \cdot \cos[\omega(t - t_0)]$$





$\Rightarrow$ **Annual modulation signal peaked on June 2** [Freese *et al.*, PRD 37, 3388 (1988)]

Nuclear Recoil

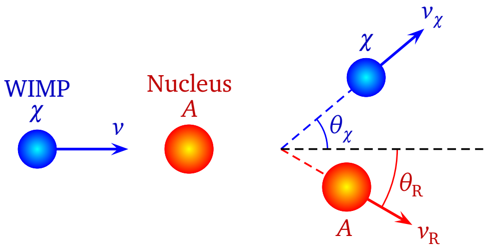
Energy conservation:

$$\frac{1}{2}m_\chi v^2 = \frac{1}{2}m_\chi v_\chi^2 + \frac{1}{2}m_A v_\mathrm{R}^2$$

Momentum conservation:

$$m_\chi v = m_\chi v_\chi \cos\theta_\chi + m_A v_\mathrm{R} \cos\theta_\mathrm{R}$$

$$m_\chi v_\chi \sin\theta_\chi = m_A v_\mathrm{R} \sin\theta_\mathrm{R}$$



$\Rightarrow$ Recoil velocity $v_\mathrm{R} = \dfrac{2m_\chi v \cos\theta_\mathrm{R}}{m_\chi + m_A}$

$\Rightarrow$ **Recoil momentum** (momentum transfer) $q_\mathrm{R} = m_A v_\mathrm{R} = 2\mu_{\chi A} v \cos\theta_\mathrm{R}$

Reduced mass of the χA system $\mu_{\chi A} \equiv \dfrac{m_\chi m_A}{m_\chi + m_A} = \begin{cases} m_A, & \text{for } m_\chi \gg m_A \\ \frac{1}{2}m_\chi, & \text{for } m_\chi = m_A \\ m_\chi, & \text{for } m_\chi \ll m_A \end{cases}$

Forward scattering ($\theta_\mathrm{R} = 0$) $\Rightarrow$ maximal momentum transfer $q_\mathrm{R}^{\max} = 2\mu_{\chi A} v$

Nuclear Recoil

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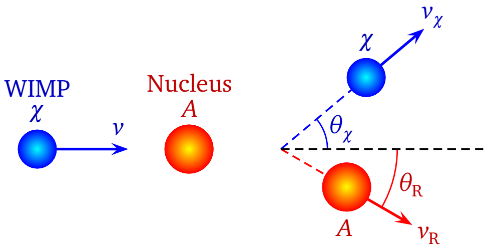

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$\Rightarrow$ **Recoil momentum** (momentum transfer) $q_\mathrm{R} = m_A v_\mathrm{R} = 2\mu_{\chi A} v\cos\theta_\mathrm{R}$

$\Rightarrow$ **Kinetic energy** of the recoiled nucleus $E_\mathrm{R} = \dfrac{q_\mathrm{R}^2}{2m_A} = \dfrac{2\mu_{\chi A}^2}{m_A}v^2\cos^2\theta_\mathrm{R}$

As $v \sim 10^{-3}c$, for $m_\chi = m_A \simeq 100$ GeV and $\theta_\mathrm{R} = 0$,

$$q_\mathrm{R} = m_\chi v \sim 100\ \mathrm{MeV}, \quad E_\mathrm{R} = \frac{1}{2}m_\chi v^2 \sim 50\ \mathrm{keV}$$

Event Rate

Event rate per unit time per unit energy interval:

$$\frac{dR}{dE_\mathrm{R}} = N_A \frac{\rho_\oplus}{m_\chi} \int_{v_\mathrm{min}}^{v_\mathrm{max}} d^3v\, f(\mathbf{v}) v \frac{d\sigma_{\chi A}}{dE_\mathrm{R}}$$

Astrophysics factors
Particle physics factors
Detector factors

N_A: **target nucleus number**

$\rho_\oplus \simeq 0.4$ GeV/cm^3: DM **mass density** around the Earth

($\rho_\oplus/m_\chi$ is the DM particle **number density** around the Earth)

$\sigma_{\chi A}$: DM-nucleus **scattering cross section**

Minimal velocity $v_\mathrm{min} = \left(\frac{m_A E_\mathrm{R}^\mathrm{th}}{2\mu_{\chi A}^2}\right)^{1/2}$: determined by the **detector threshold** of nuclear recoil energy, E_R^th

Maximal velocity v_max: determined by the DM **escape velocity** v_esc

($v_\mathrm{esc} \simeq 544$ km/s [Smith *et al.*, MNRAS 379, 755])

Cross Section Dependence on Nucleus Spin

There are two kinds of DM-nucleus scattering

Spin-independent (SI) cross section: $\sigma_{\chi A}^{\mathrm{SI}} \propto \mu_{\chi A}^2 [Z G_p + (A - Z) G_n]^2$

Spin-dependent (SD) cross section: $\sigma_{\chi A}^{\mathrm{SD}} \propto \mu_{\chi A}^2 \frac{J_A + 1}{J_A} (S_p^A G_p' + S_n^A G_n')^2$

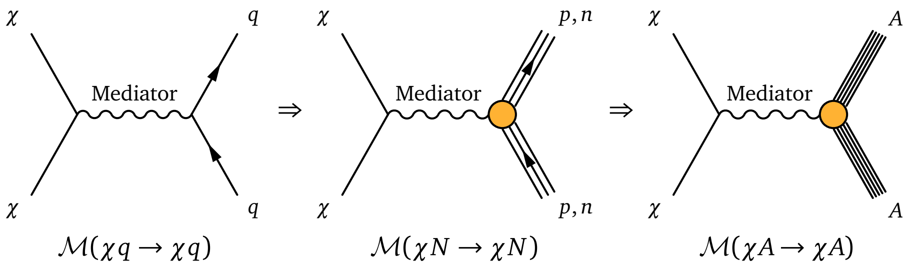
Nucleus properties: **mass number** A, **atomic number** Z, **spin** J_A, expectation value of the **proton (neutron) spin content** in the nucleus S_p^A (S_n^A)

$G_p^{(\prime)}$ and $G_n^{(\prime)}$: **DM effective couplings** to the proton and the neutron

- $Z \simeq A/2 \;\Rightarrow\; \sigma_{\chi A}^{\mathrm{SI}} \propto A^2 [(G_p + G_n)/2]^2$
  Strong **coherent enhancement** for **heavy** nuclei
- Spins of nucleons tend to **cancel out** among themselves:
  - $S_N^A \simeq 1/2$ ($N = p$ or n) for a nucleus with an **odd** number of N
  - $S_N^A \simeq 0$ for a nucleus with an **even** number of N

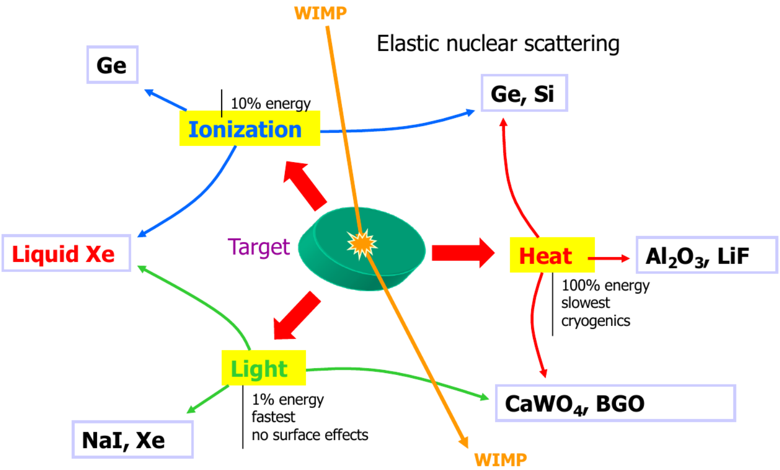
Three Levels of Interaction

DM-parton interaction **DM-nucleon interaction** **DM-nucleus interaction**



- As a variety of target nuclei are used in direct detection experiments, results are usually compared with each other at the **DM-nucleon level**
- The DM-nucleon level is related to the DM-parton level via **form factors**, which describe the probabilities of finding partons inside nucleons
- Relevant partons involve not only valence quarks, but also **sea quarks and gluons**

Technologies and Detector Material



[From M. Lindner's talk (2016)]

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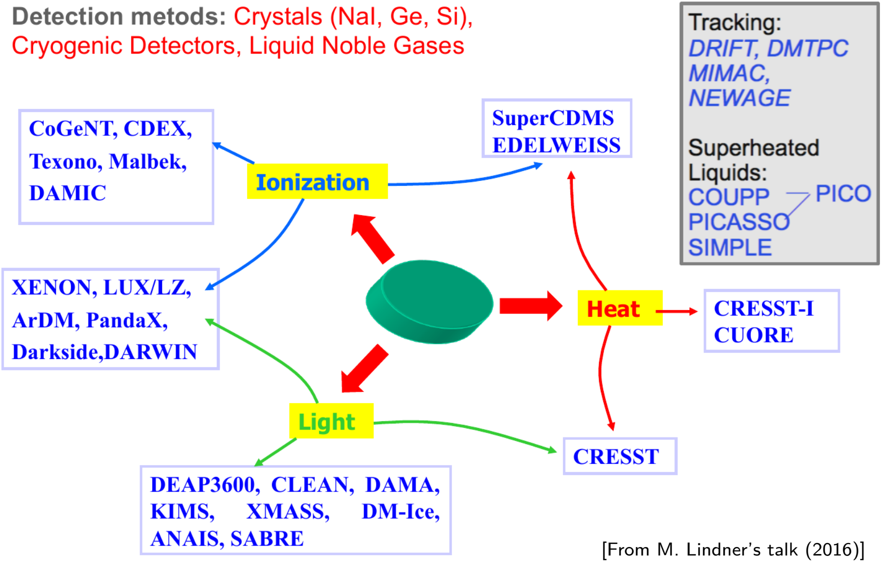


[From M. Lindner's talk (2016)]

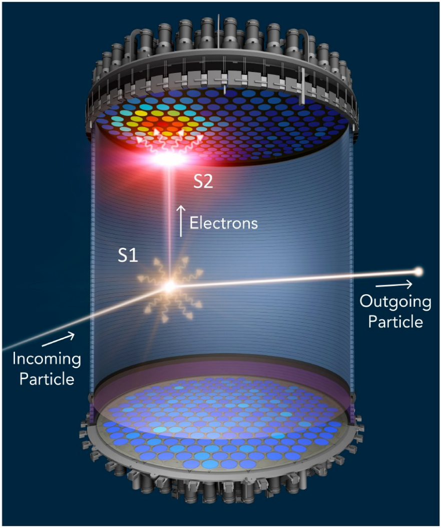
Example: Dual-phase Xenon Time Projection Chamber

Upper: Xenon gas
Lower: Liquid Xenon

UV scintillation photons recorded by photomultiplier tube (PMT) arrays on top and bottom

- **Primary scintillation (S1):** Scintillation light promptly emitted from the interaction vertex
- **Secondary scintillation (S2):** Ionization electrons emitted from the interaction are drifted to the surface and into the gas, where they emit proportional scintillation light

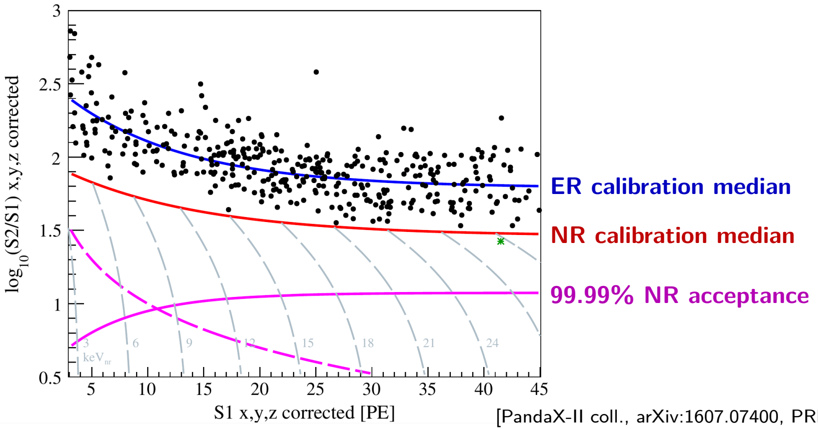
Experiments: XENON, LUX, PandaX



[From A. Cottle's talk (2017)]

PandaX-II Real Data: S1 versus S2

- S1 and S2: characterized by numbers of **photoelectrons (PEs)** in PMTs
- The γ **background**, which produces **electron recoil (ER)** events, can be distinguished from **nuclear recoil (NR)** events using the S2-to-S1 ratio



[PandaX-II coll., arXiv:1607.07400, PRL]

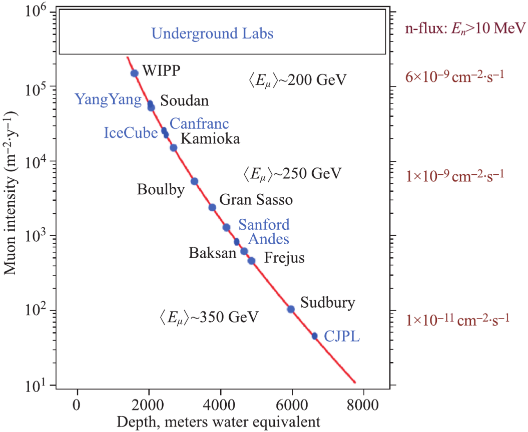
Backgrounds

Background suppression:

Deep underground

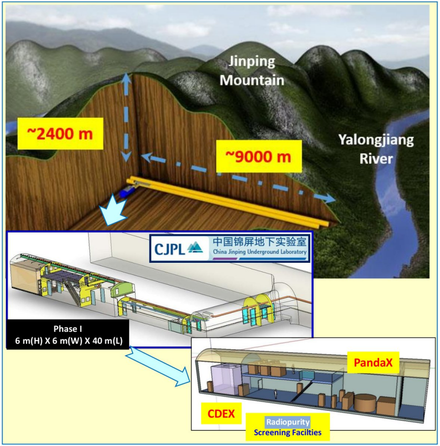
Shielded environments

- **Cosmogenic backgrounds:**
  - Cosmic rays and secondary reactions
  - Activation products in shields and detectors
- **Radiogenic backgrounds:**
  - External natural radioactivity: walls, structures of site, radon
  - Internal radioactivity: shield and construction materials, detector contamination in manufacture, naturally occurring radio-isotopes in target material



[From P. Cushman's talk (2014)]

China JinPing Underground Laboratory (CJPL)

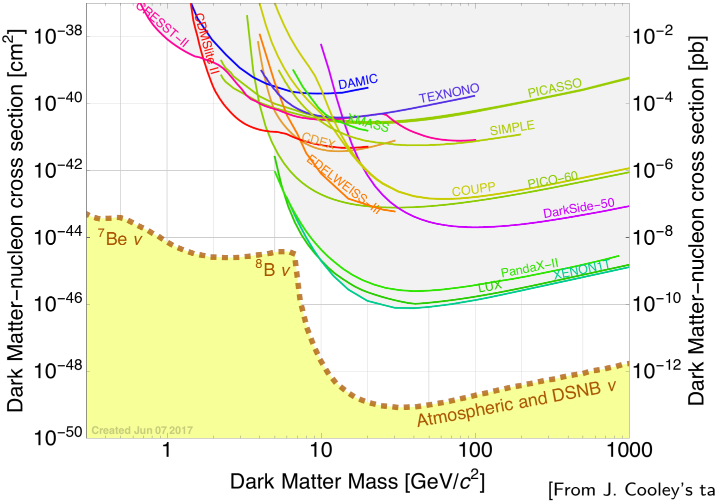


[Yue *et al.*, arXiv:1602.02462]

Experiments: CDEX, PandaX

Exclusion Limits for SI Scattering

For **SI scattering**, the **coherent enhancement** allows us to treat protons and neutrons as the same species, **"nucleons"**



[From J. Cooley's talk (2017)]

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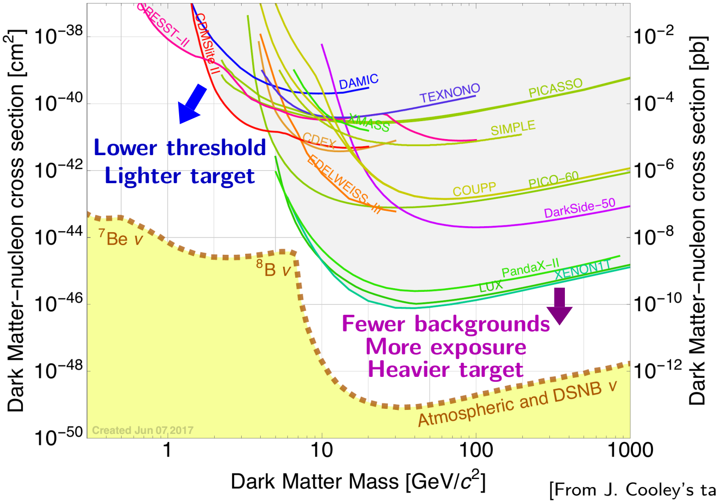
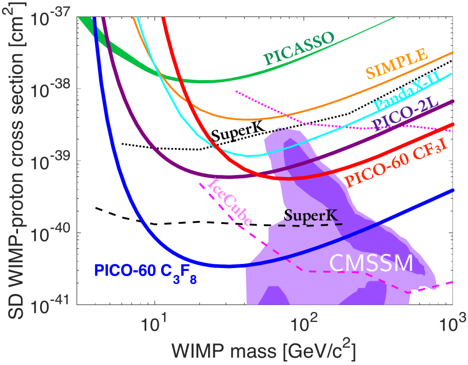


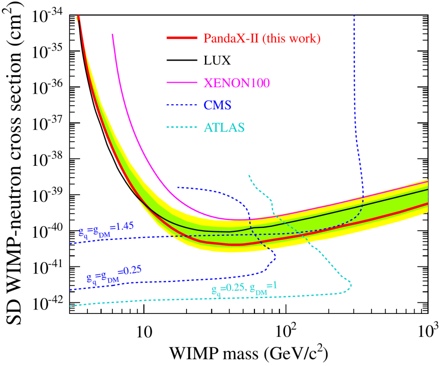
[From J. Cooley's talk (2017)]

Exclusion Limits for SD Scattering

- For **SD scattering**, specific detection material usually has **very different** sensitivities to WIMP-proton and WIMP-neutron cross sections
- As there is no coherent enhancement for SD scattering, the sensitivity is **lower** than the SI case by **several orders of magnitude**



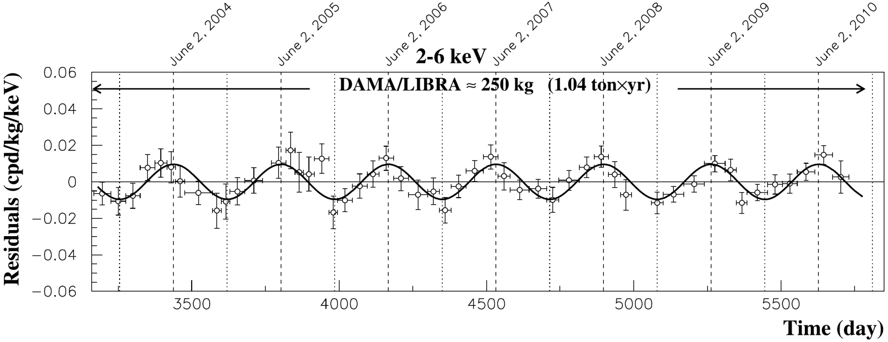
[PICO coll., arXiv:1702.07666, PRL]



[PandaX-II coll., arXiv:1611.06553, PRL]

DAMA/LIBRA Annual Modulation "Signal"

- Highly radio-pure scintillating **NaI(TI) crystals** at Gran Sasso, Italy
- **Annual modulation signal** observed over 14 cycles at 9.3σ **significance**
- **No background/signal discrimination**



[Bernabei *et al.*, arXiv:1308.5109, EPJC]

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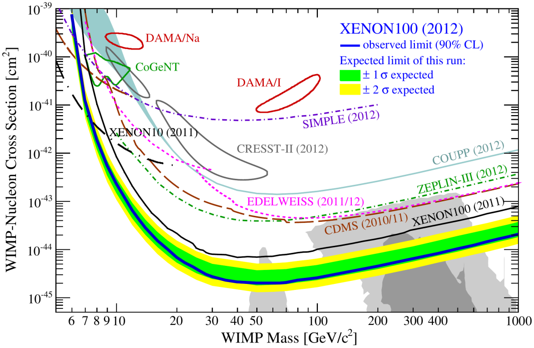


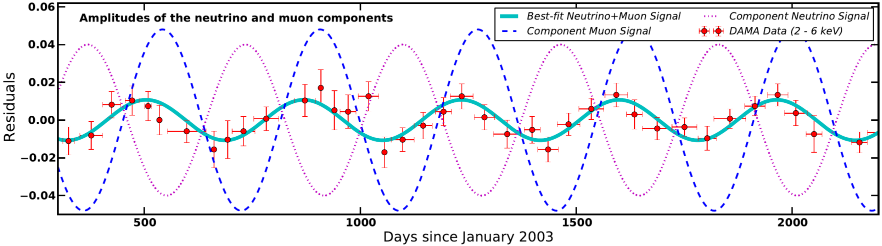
[XENON100 coll., arXiv:1207.5988, PRL]

Favored regions excluded by other direct detection experiments

Other Sources for DAMA/LIBRA Signal

☹ The DAMA/LIBRA signal might be composed of **neutrons** liberated in the material surrounding the detector by **two sources** [Davis, arXiv:1407.1052, PRL]

- **Atmospheric muons:** flux depends on the **temperature of the atmosphere**, peaked on **June 21st**
- **Solar neutrinos:** flux depends on the **distance between the Earth and the Sun**, peaked on **January 4th**



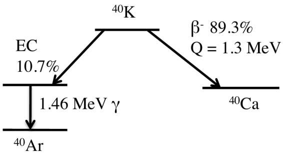
☺ **Objection:** Klinger & Kudryavtsev, "muon-induced neutrons do not explain the DAMA data," arXiv:1503.07225, PRL

Further Test: SABRE Project

SABRE: Sodium iodide with Active Background REjection

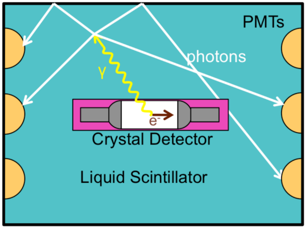
- Complementary tests in **both hemispheres**: one part in Gran Sasso (Italy) and one part in Stawell (Australia)
- Developing **low background** scintillating NaI(Tl) crystals that exceed the radio-purity of DAMA/LIBRA
- A well-shielded **active veto** to reduce internal and external backgrounds

Ultra-high purity NaI(Tl) detector + Active veto detector



^{40}K→^{40}Ar, ~11% branch ratio

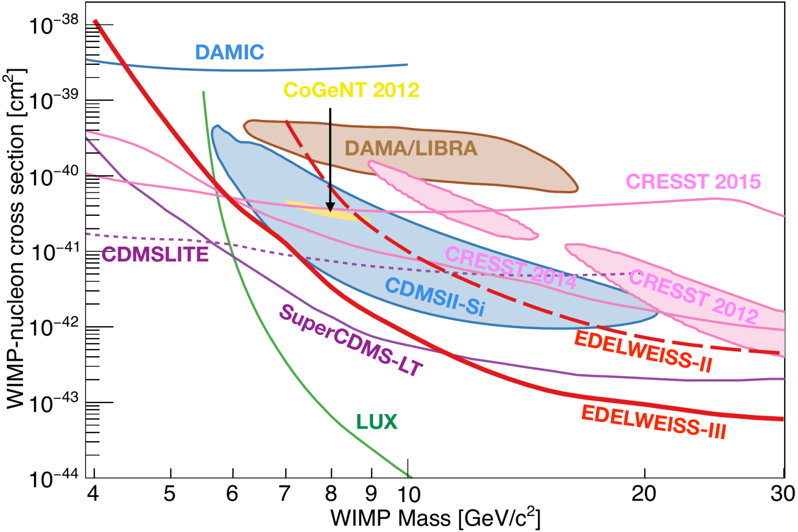
- 3 keV K shell X-ray, Auger e^-
- Background at ~3 keV if γ escapes



1.46MeV γ can be detected by a veto.
^{40}K background can be rejected.

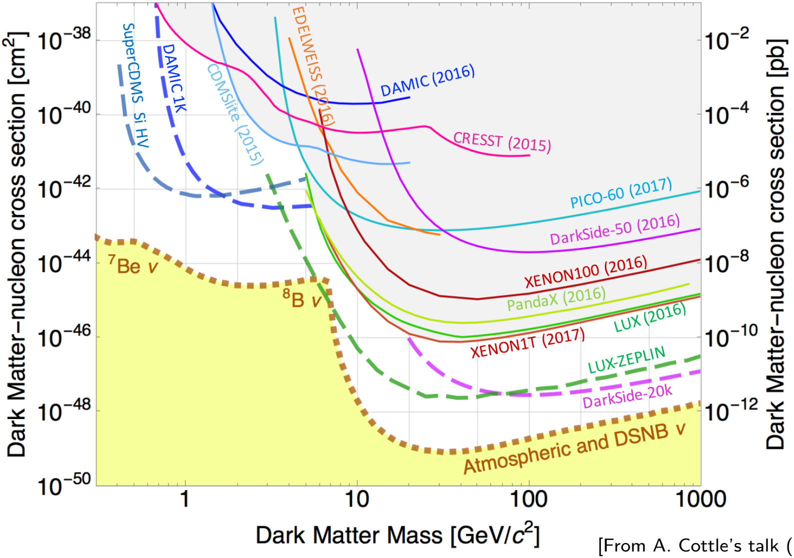
[From E. Barberio's talk]

Low Mass Situation



[From J. Billard's talk (2016)]

Near Future Prospect

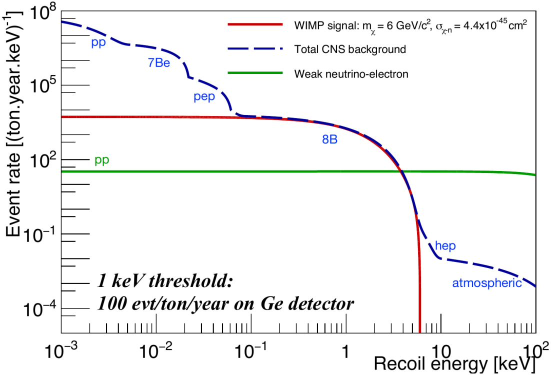


[From A. Cottle's talk (2017)]

Neutrino Backgrounds

Direct detection experiments will be sensitive to **coherent neutrino-nucleus scattering (CNS)** due to astrophysical neutrinos [Billard *et al.*, arXiv:1307.5458, PRD]

- **Solar neutrinos**
  - pp neutrinos:
    $p + p \to \mathrm{D} + e^+ + \nu_e$
  - ^{7}Be neutrinos:
    $e^- + {}^7\mathrm{Be} \to {}^7\mathrm{Li} + \nu_e$
  - pep neutrinos:
    $p + e^- + p \to \mathrm{D} + \nu_e$
  - ^{8}B neutrinos:
    $^8\mathrm{B} \to {}^8\mathrm{Be}^* + e^+ + \nu_e$
  - Hep neutrinos:
    $^3\mathrm{He} + p \to {}^4\mathrm{He} + e^+ + \nu_e$



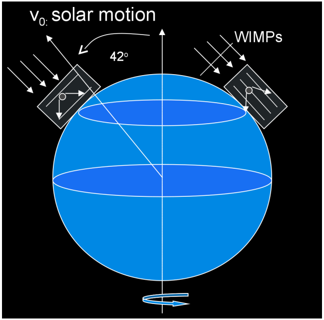
[From J. Billard's talk (2016)]

- **Atmospheric neutrinos**
  Cosmic-ray collisions in the atmosphere
- **Diffuse supernova neutrino background (DSNB)**
  All supernova explosions in the past history of the Universe

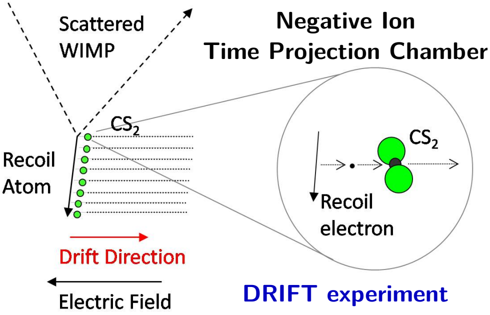
Going beyond the Neutrino Floor

Possible ways to reduce the impact of neutrino backgrounds:

- Reduction of **systematic uncertainties** on neutrino fluxes
- Utilization of **different target nuclei** [Ruppin *et al.*, arXiv:1408.3581, PRD]
- Measurement of **annual modulation** [Davis, arXiv:1412.1475, JCAP]
- Measurement of **nuclear recoil direction** [O'Hare, *et al.*, arXiv:1505.08061, PRD]



Diurnal modulation

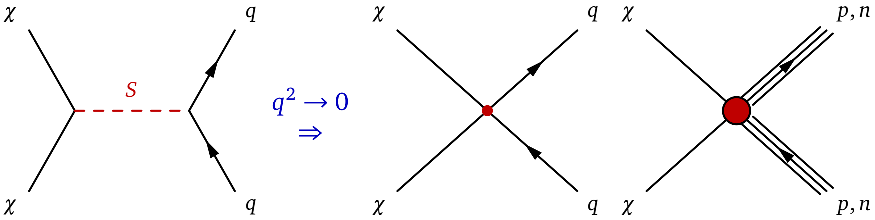


DRIFT experiment

[From J. Spooner's talk (2010)]

Zero Momentum Transfer Limit

- As the momentum transfer (q_R in the nucleus rest frame) is typically much smaller than the underlying energy scale (*e.g.*, mediator mass), the **zero momentum transfer limit** is a good approximation for calculation
- In this limit, the mediator field can be integrated out, and the interaction can be described by **effective operators** in **effective field theory**



Scalar mediator propagator: $\dfrac{i}{q^2 - m_S^2} \quad \Rightarrow \quad -\dfrac{i}{m_S^2}$

Lagrangian: $\mathcal{L}_\mathrm{int} = g_\chi S \bar{\chi}\chi + g_q S \bar{q} q \quad \Rightarrow \quad \mathcal{L}_\mathrm{eff} = G_\mathrm{eff} \bar{\chi}\chi \bar{q} q, \quad G_\mathrm{eff} = \dfrac{g_\chi g_q}{m_S^2}$

Effective Operators for DM-nucleon interactions

Assuming the DM particle is a **Dirac fermion** χ and using **Dirac fields** p and n to describe the proton and the neutron, the effective Lagrangian reads

$$\mathcal{L}_{\text{eff},N} = \sum_{N=p,n} \sum_{ij} G_{N,ij} \bar{\chi} \Gamma^i \chi \bar{N} \Gamma_j N, \quad \Gamma^i, \Gamma^j \in \{1, i\gamma_5, \gamma^\mu, \gamma^\mu \gamma_5, \sigma^{\mu\nu}\}$$

[Bélanger *et al.*, arXiv:0803.2360, Comput.Phys.Commun.]

- **Lorentz indices** in Γ^i and Γ_j should be contracted in pair
- Effective couplings $G_{N,ij}$ have a mass dimension of -2: $[G_{N,ij}] = [\text{Mass}]^{-2}$
- $\bar{\chi}\chi\bar{N}N$ and $\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu N$ lead to **SI** DM-nucleon scattering
- $\bar{\chi}\gamma^\mu\gamma_5\chi\bar{N}\gamma_\mu\gamma_5 N$ and $\bar{\chi}\sigma^{\mu\nu}\chi\bar{N}\sigma_{\mu\nu}N$ lead to **SD** DM-nucleon scattering
- The following operators lead to scattering cross sections $\sigma_{\chi N} \propto v^2$:

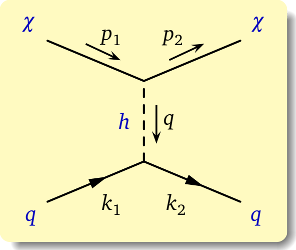
  $$\bar{\chi}i\gamma_5\chi\bar{N}i\gamma_5 N, \quad \bar{\chi}\chi\bar{N}i\gamma_5 N, \quad \bar{\chi}i\gamma_5\chi\bar{N}N, \quad \bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu\gamma_5 N, \quad \bar{\chi}\gamma^\mu\gamma_5\chi\bar{N}\gamma_\mu N$$
- For a **Majorana fermion** χ instead, we have $\bar{\chi}\gamma^\mu\chi = 0$ and $\bar{\chi}\sigma^{\mu\nu}\chi = 0$, and hence the related operators vanish

Higgs Portal for Majorana Fermionic DM

Interactions for a **Majorana fermion** χ, the **SM Higgs boson** h, and quarks q:

$$\mathcal{L}_{\mathrm{DM}} \supset \frac{1}{2} g_\chi h \bar{\chi}\chi$$

$$\mathcal{L}_{\mathrm{SM}} \supset -\sum_q \frac{m_q}{v} h \bar{q} q, \quad q = d, u, s, c, b, t$$



The amplitude for $\chi(p_1) + q(k_1) \to \chi(p_2) + q(k_2)$:

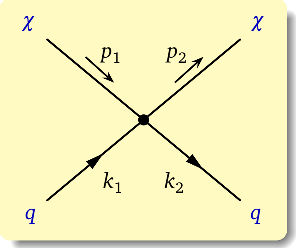
$$i\mathcal{M} = i g_\chi \bar{u}(p_2) u(p_1) \frac{i}{q^2 - m_h^2} \left(-i \frac{m_q}{v}\right) \bar{u}(k_2) u(k_1)$$

Zero momentum transfer $\Downarrow$ $q^2 = (k_2 - k_1)^2 \to 0$

$$i\mathcal{M} = -i \frac{g_\chi m_q}{v m_h^2} \bar{u}(p_2) u(p_1) \bar{u}(k_2) u(k_1)$$

$$\Downarrow$$

$$\mathcal{L}_{\mathrm{eff},q} = \sum_q G_{\mathrm{S},q} \bar{\chi}\chi \bar{q} q, \quad G_{\mathrm{S},q} = -\frac{g_\chi m_q}{2 v m_h^2}$$



Effective Lagrangian: Scalar Type

Scalar-type effective Lagrangian for a **spin-1/2 fermion** χ:

$$\mathcal{L}_{S,q} = \sum_q G_{S,q} \bar{\chi}\chi \bar{q}q \quad \Rightarrow \quad \mathcal{L}_{S,N} = \sum_{N=p,n} G_{S,N} \bar{\chi}\chi \bar{N}N$$

$$G_{S,N} = m_N \left(\sum_{q=u,d,s} \frac{G_{S,q}}{m_q} f_q^N + \sum_{q=c,b,t} \frac{G_{S,q}}{m_q} f_Q^N \right)$$

The second term accounts for DM interactions with gluons through loops of heavy quarks (c, b, and t): $f_Q^N = \frac{2}{27}\left(1 - \sum_{q=u,d,s} f_q^N\right)$

Form factor f_q^N is the contribution of q to m_N: $\langle N| m_q \bar{q}q |N\rangle = f_q^N m_N$

$$f_u^p \simeq 0.020, \quad f_d^p \simeq 0.026, \quad f_u^n \simeq 0.014, \quad f_d^n \simeq 0.036, \quad f_s^p = f_s^n \simeq 0.118$$

[Ellis *et al.*, arXiv:hep-ph/0001005, PLB]

The scalar type induces **SI** DM-nucleon scattering with a cross section of

$$\sigma_{\chi N}^{\mathrm{SI}} = \frac{n_\chi}{\pi} \mu_{\chi N}^2 G_{S,N}^2, \quad \mu_{\chi N} \equiv \frac{m_\chi m_N}{m_\chi + m_N}, \quad n_\chi = \begin{cases} 1, & \text{for Dirac fermion } \chi \\ 4, & \text{for Majorana fermion } \chi \end{cases}$$

Z Portal for Majorana Fermionic DM

Interactions for a **Majorana fermion** χ, the Z **boson**, and quarks q:

$$\mathcal{L}_{\rm DM} \supset \frac{1}{2} g_\chi Z_\mu \bar{\chi}\gamma^\mu\gamma_5\chi, \quad \mathcal{L}_{\rm SM} \supset \frac{g}{2c_{\rm W}} Z_\mu \sum_q \bar{q}\gamma^\mu(g_{\rm V}^q - g_{\rm A}^q\gamma_5)q$$

$$g_{\rm V}^{u_i} = \frac{1}{2} - \frac{4}{3}s_{\rm W}^2, \quad g_{\rm V}^{d_i} = -\frac{1}{2} + \frac{2}{3}s_{\rm W}^2, \quad g_{\rm A}^{u_i} = \frac{1}{2} = -g_{\rm A}^{d_i}, \quad c_{\rm W} \equiv \cos\theta_{\rm W}, \quad s_{\rm W} \equiv \sin\theta_{\rm W}$$

Z **boson propagator** $\displaystyle \frac{-i}{q^2 - m_Z^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2}\right) \xrightarrow{q^2 \to 0} \frac{i}{m_Z^2} g_{\mu\nu}$

Effective Lagrangian in the zero momentum transfer limit:

$$\mathcal{L}_{{\rm eff},q} = \sum_q \bar{\chi}\gamma^\mu\gamma_5\chi(G_{{\rm A},q}\bar{q}\gamma_\mu\gamma_5 q + G_{{\rm AV},q}\bar{q}\gamma_\mu q), \quad G_{{\rm A},q} = \frac{g_\chi g g_{\rm A}^q}{4c_{\rm W} m_Z^2}$$

$G_{{\rm AV},q} = -\dfrac{g_\chi g g_{\rm V}^q}{4c_{\rm W} m_Z^2}$ leads to $\sigma_{\chi N} \propto v^2$ and can be neglected for direct detection

Effective Lagrangian: Axial Vector Type

Axial-vector-type effective Lagrangian for a **spin-1/2 fermion χ**:

$$\mathcal{L}_{\mathrm{A},q} = \sum_q G_{\mathrm{A},q} \bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu\gamma_5 q \quad \Rightarrow \quad \mathcal{L}_{\mathrm{A},N} = \sum_{N=p,n} G_{\mathrm{A},N} \bar{\chi}\gamma^\mu\gamma_5\chi\bar{N}\gamma_\mu\gamma_5 N$$

$$G_{\mathrm{A},N} = \sum_{q=u,d,s} G_{\mathrm{A},q}\Delta_q^N, \quad 2\Delta_q^N s_\mu \equiv \langle N|\,\bar{q}\gamma_\mu\gamma_5 q\,|N\rangle$$

Form factors Δ_q^N account the contributions of quarks and anti-quarks to the nucleon spin vector s_μ, and can be extracted from lepton-proton scattering data:

$$\Delta_u^p = \Delta_d^n \simeq 0.842, \quad \Delta_d^p = \Delta_u^n \simeq -0.427, \quad \Delta_s^p = \Delta_s^n \simeq -0.085$$

[HERMES coll., arXiv:hep-ex/0609039, PRD]

Neutron form factors are related to proton form factors by **isospin symmetry**

The axial vector type induces **SD** DM-nucleon scattering:

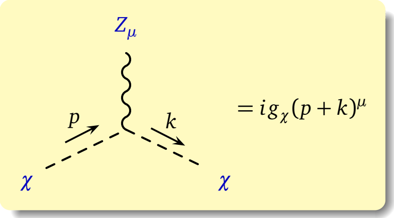
$$\sigma_{\chi N}^{\mathrm{SD}} = \frac{3n_\chi}{\pi}\mu_{\chi N}^2 G_{\mathrm{A},N}^2, \quad n_\chi = \begin{cases} 1, & \text{for Dirac fermion } \chi \\ 4, & \text{for Majorana fermion } \chi \end{cases}$$

Z Portal for Complex Scalar DM

Interactions for a **complex scalar** χ, the Z **boson**, and quarks q:

$$\mathcal{L}_{\text{DM}} \supset g_\chi Z_\mu (\chi^* i \overleftrightarrow{\partial^\mu} \chi)$$

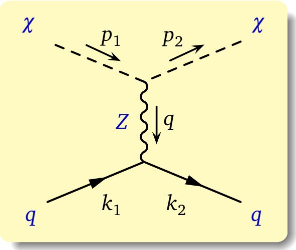
$$\mathcal{L}_{\text{SM}} \supset \frac{g}{2c_{\text{W}}} Z_\mu \sum_q \bar{q}\gamma^\mu (g_{\text{V}}^q - g_{\text{A}}^q \gamma_5) q$$



$$i\mathcal{M} = i g_\chi (p_1 + p_2)^\mu \frac{-i(g_{\mu\nu} - q_\mu q_\nu / m_Z^2)}{q^2 - m_Z^2}$$

$$\times i \frac{g}{2c_{\text{W}}} \bar{u}(k_2) \gamma^\nu (g_{\text{V}}^q - g_{\text{A}}^q \gamma_5) u(k_1)$$

$$\xrightarrow{q^2 \to 0} -i \frac{g_\chi g}{2c_{\text{W}} m_Z^2} (p_1 + p_2)^\mu \bar{u}(k_2) \gamma_\mu (g_{\text{V}}^q - g_{\text{A}}^q \gamma_5) u(k_1)$$



$$\mathcal{L}_{\text{eff},q} = \sum_q (\chi^* i \overleftrightarrow{\partial^\mu} \chi)(F_{\text{V},q} \bar{q}\gamma_\mu q + F_{\text{VA},q} \bar{q}\gamma_\mu \gamma_5 q)$$

$$F_{\text{V},q} = -\frac{g_\chi g g_{\text{V}}^q}{2c_{\text{W}} m_Z^2}, \quad F_{\text{VA},q} = \frac{g_\chi g g_{\text{A}}^q}{2c_{\text{W}} m_Z^2} (\Rightarrow \sigma_{\chi N} \propto v^2)$$

Effective Lagrangian: Vector Type

Vector-type effective Lagrangian for a **complex scalar** χ:

$$\mathcal{L}_{\mathrm{V},q} = \sum_q F_{\mathrm{V},q}(\chi^* i \overleftrightarrow{\partial^\mu} \chi)\bar{q}\gamma_\mu q \quad \Rightarrow \quad \mathcal{L}_{\mathrm{A},N} = \sum_{N=p,n} F_{\mathrm{V},N}(\chi^* i \overleftrightarrow{\partial^\mu} \chi)\bar{N}\gamma_\mu N$$

The relation between $F_{\mathrm{V},N}$ and $F_{\mathrm{V},q}$ reflects the valence quark numbers in N:

$$F_{\mathrm{V},p} = 2F_{V,u} + F_{V,d}, \quad F_{\mathrm{V},n} = F_{V,u} + 2F_{V,d}$$

The vector type induces **SI** DM-nucleon scattering: $\sigma^{\mathrm{SI}}_{\chi N} = \dfrac{1}{\pi}\mu^2_{\chi N} F^2_{\mathrm{V},N}$

Vector-type effective Lagrangian for a **Dirac fermion** χ:

$$\mathcal{L}_{\mathrm{V},q} = \sum_q G_{\mathrm{V},q}\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q \quad \Rightarrow \quad \mathcal{L}_{\mathrm{A},N} = \sum_{N=p,n} G_{\mathrm{V},N}\bar{\chi}\gamma^\mu\chi\bar{N}\gamma_\mu N$$

It also induces **SI** DM-nucleon scattering:

$$\sigma^{\mathrm{SI}}_{\chi N} = \frac{1}{\pi}\mu^2_{\chi N} G^2_{\mathrm{V},N}, \quad G_{\mathrm{V},p} = 2G_{V,u} + G_{V,d}, \quad G_{\mathrm{V},n} = G_{V,u} + 2G_{V,d}$$

Effective Operators for DM-quark Interactions

	Spin-1/2 DM	Spin-0 DM
SI	$\bar{\chi}\chi\bar{q}q,\quad \bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	$\chi^*\chi\bar{q}q,\quad (\chi^* i\overleftrightarrow{\partial^\mu}\chi)\bar{q}\gamma_\mu q$
SD	$\bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu\gamma_5 q,\quad \bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu}q$	
$\sigma_{\chi N}\propto v^2$	$\bar{\chi}i\gamma_5\chi\bar{q}i\gamma_5 q,\quad \bar{\chi}\chi\bar{q}i\gamma_5 q$ $\bar{\chi}i\gamma_5\chi\bar{q}q,\quad \bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma_5 q$ $\bar{\chi}\gamma^\mu\gamma_5\chi\bar{q}\gamma_\mu q,\quad \varepsilon^{\mu\nu\rho\sigma}\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\rho\sigma}q$	$\chi^*\chi\bar{q}i\gamma_5 q$ $(\chi^* i\overleftrightarrow{\partial^\mu}\chi)\bar{q}\gamma_\mu\gamma_5 q$

	Spin-3/2 DM	Spin-1 DM
SI	$\bar{\chi}^\mu\chi_\mu\bar{q}q,\quad \bar{\chi}^\nu\gamma^\mu\chi_\nu\bar{q}\gamma_\mu q$	$\chi_\mu^*\chi^\mu\bar{q}q,\quad (\chi_\nu^* i\overleftrightarrow{\partial^\mu}\chi^\nu)\bar{q}\gamma_\mu q$
SD	$\bar{\chi}^\nu\gamma^\mu\gamma_5\chi_\nu\bar{q}\gamma_\mu\gamma_5 q,\quad \bar{\chi}^\rho\sigma^{\mu\nu}\chi_\rho\bar{q}\sigma_{\mu\nu}q$ $i(\bar{\chi}^\mu\chi^\nu-\bar{\chi}^\nu\chi^\mu)\bar{q}\sigma_{\mu\nu}q$	$i(\chi_\mu^*\chi_\nu-\chi_\nu^*\chi_\mu)\bar{q}\sigma^{\mu\nu}q$ $\varepsilon^{\mu\nu\rho\sigma}(\chi_\mu^*\overleftrightarrow{\partial_\nu}\chi_\rho)\bar{q}\gamma_\sigma\gamma_5 q$
$\sigma_{\chi N}\propto v^2$	$\bar{\chi}^\mu i\gamma_5\chi_\mu\bar{q}i\gamma_5 q,\quad \bar{\chi}^\mu\chi_\mu\bar{q}i\gamma_5 q$ $\bar{\chi}^\mu i\gamma_5\chi_\mu\bar{q}q,\quad \bar{\chi}^\nu\gamma^\mu\chi_\nu\bar{q}\gamma_\mu\gamma_5 q$ $\bar{\chi}^\mu\gamma^\mu\gamma_5\chi_\nu\bar{q}\gamma_\mu q,\quad \varepsilon^{\mu\nu\rho\sigma}i(\bar{\chi}_\mu\chi_\nu-\bar{\chi}_\nu\chi_\mu)\bar{q}\sigma_{\rho\sigma}q$ $\varepsilon^{\mu\nu\rho\sigma}\bar{\chi}^\alpha\sigma_{\mu\nu}\chi_\alpha\bar{q}\sigma_{\rho\sigma}q,\quad (\bar{\chi}^\mu\gamma_5\chi^\nu-\bar{\chi}^\nu\gamma_5\chi^\mu)\bar{q}\sigma_{\mu\nu}q$ $\varepsilon^{\mu\nu\rho\sigma}(\bar{\chi}_\mu\gamma_5\chi_\nu-\bar{\chi}_\nu\gamma_5\chi_\mu)\bar{q}\sigma_{\rho\sigma}q$	$\chi_\mu^*\chi^\mu\bar{q}i\gamma_5 q$ $(\chi_\nu^* i\overleftrightarrow{\partial^\mu}\chi^\nu)\bar{q}\gamma_\mu\gamma_5 q$ $\varepsilon^{\mu\nu\rho\sigma}(\chi_\mu^*\overleftrightarrow{\partial_\nu}\chi_\rho)\bar{q}\gamma_\sigma q$ $\varepsilon^{\mu\nu\rho\sigma}i(\chi_\mu^*\chi_\nu-\chi_\nu^*\chi_\mu)\bar{q}\sigma_{\rho\sigma}q$

[Zheng, ZHY, Shao, Bi, Li, Zhang, arXiv:1012.2022, NPB; ZHY, Zheng, Bi, Li, Yao, Zhang, arXiv:1112.6052, NPB; Ding & Liao, arXiv:1201.0506, JHEP]

Homework

1. Derive the speed distribution $f(v)$ in Page 8 from $f(\mathbf{v}) = \tilde{f}(\mathbf{v} + \mathbf{v}_{\text{obs}})$
2. Calculate the normalization factor for the velocity distribution $\tilde{f}(\tilde{\mathbf{v}})$ in Page 7 if the escape velocity v_{esc} is taken into account
3. Derive the recoil velocity v_{R} in Page 9 from the laws of energy and momentum conservation
4. Examine the conservation of electric charge, lepton number, and baryon number for the reactions producing solar neutrinos in Page 26
5. Evaluate the values of DM-nucleon effective couplings $G_{\text{S},p}$ ($G_{\text{A},p}$) and $G_{\text{S},n}$ ($G_{\text{A},n}$) for the Higgs-portal (Z-portal) model in Page 30 (32) using the values of form factors listed in Page 31 (33)
6. Proof the expressions for $\sigma_{\chi N}^{\text{SI}}$ and $\sigma_{\chi N}^{\text{SD}}$ shown in Pages 31, 33, and 35
7. Examine the hermiticity of the operators tabulated in Page 36