Neutrinos——The Basics & Hot Topics

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- ★ A brief history of neutrinos
- ★ Basic neutrino interactions
- ★ Dirac and Majorana masses
- ★ Flavor mixing & CP violation
- ★ Oscillation phenomenology
- ★ Neutrinoless double-β decay
- ★ Typical seesaw mechanisms
- ★ Two types of cosmic neutrinos
- ★ Matter-antimatter asymmetry

Leptons: a partial list

1897: Discovery of electron (J.J. Thomson) 

1928: Prediction of positron (P.A.M. Dirac) 

1930: Postulation of neutrino (W. Pauli)

1932: Discovery of positron (C.D. Anderson) 

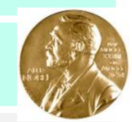
1933: Effective theory of beta decay (E. Fermi)

1936: Discovery of muon (J.C. Street et al; C.D. Anderson et al)

1956: Discovery of electron antineutrino (C.L. Cowan et al) 

1956: Postulation of parity violation (T.D. Lee, C.N. Yang) 

1957: Discovery of parity violation (C.S. Wu et al)

1962: Discovery of muon neutrino (G. Danby et al) 

1962: Postulation of neutrino flavor conversion (Z. Maki et al)

1967: Standard model of leptons (S. Weinberg) 

1975: Discovery of tau (M. Perl et al) 

2000: Discovery of tau neutrino (K. Kodama et al)

Quarks: a partial list

1917: Discovery of proton (E. Rutherford) — **up and down**

1932: Discovery of neutron (J. Chadwick) — **up and down** 

1947: Discovery of Kaon (G. Rochester, C. Butler) — **strange**

1960: The quark model (M. Gell-Mann; G. Zweig) 

1963: The Cabibbo angle of quark mixing (N. Cabibbo)

1964: Discovery of CP violation (J.W. Cronin, V.L. Fitch) 

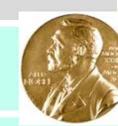
1964: The Higgs mechanism (F. Englert, R. Brout; P. Higgs) 

1967: The standard model (S. Weinberg) 

1970: The GIM mechanism (S. Glashow et al)

1973: Asymptotic freedom (F. Wilczek, D. Gross; H. Politzer) 

1973: The origin of CP violation (M. Kobayashi, T. Maskawa) 

1974: Discovery of charm (C.C. Ting; B. Richter) 

1977: Discovery of bottom (L. Lederman et al)

1995: Discovery of top (F. Abe et al)

Origin of “flavor”

The term Flavor was coined by Harald Fritzsch and Murray Gell-Mann at a Baskin-Robbins ice-cream store in Pasadena in 1971.

One of the most puzzling things in particle physics is *flavor mixing* ! But this is normal for ice creams!

Lecture A4

- ★ Flavor mixing and CP violation
- ★ The PMNS flavor mixing matrix
- ★ What is behind: μ-τ symmetry

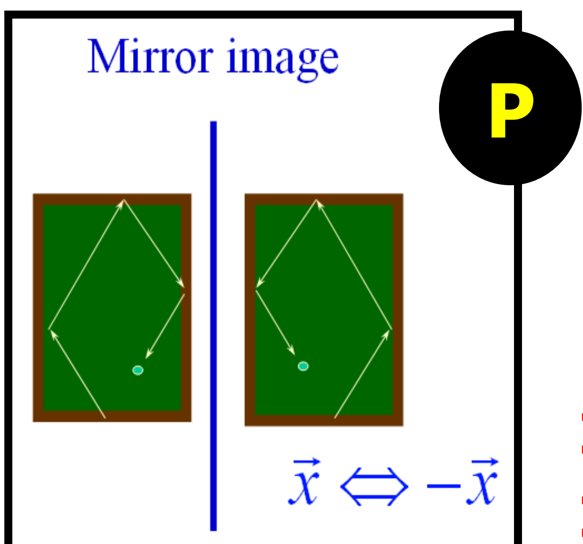
Flavor mixing

Flavor mixing: mismatch between **weak/flavor** eigenstates and **mass** eigenstates of fermions due to coexistence of **2** types of interactions.

Weak eigenstates: members of weak isospin doublets transforming into each other through the interaction with the ***W*** boson;

Mass eigenstates: states of definite masses that are created by the interaction with the Higgs boson (**Yukawa** interactions).

CP violation: **matter** and **antimatter**, or a reaction & its CP-conjugate process, are distinguishable --- coexistence of **2** types of interactions.

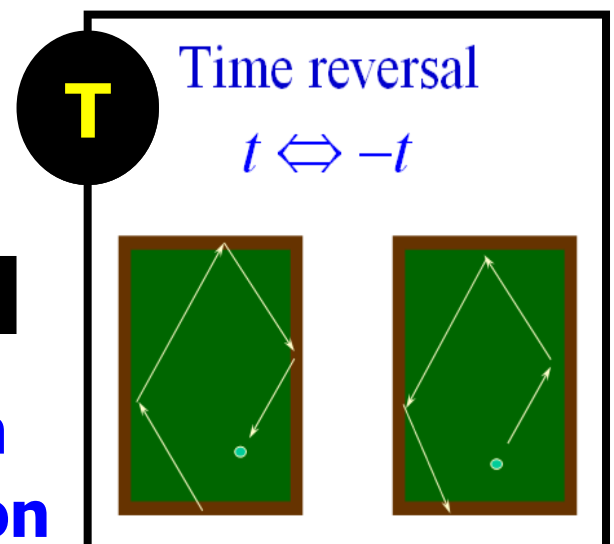


C

Charge-conjugation

1957: P violation

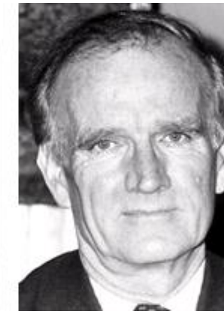
1964: CP violation



Towards the KM paper

1964: **Discovery of CP violation in K decays (J.W. Cronin, Val L. Fitch)**

NP 1980



1967: **Sakharov conditions for cosmological matter-antimatter asymmetry (A. Sakharov)**

NP 1975

1967: **The standard model of electromagnetic and weak interactions without quarks (S. Weinberg)**

NP 1979

0 citation for the first 4 yrs

1971: **The first proof of the renormalizability of the standard model (G. 't Hooft)**

NP 1999

KM in 1972

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

3 families allow for CP violation: Maskawa's bathtub idea!

"as I was getting out of the bathtub, an idea came to me"

Where or why

In the standard model, plus 3 right-handed ν's, where/why can flavor mixing and CP violation arise?

$$\mathcal{L}_{\nu\rm SM} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm Y}$$

$$\mathcal{L}_{\rm G} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right)$$

$$\mathcal{L}_{\rm H} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - \mu^{2}H^{\dagger}H - \lambda\left(H^{\dagger}H\right)^{2}$$

$$\mathcal{L}_{\rm F} = \overline{Q_{\rm L}}\,i\not{D}\,Q_{\rm L} + \overline{\ell_{\rm L}}\,i\not{D}\,\ell_{\rm L} + \overline{U_{\rm R}}\,i\not{\partial}'\,U_{\rm R} + \overline{D_{\rm R}}\,i\not{\partial}'\,D_{\rm R} + \overline{E_{\rm R}}\,i\not{\partial}'\,E_{\rm R} + \underline{\overline{N_{\rm R}}\,i\not{\partial}'\,N_{\rm R}}$$

$$\mathcal{L}_{\rm Y} = -\overline{Q_{\rm L}}\,Y_{\rm u}\tilde{H}U_{\rm R} - \overline{Q_{\rm L}}\,Y_{\rm d}HD_{\rm R} - \overline{\ell_{\rm L}}\,Y_{l}HE_{\rm R} - \underline{\overline{\ell_{\rm L}}\,Y_{\nu}\tilde{H}N_{\rm R}} + {\rm h.c.}$$

ν's Dirac mass

The strategy of diagnosis:

♣ **Flavor mixing:** transform the **flavor eigenstates** of fermions to their **mass eigenstates**, to see whether a kind of "**mismatch**" can occur.

♣ **CP violation:** given proper **CP** transformations of gauge, Higgs and fermion fields, one may prove that **1st, 2nd and 3rd** terms are formally invariant, and the **4th** term can be invariant only if the corresponding **Yukawa coupling matrices** are real. Otherwise, CP violation occurs.

CP transformations

Gauge fields:

$$[B_\mu,\ W^1_\mu,\ W^2_\mu,\ W^3_\mu] \xrightarrow{\text{CP}} [-B^\mu,\ -W^{1\mu},\ +W^{2\mu},\ -W^{3\mu}]$$

$$[B_{\mu\nu},\ W^1_{\mu\nu},\ W^2_{\mu\nu},\ W^3_{\mu\nu}] \xrightarrow{\text{CP}} [-B^{\mu\nu},\ -W^{1\mu\nu},\ +W^{2\mu\nu},\ -W^{3\mu\nu}]$$

Higgs fields:

$$H(t,\mathbf{x}) = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \xrightarrow{\text{CP}} H^*(t,-\mathbf{x}) = \begin{pmatrix}\phi^-\\ \phi^{0*}\end{pmatrix}$$

Lepton or quark fields:

$$\overline{\psi_1}\gamma_\mu(1\pm\gamma_5)\psi_2 \xrightarrow{\text{CP}} -\overline{\psi_2}\gamma^\mu(1\pm\gamma_5)\psi_1$$

$$\overline{\psi_1}\gamma_\mu(1\pm\gamma_5)\partial^\mu\psi_2 \xrightarrow{\text{CP}} \overline{\psi_2}\gamma^\mu(1\pm\gamma_5)\partial_\mu\psi_1$$

Spinor bilinears:

	$\overline{\psi_1}\psi_2$	$i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma_\mu\psi_2$	$\overline{\psi_1}\gamma_\mu\gamma_5\psi_2$	$\overline{\psi_1}\sigma_{\mu\nu}\psi_2$
C	$\overline{\psi_2}\psi_1$	$i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma_\mu\psi_1$	$\overline{\psi_2}\gamma_\mu\gamma_5\psi_1$	$-\overline{\psi_2}\sigma_{\mu\nu}\psi_1$
P	$\overline{\psi_1}\psi_2$	$-i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma^\mu\psi_2$	$-\overline{\psi_1}\gamma^\mu\gamma_5\psi_2$	$\overline{\psi_1}\sigma^{\mu\nu}\psi_2$
T	$\overline{\psi_1}\psi_2$	$-i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma^\mu\psi_2$	$\overline{\psi_1}\gamma^\mu\gamma_5\psi_2$	$-\overline{\psi_1}\sigma^{\mu\nu}\psi_2$
CP	$\overline{\psi_2}\psi_1$	$-i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma^\mu\psi_1$	$-\overline{\psi_2}\gamma^\mu\gamma_5\psi_1$	$-\overline{\psi_2}\sigma^{\mu\nu}\psi_1$
CPT	$\overline{\psi_2}\psi_1$	$i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma_\mu\psi_1$	$-\overline{\psi_2}\gamma_\mu\gamma_5\psi_1$	$\overline{\psi_2}\sigma_{\mu\nu}\psi_1$

$\mathcal{L}_\text{G}$

$\mathcal{L}_\text{H}$

$\mathcal{L}_\text{F}$

formally invariant under CP

The source

The **Yukawa** interactions of fermions are **formally invariant** under **CP** if and only if

$$Y_{\rm u} = Y_{\rm u}^* \; , \qquad Y_{\rm d} = Y_{\rm d}^*$$
$$Y_l = Y_l^* \; , \qquad Y_\nu = Y_\nu^*$$

If the effective **Majorana** mass term is added into the SM, then the **Yukawa** interactions of leptons can be **formally invariant** under **CP** if

$$M_{\rm L} = M_{\rm L}^* \; , \qquad Y_l = Y_l^*$$

If the **flavor eigenstates** are transformed into the **mass eigenstates**, flavor mixing and **CP** violation will show up in the **CC** interactions:

quarks

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(u \;\, c \;\, t)_{\rm L}} \; \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W^+_\mu + {\rm h.c.}$$

leptons

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \;\, \mu \;\, \tau)_{\rm L}} \; \gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

Comment A: flavor mixing and **CP** violation take place since fermions interact with both the **gauge bosons** and the **Higgs boson**.

Comment B: both the **CC** and **Yukawa** interactions have been verified.

Comment C: the **CKM** matrix ***V*** is unitary, the **PMNS** matrix ***U*** is too?

Parameter counting

The 3×3 unitary matrix V can always be parametrized as a product of 3 unitary rotation matrices in the complex planes:

$$O_1(\theta_1,\alpha_1,\beta_1,\gamma_1) = \begin{pmatrix} c_1 e^{i\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$O_2(\theta_2,\alpha_2,\beta_2,\gamma_2) = \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{i\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix}$$

$$O_3(\theta_3,\alpha_3,\beta_3,\gamma_3) = \begin{pmatrix} c_3 e^{i\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix}$$

where $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ (for $i = 1, 2, 3$)

Category A: 3 possibilities

$$V = O_i O_j O_i \quad (i \neq j)$$

Category B: 6 possibilities

$$V = O_i O_j O_k \quad (i \neq j \neq k)$$

Phases

For instance, the standard parametrization is given below:

V

$$= \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix} \begin{pmatrix} c_3 e^{\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix} \begin{pmatrix} c_1 e^{\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 c_3 e^{i(\alpha_1+\gamma_2+\alpha_3)} & s_1 c_3 e^{i(-\beta_1+\gamma_2+\alpha_3)} & s_3 e^{i(\gamma_1+\gamma_2-\beta_3)} \\ -s_1 c_2 e^{i(\beta_1+\alpha_2+\gamma_3)} - c_1 s_2 s_3 e^{i(\alpha_1-\beta_2+\beta_3)} & c_1 c_2 e^{i(-\alpha_1+\alpha_2+\gamma_3)} - s_1 s_2 s_3 e^{i(-\beta_1-\beta_2+\beta_3)} & s_2 c_3 e^{i(\gamma_1-\beta_2-\alpha_3)} \\ s_1 s_2 e^{i(\beta_1+\beta_2+\gamma_3)} - c_1 c_2 s_3 e^{i(\alpha_1-\alpha_2+\beta_3)} & -c_1 s_2 e^{i(-\alpha_1+\beta_2+\gamma_3)} - s_1 c_2 s_3 e^{i(-\beta_1-\alpha_2+\beta_3)} & c_2 c_3 e^{i(\gamma_1-\alpha_2-\alpha_3)} \end{pmatrix}$$

$$= \begin{pmatrix} e^{ia} & 0 & 0 \\ 0 & e^{ib} & 0 \\ 0 & 0 & e^{ic} \end{pmatrix} \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix}$$

$$a = (\alpha_1 - \beta_1) - (\alpha_2 + \beta_2 - \gamma_2) - \gamma_3 \,, \quad b = -\beta_2 - \alpha_3 \,, \quad c = -\alpha_2 - \alpha_3 \,;$$

$$x = \beta_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3) \,, \quad y = -\alpha_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3) \,, \quad z = \gamma_1 \,.$$

$$\delta = \beta_3 - \gamma_1 - \gamma_2$$

Physical phases

If neutrinos are Dirac particles, the phases *x*, *y* and *z* can be removed. Then the neutrino mixing matrix is

Dirac neutrino mixing matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

If neutrinos are Majorana particles, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g., $z = 0$). Then

Majorana neutrino mixing matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Global fit of current data

F. Capozzi et al (1703.04471): a global fit of current ν-oscillation data

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2	NO, IO, any	7.37	7.21–7.54	7.07–7.73	6.93–7.96
$\sin^2\theta_{12}/10^{-1}$	NO, IO, any	2.97	2.81–3.14	2.65–3.34	2.50–3.54
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.525	2.495–2.567	2.454–2.606	2.411–2.646
	IO	2.505	2.473–2.539	2.430–2.582	2.390–2.624
	Any	2.525	2.495–2.567	2.454–2.606	2.411–2.646
$\sin^2\theta_{13}/10^{-2}$	NO	2.15	2.08–2.22	1.99–2.31	1.90–2.40
	IO	2.16	2.07–2.24	1.98–2.33	1.90–2.42
	Any	2.15	2.08–2.22	1.99–2.31	1.90–2.40
$\sin^2\theta_{23}/10^{-1}$	NO	4.25	4.10–4.46	3.95–4.70	3.81–6.15
	IO	5.89	4.17–4.48 ⊕ 5.67–6.05	3.99-4.83 ⊕ 5.33–6.21	3.84–6.36
	Any	4.25	4.10–4.46	3.95–4.70 ⊕ 5.75–6.00	3.81–6.26
δ/π	NO	1.38	1.18–1.61	1.00–1.90	0–0.17 ⊕ 0.76–2
	IO	1.31	1.12–1.62	0.92–1.88	0–0.15 ⊕ 0.69–2
	Any	1.38	1.18–1.61	1.00–1.90	0–0.17 ⊕ 0.76–2

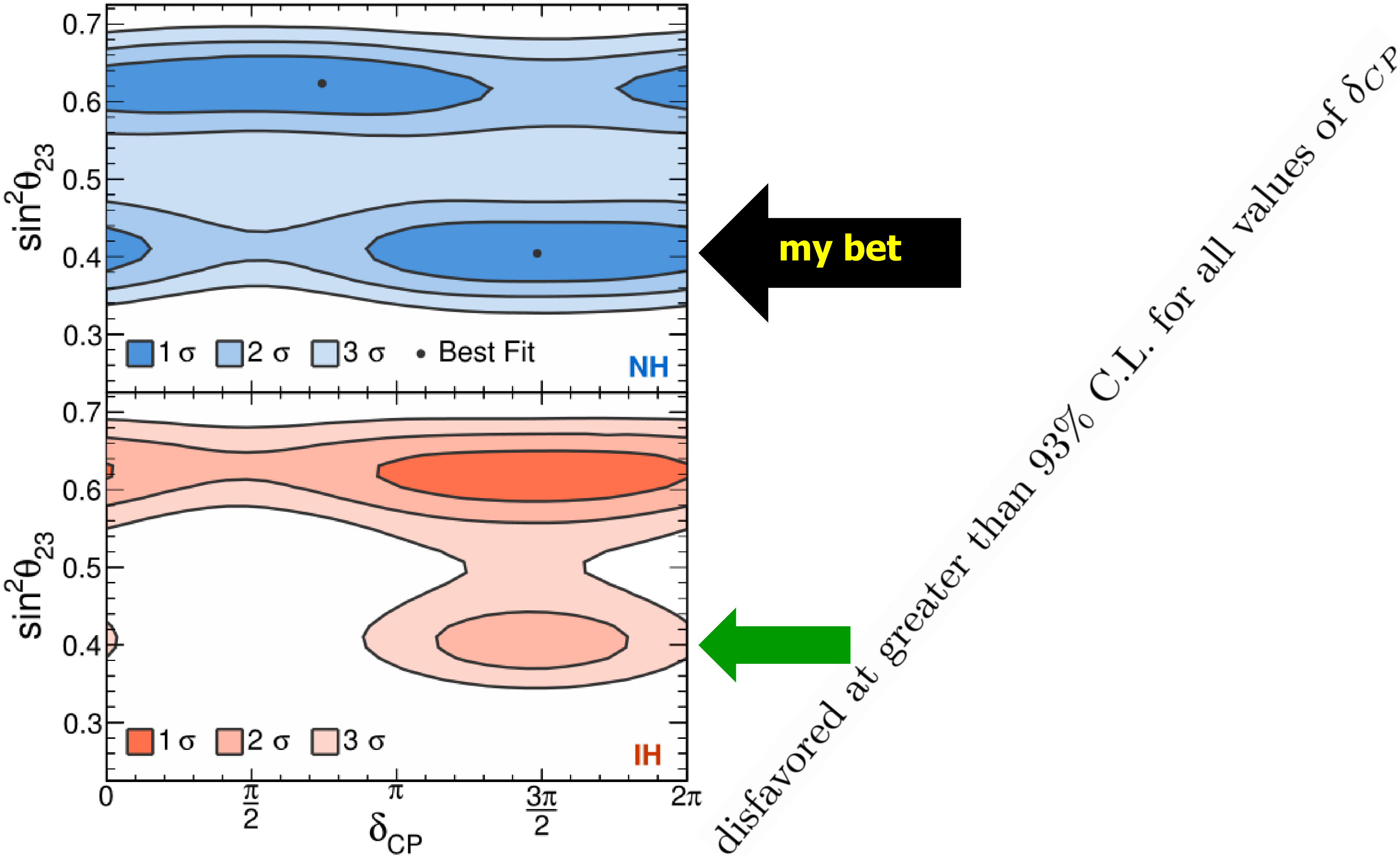
"Summarizing, the SK (+T2K) official results and ours suggest, at face value, that global 3ν oscillation analysis may have reached an overall ~2σ sensitivity to the mass ordering, with a preference for NO driven by atmospheric data (= multi-GeV e-like events) and corroborated by accelerator data, together with reactor constraints."

Terrestrial matter effects play the crucial role.
T2K, NOνA, SK, PINGU, INO, …

$$\Delta m^2_{31} \mp 2\sqrt{2}G_F N_e E$$

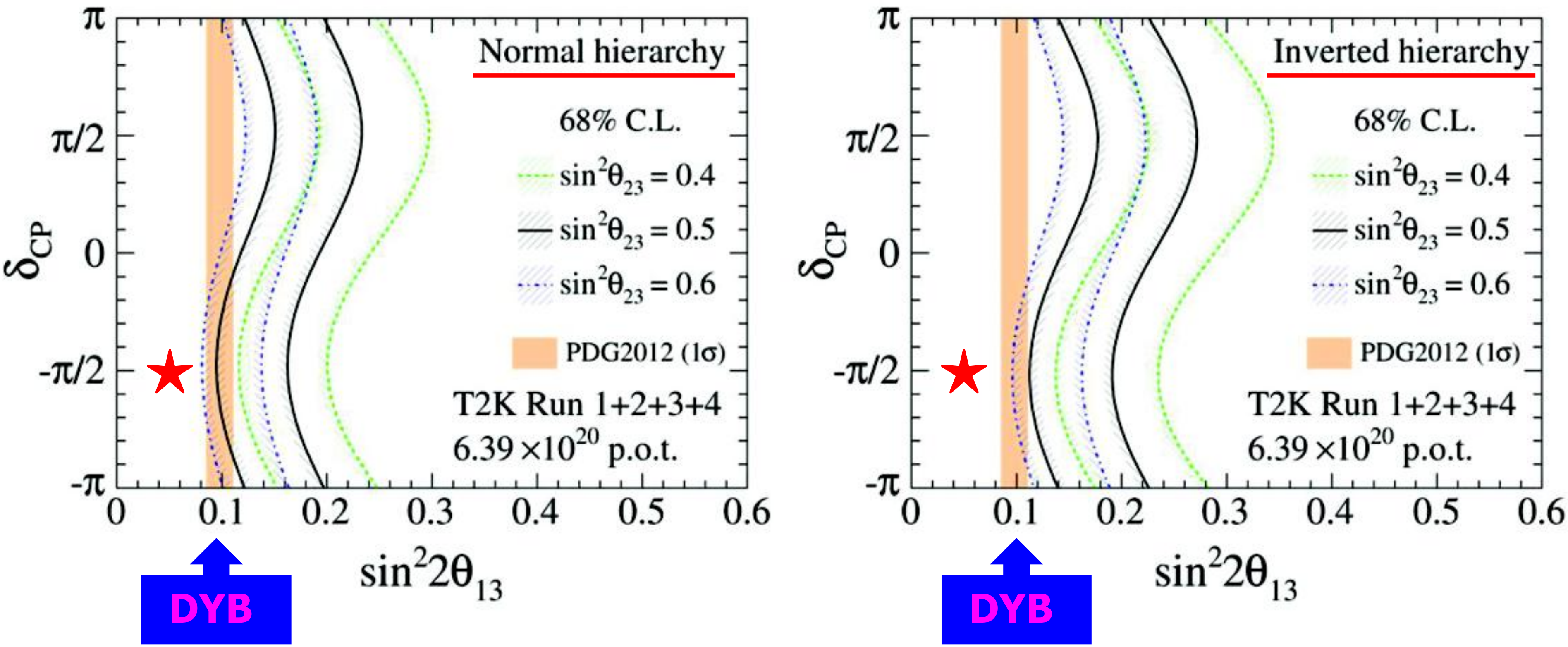
Normal ordering?

NOνA (arXiv:1703.03328, 9 March 2017):



Hint for the CP phase

The **T2K** observation of a relatively strong $\nu_\mu \to \nu_e$ appearance plays a crucial role in the global fit to make θ_{13} consistent with the **Daya Bay** result and drive a slight but intriguing preference for $\delta \sim -\pi/2$.



DYB's good news: θ_{13} unsuppressed

precision measurements

T2K's good news: δ unsuppressed

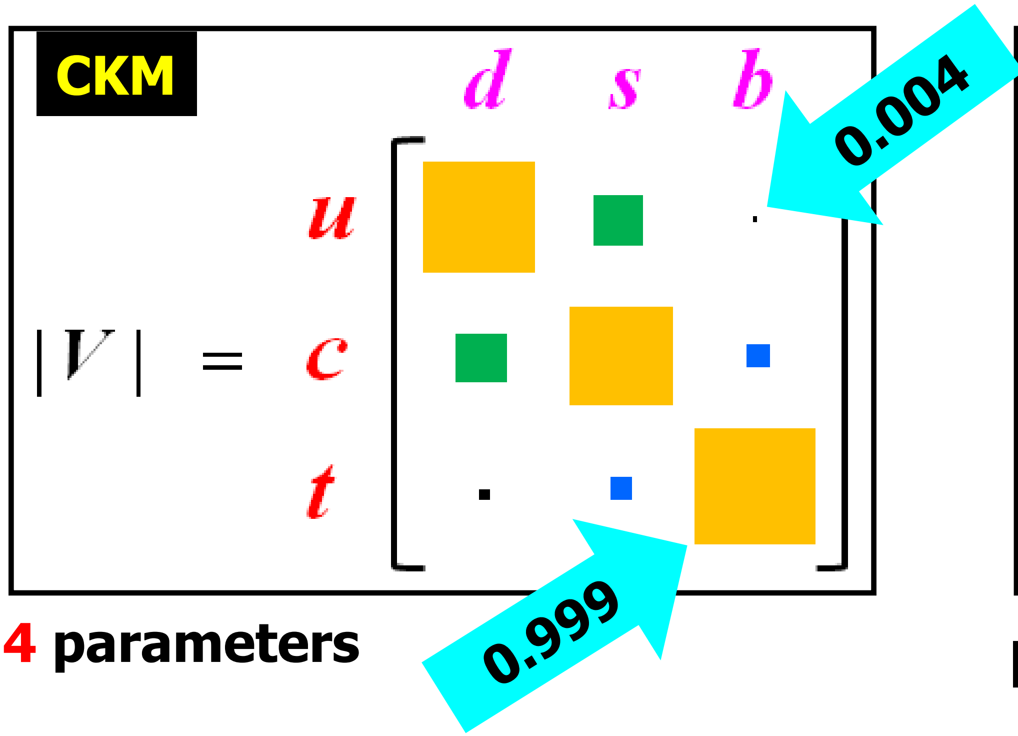
Life is easier for probing CP violation, ν mass hierarchy ....

Flavor mixing puzzle

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[\overline{(u \;\; c \;\; t)_L} \, \gamma^\mu \, V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu + \overline{(e \;\; \mu \;\; \tau)_L} \, \gamma^\mu \, U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu \right] + \text{h.c.}$$

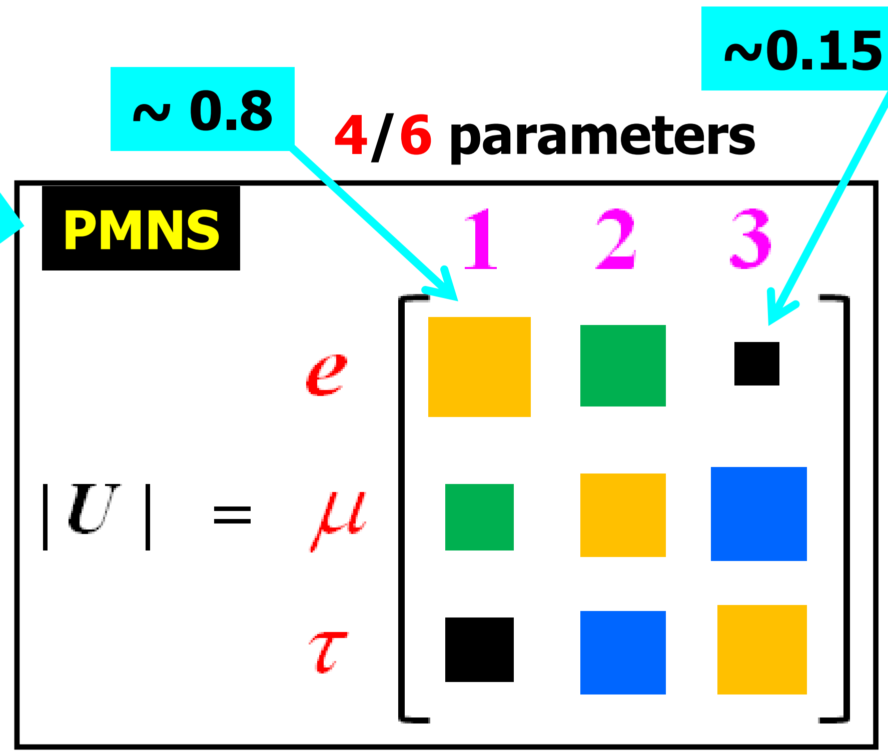
CKM **PMNS**

Quark mixing: hierarchy!



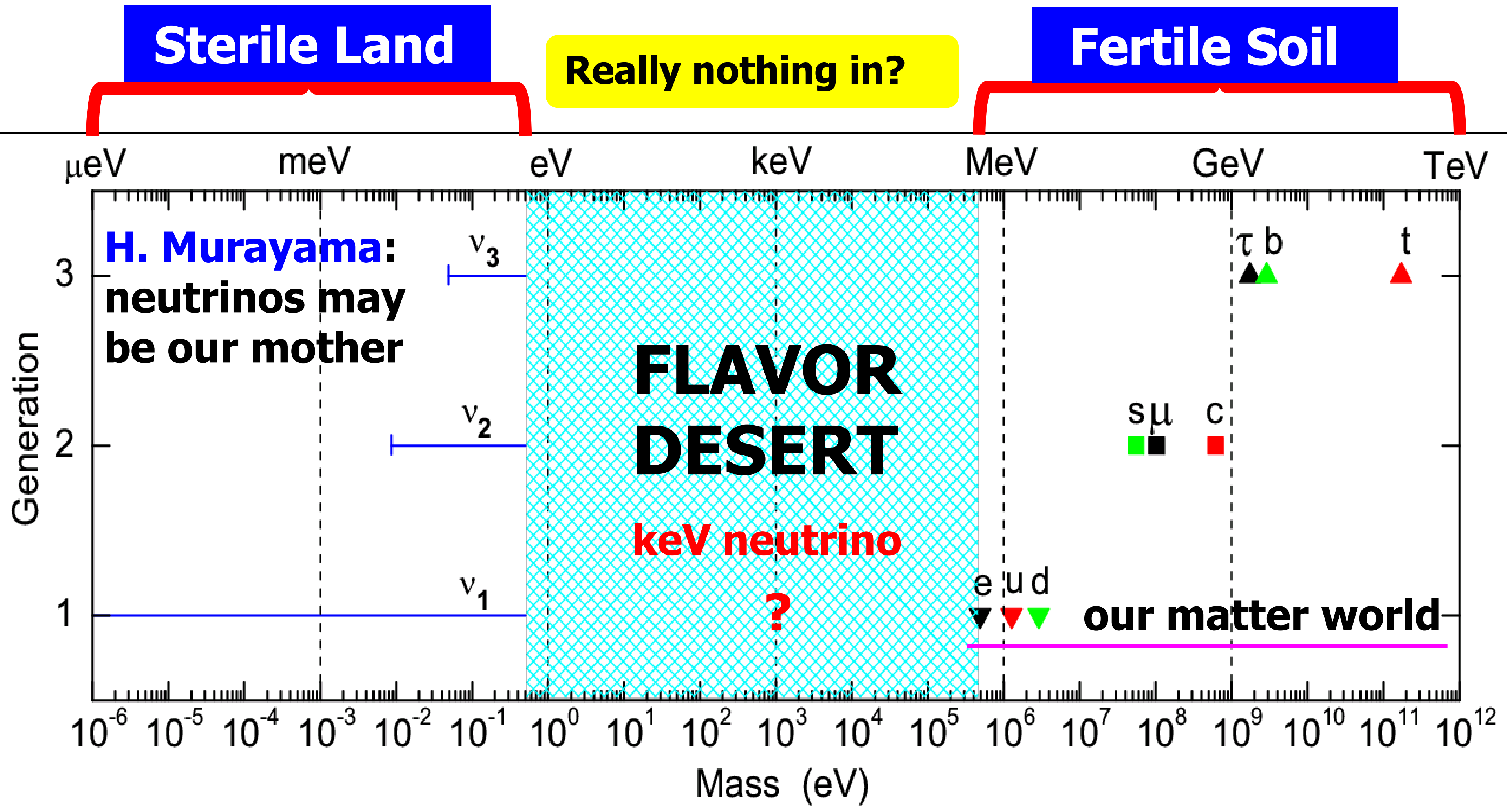
4 parameters

4/6 parameters



Lepton mixing: anarchy?

Flavor mass puzzle



Gauge Hierarchy & Desert Puzzles / Flavor Hierarchy & Desert Puzzles

Implications of electron mass < u quark mass < d quark mass on …….

What is behind?

What distinguishes different families of fermions?

----- **they have the same gauge quantum numbers,**
yet they are quite different from one another,
in their masses, flavor mixing strengths, ...

We are blind today: no convincing predictive flavor theory

The structure of flavors should determine their properties

Bottom-Up Way

Although nature commences with reason and ends in experience, it is necessary for us to do the opposite, that is, to commence with experience and from this to proceed to investigate the reason

We will see: the minimal symmetry behind: μ-τ symmetry!

Lessons learnt before

Symmetries: crucial for understanding the laws of Nature.

Examples: they help simplify problems, classify complicated systems, fix conservation laws and even determine dynamics of interactions.

- **Continuous space-time (translational/rotational) symmetries**
  $\Rightarrow$ **energy-momentum conservation laws**
- **Gauge symmetries $\Rightarrow$ electroweak and strong interactions**
- **SU(3) quark flavor symmetry $\Rightarrow$ the quark model** ♣♣♣

Symmetries may keep exact or be broken: both important!

- **Continuous space-time symmetries: exact**
- **U(1) electromagnetic gauge symmetry: exact (massless photon)**
- **SU(2) weak gauge symmetry: broken (massive W, Z, *etc*)**
- **SU(3) color gauge symmetry: exact (massless gluons)**
- **SU(3) quark flavor symmetry: broken** ♣♣♣

What the data tell?

Given the global-fit results at the 3σ level, the elements of the PMNS matrix are:

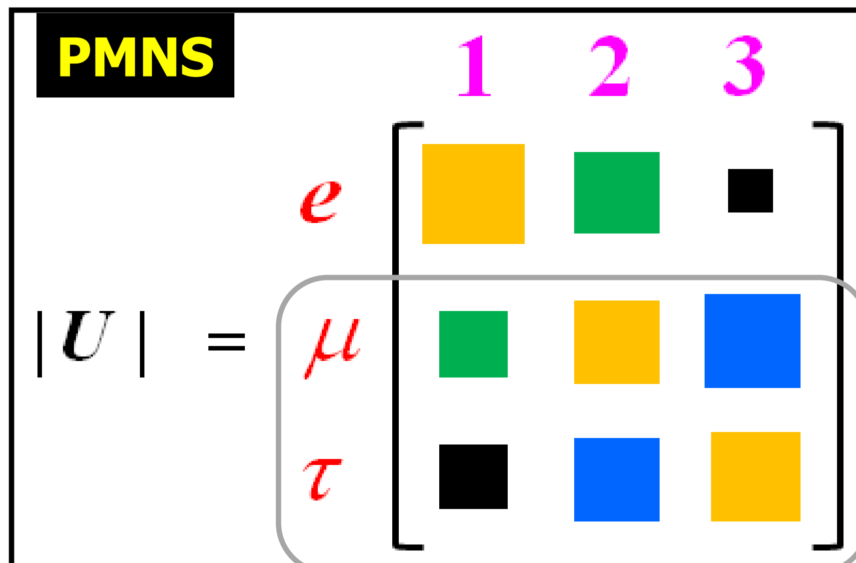
The normal ordering:

$$|U| \simeq \begin{pmatrix} 0.79-0.85 & 0.50-0.59 & 0.13-0.17 \\ 0.19-0.56 & 0.41-0.74 & 0.60-0.78 \\ 0.19-0.56 & 0.41-0.74 & 0.60-0.78 \end{pmatrix}$$

The inverted ordering:

$$|U| \simeq \begin{pmatrix} 0.89-0.85 & 0.50-0.59 & 0.13-0.17 \\ 0.19-0.56 & 0.40-0.73 & 0.61-0.79 \\ 0.20-0.56 & 0.41-0.74 & 0.59-0.78 \end{pmatrix}$$

$$|U_{\mu 1}| \simeq |U_{\tau 1}|$$
$$|U_{\mu 2}| \simeq |U_{\tau 2}|$$
$$|U_{\mu 3}| \simeq |U_{\tau 3}|$$



Behind the PMNS matrix

Behind the observed pattern of lepton flavor mixing is an approximate (or a partial) μ-τ flavor symmetry!

$$|U_{\mu 1}| \simeq |U_{\tau 1}| \; , \;\; |U_{\mu 2}| \simeq |U_{\tau 2}| \; , \;\; |U_{\mu 3}| \simeq |U_{\tau 3}|$$

It is very likely that the PMNS matrix possesses an exact μ-τ symmetry at a given energy scale, and this symmetry must be softly broken —— shed light on flavor structures

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu$$

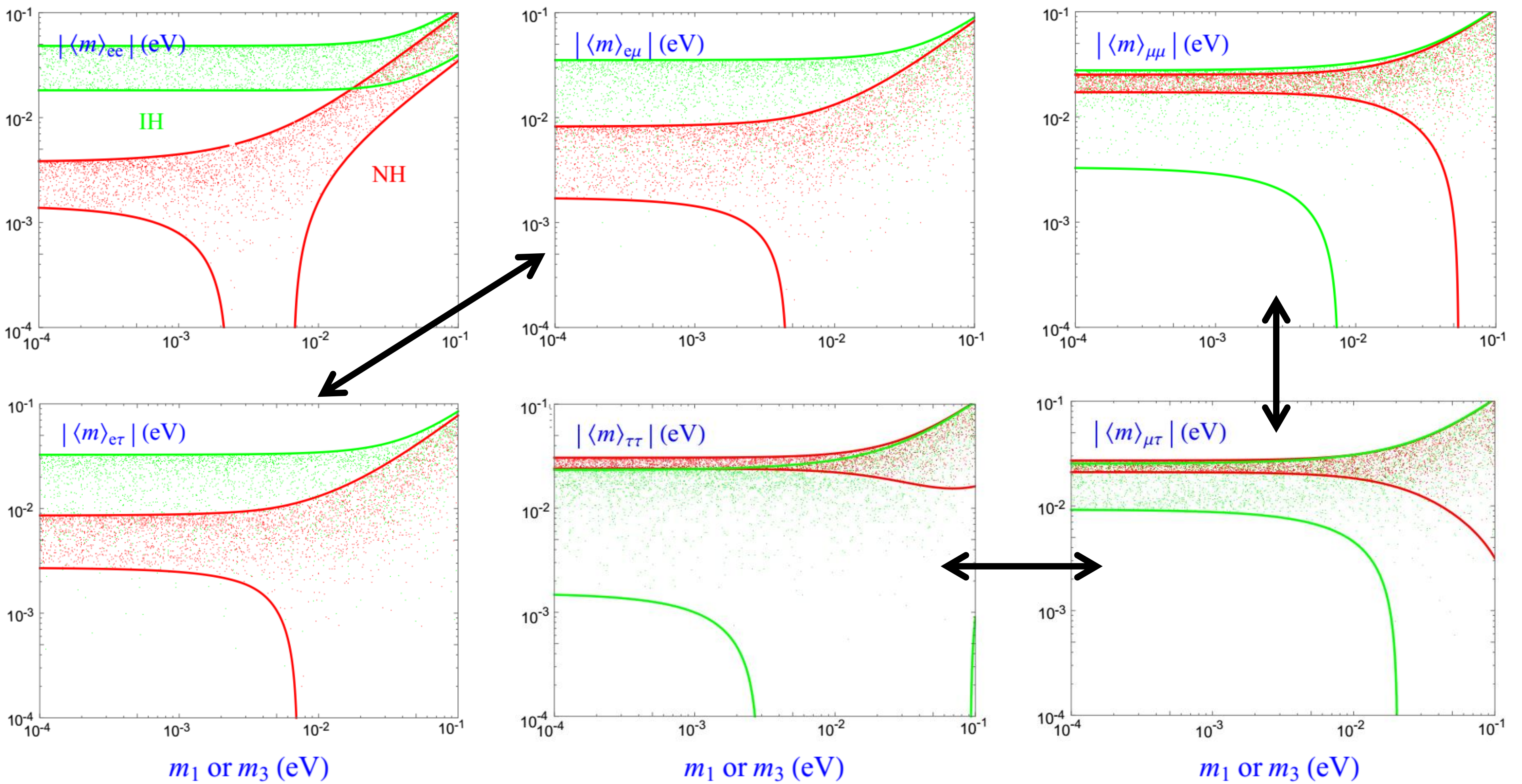
Conditions for the exact μ-τ symmetry in the PMNS matrix:

$$|U_{\mu i}| = |U_{\tau i}| \quad \Longrightarrow \quad \left\{ \begin{array}{l} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{array} \right. \text{ or } \left\{ \begin{array}{l} \delta = +\pi/2 \\ \theta_{23} = \pi/4 \end{array} \right. \text{ or } \left\{ \begin{array}{l} \delta = -\pi/2 \\ \theta_{23} = \pi/4 \end{array} \right.$$

Current data: ruled out not sure favored

Neutrino mass matrix

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T = \begin{pmatrix} \langle m \rangle_{ee} & \langle m \rangle_{e\mu} & \langle m \rangle_{e\tau} \\ \langle m \rangle_{e\mu} & \langle m \rangle_{\mu\mu} & \langle m \rangle_{\mu\tau} \\ \langle m \rangle_{e\tau} & \langle m \rangle_{\mu\tau} & \langle m \rangle_{\tau\tau} \end{pmatrix}$$



μ-τ flavor symmetry

In the flavor basis, the **Majorana** ν mass matrix can be reconstructed:

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T$$

→ **μ-τ symmetry**

μ-τ permutation symmetry

$$M_\nu = \begin{pmatrix} C & D & D \\ D & A & B \\ D & B & A \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau$

$$\begin{cases} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{cases}$$

T. Fukuyama, H. Nishiura
hep-ph/9702253

Bimaximal, Tribimaximal …

μ-τ reflection symmetry

$$M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$

$$\begin{cases} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{cases}$$

K. Babu, E. Ma, J. Valle
hep-ph/0206292

TM1, Tetramaximal …

$$\frac{1}{2}\,\overline{\left(\nu_e \;\; \nu_\mu \;\; \nu_\tau\right)_{\rm L}}\, M_\nu \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix}_{\rm R}$$

Current data

Larger ← μ-τ symmetry breaking → **Softer**

A proof: permutation

A generic (symmetric) Majorana neutrino mass term reads as follows:

$$\begin{aligned}-\mathcal{L}_{\rm mass} = {} & M_{ee}\overline{\nu_{e\rm L}}(\nu_{e\rm L})^c + M_{e\mu}\overline{\nu_{e\rm L}}(\nu_{\mu\rm L})^c + M_{e\tau}\overline{\nu_{e\rm L}}(\nu_{\tau\rm L})^c \\ & + M_{e\mu}\overline{\nu_{\mu\rm L}}(\nu_{e\rm L})^c + M_{\mu\mu}\overline{\nu_{\mu\rm L}}(\nu_{\mu\rm L})^c + M_{\mu\tau}\overline{\nu_{\mu\rm L}}(\nu_{\tau\rm L})^c \\ & + M_{e\tau}\overline{\nu_{\tau\rm L}}(\nu_{e\rm L})^c + M_{\mu\tau}\overline{\nu_{\tau\rm L}}(\nu_{\mu\rm L})^c + M_{\tau\tau}\overline{\nu_{\tau\rm L}}(\nu_{\tau\rm L})^c + \text{h.c.}\end{aligned}$$

Under μ-τ permutation, the above term changes to

$$\nu_{\mu\rm L} \leftrightarrow \nu_{\tau\rm L}$$

$$\begin{aligned}-\mathcal{L}_{\rm mass} = {} & M_{ee}\overline{\nu_{e\rm L}}(\nu_{e\rm L})^c + M_{e\mu}\overline{\nu_{e\rm L}}(\nu_{\tau\rm L})^c + M_{e\tau}\overline{\nu_{e\rm L}}(\nu_{\mu\rm L})^c \\ & + M_{e\mu}\overline{\nu_{\tau\rm L}}(\nu_{e\rm L})^c + M_{\mu\mu}\overline{\nu_{\tau\rm L}}(\nu_{\tau\rm L})^c + M_{\mu\tau}\overline{\nu_{\tau\rm L}}(\nu_{\mu\rm L})^c \\ & + M_{e\tau}\overline{\nu_{\mu\rm L}}(\nu_{e\rm L})^c + M_{\mu\tau}\overline{\nu_{\mu\rm L}}(\nu_{\tau\rm L})^c + M_{\tau\tau}\overline{\nu_{\mu\rm L}}(\nu_{\mu\rm L})^c + \text{h.c.}\end{aligned}$$

Invariance of this transformation requires: $M_{e\mu} = M_{e\tau}$ and $M_{\mu\mu} = M_{\tau\tau}$

$$M_\nu = \begin{pmatrix} C & D & D \\ D & A & B \\ D & B & A \end{pmatrix}$$

ν_e $\quad \nu_\mu \leftrightarrow \nu_\tau$

$$\Rightarrow \begin{cases} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{cases}$$

reflection

A generic Majorana neutrino mass term reads as follows:

$$
\begin{aligned}
-\mathcal{L}_{\rm mass} = & \; M_{ee}\overline{\nu_{e\rm L}}(\nu_{e\rm L})^c + M_{e\mu}\overline{\nu_{e\rm L}}(\nu_{\mu\rm L})^c + M_{e\tau}\overline{\nu_{e\rm L}}(\nu_{\tau\rm L})^c \\
& + M_{e\mu}\overline{\nu_{\mu\rm L}}(\nu_{e\rm L})^c + M_{\mu\mu}\overline{\nu_{\mu\rm L}}(\nu_{\mu\rm L})^c + M_{\mu\tau}\overline{\nu_{\mu\rm L}}(\nu_{\tau\rm L})^c \\
& + M_{e\tau}\overline{\nu_{\tau\rm L}}(\nu_{e\rm L})^c + M_{\mu\tau}\overline{\nu_{\tau\rm L}}(\nu_{\mu\rm L})^c + M_{\tau\tau}\overline{\nu_{\tau\rm L}}(\nu_{\tau\rm L})^c \\
& + M^*_{ee}\overline{(\nu_{e\rm L})^c}\nu_{e\rm L} + M^*_{e\mu}\overline{(\nu_{\mu\rm L})^c}\nu_{e\rm L} + M^*_{e\tau}\overline{(\nu_{\tau\rm L})^c}\nu_{e\rm L} \\
& + M^*_{e\mu}\overline{(\nu_{e\rm L})^c}\nu_{\mu\rm L} + M^*_{\mu\mu}\overline{(\nu_{\mu\rm L})^c}\nu_{\mu\rm L} + M^*_{\mu\tau}\overline{(\nu_{\tau\rm L})^c}\nu_{\mu\rm L} \\
& + M^*_{e\tau}\overline{(\nu_{e\rm L})^c}\nu_{\tau\rm L} + M^*_{\mu\tau}\overline{(\nu_{\mu\rm L})^c}\nu_{\tau\rm L} + M^*_{\tau\tau}\overline{(\nu_{\tau\rm L})^c}\nu_{\tau\rm L}
\end{aligned}
$$

Under μ-τ reflection, the mass term is

$$
\begin{aligned}
\nu_{e\rm L} & \leftrightarrow (\nu_{e\rm L})^c \\
\nu_{\mu\rm L} & \leftrightarrow (\nu_{\tau\rm L})^c \\
\nu_{\tau\rm L} & \leftrightarrow (\nu_{\mu\rm L})^c
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_{\rm mass} = & \; M_{ee}\overline{(\nu_{e\rm L})^c}\nu_{e\rm L} + M_{e\mu}\overline{(\nu_{e\rm L})^c}\nu_{\tau\rm L} + M_{e\tau}\overline{(\nu_{e\rm L})^c}\nu_{\mu\rm L} \\
& + M_{e\mu}\overline{(\nu_{\tau\rm L})^c}\nu_{e\rm L} + M_{\mu\mu}\overline{(\nu_{\tau\rm L})^c}\nu_{\tau\rm L} + M_{\mu\tau}\overline{(\nu_{\tau\rm L})^c}\nu_{\mu\rm L} \\
& + M_{e\tau}\overline{(\nu_{\mu\rm L})^c}\nu_{e\rm L} + M_{\mu\tau}\overline{(\nu_{\mu\rm L})^c}\nu_{\tau\rm L} + M_{\tau\tau}\overline{(\nu_{\mu\rm L})^c}\nu_{\mu\rm L} \\
& + M^*_{ee}\overline{\nu_{e\rm L}}(\nu_{e\rm L})^c + M^*_{e\mu}\overline{\nu_{\tau\rm L}}(\nu_{e\rm L})^c + M^*_{e\tau}\overline{\nu_{\mu\rm L}}(\nu_{e\rm L})^c \\
& + M^*_{e\mu}\overline{\nu_{e\rm L}}(\nu_{\tau\rm L})^c + M^*_{\mu\mu}\overline{\nu_{\tau\rm L}}(\nu_{\tau\rm L})^c + M^*_{\mu\tau}\overline{\nu_{\mu\rm L}}(\nu_{\tau\rm L})^c \\
& + M^*_{e\tau}\overline{\nu_{e\rm L}}(\nu_{\mu\rm L})^c + M^*_{\mu\tau}\overline{\nu_{\tau\rm L}}(\nu_{\mu\rm L})^c + M^*_{\tau\tau}\overline{\nu_{\mu\rm L}}(\nu_{\mu\rm L})^c
\end{aligned}
$$

Invariance of this transformation:

$$
\begin{aligned}
M_{ee} &= M^*_{ee} \\
M_{\mu\tau} &= M^*_{\mu\tau} \\
M_{e\mu} &= M^*_{e\tau} \\
M_{\mu\mu} &= M^*_{\tau\tau}
\end{aligned}
$$

$$
M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix} \Rightarrow \begin{cases} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{cases}
$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$

Model building strategies

The **flavor symmetry** is a powerful **guiding principle** of model building.

The **flavor symmetry** could be

- ♣ Abelian or non-Abelian
- ♣ Continuous or discrete
- ♣ Local or global
- ♣ Spontaneously or explicitly broken

S_3 , S_4 , A_4 , Z_2 , $U(1)_F$, $SU(2)_F$, ...

Advantages of choosing a **global + discrete** flavor symmetry group G_F .

- ♣ No Goldstone bosons
- ♣ No additional gauge bosons mediating harmful FCNC processes
- ♣ No family-dependent D-terms contributing to sfermion masses
- ♣ Discrete G_F could come from some string compactifications
- ♣ Discrete G_F could be embedded in a continuous symmetry group

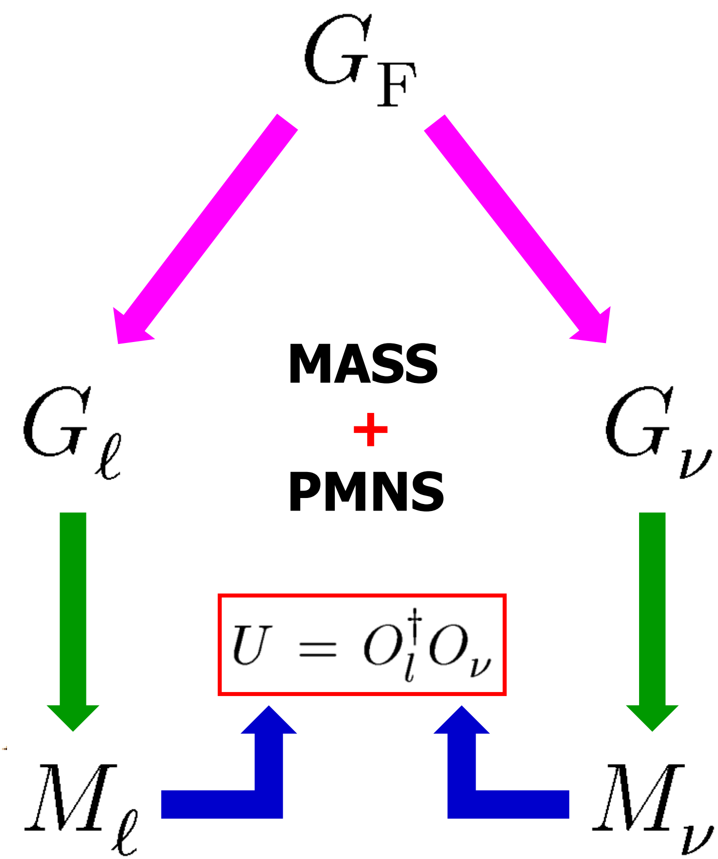
SUSY

Flavor symmetry groups

Some small discrete groups for model building (Altarelli, Feruglio 2010).

Group	d	Irreducible representation
$D_3 \sim S_3$	6	$1, 1', 2$
D_4	8	$1_1, \ldots, 1_4, 2$
D_7	14	$1, 1', 2, 2', 2''$
A_4	12	$1, 1', 1'', 3$
$A_5 \sim PSL_2(5)$	60	$1, 3, 3', 4, 5$
T'	24	$1, 1', 1'', 2, 2', 2'', 3$
S_4	24	$1, 1', 2, 3, 3'$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, 1_9, 3, \bar{3}$
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1}', 3, \bar{3}$

Too many possibilities, but the μ-τ symmetry inclusive



Generalized CP combined with flavor symmetry to predict the phase δ.

Phenomenology (1)

Matter effects: the behavior of neutrino oscillations is modified due to the coherent forward scattering induced by the weak charged-current interactions. The effective Hamiltonian for neutrino propagation:

$$\widetilde{\mathcal{H}}_{\rm eff} = \frac{1}{2E}\left[\widetilde{U}\begin{pmatrix}\widetilde{m}_1^2 & 0 & 0\\ 0 & \widetilde{m}_2^2 & 0\\ 0 & 0 & \widetilde{m}_3^2\end{pmatrix}\widetilde{U}^\dagger\right] = \frac{1}{2E}\left[U\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix}A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]$$

in matter **in vacuum** **correction**

Sum rules between matter and vacuum:

$$A = 2\sqrt{2}\, G_{\rm F} N_e E$$

$$\sum_{i=1}^{3}\widetilde{m}_i^2\widetilde{U}_{\alpha i}\widetilde{U}^*_{\beta i} = \sum_{i=1}^{3} m_i^2 U_{\alpha i}U^*_{\beta i} + \underline{A\delta_{\alpha e}\delta_{e\beta}}$$

$$\sum_{i=1}^{3}\widetilde{m}_i^4\widetilde{U}_{\alpha i}\widetilde{U}^*_{\beta i} = \sum_{i=1}^{3} m_i^2\left[m_i^2 + \underline{A\left(\delta_{\alpha e}+\delta_{e\beta}\right)}\right]U_{\alpha i}U^*_{\beta i} + \underline{A^2\delta_{\alpha e}\delta_{e\beta}}$$

$$\sum_{i=1}^{3}\widetilde{U}_{\alpha i}\widetilde{U}^*_{\beta i} = \sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i} = \delta_{\alpha\beta}$$

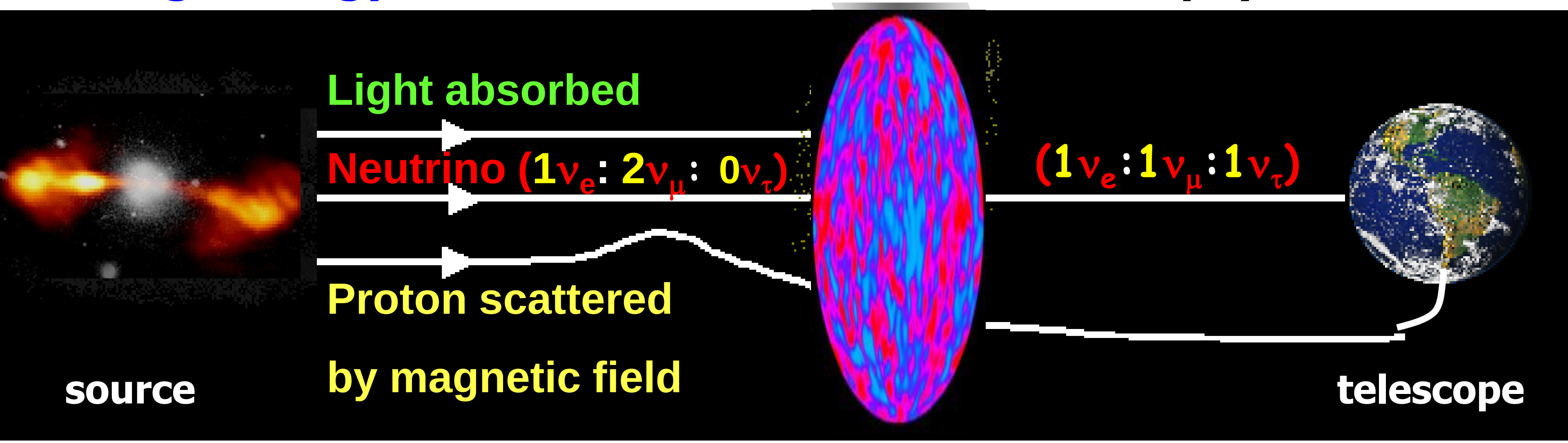
disappear when α, β = μ, τ

A proper phase convention leads us to $|\widetilde{U}_{\mu i}| = |\widetilde{U}_{\tau i}|$ from $|U_{\mu i}| = |U_{\tau i}|$**.**

Namely, matter effects (a constant profile) respect the μ-τ symmetry.

Phenomenology (2)

Ultrahigh-energy cosmic neutrinos from distant astrophysical sources



A conventional UHE cosmic neutrino source (*p* + *p* or *p* + γ collisions)

$$\Phi^{\rm T}_\mu - \Phi^{\rm T}_\tau = \frac{\Phi_0}{3}\sum_i \left(|U_{\mu i}|^2 - |U_{\tau i}|^2\right)^2$$

sensitive to μ-τ flavor symmetry

$$\Phi^{\rm T}_e : \Phi^{\rm T}_\mu : \Phi^{\rm T}_\tau = (1 + D_e) : (1 + D_\mu) : (1 + D_\tau)$$

with $D_e = -2\Delta$, $D_\mu = \Delta + \overline{\Delta}$ and $D_\tau = \Delta - \overline{\Delta}$

$$\Delta \simeq \frac{1}{2}\sin^2 2\theta_{12} \sin\varepsilon - \frac{1}{4}\sin 4\theta_{12}\sin\theta_{13}\cos\delta$$

$\varepsilon \equiv \theta_{23} - \pi/4$

$$\overline{\Delta} \simeq \left(4 - \sin^2 2\theta_{12}\right)\sin^2\varepsilon + \sin^2 2\theta_{12}\sin^2\theta_{13}\cos^2\delta + \sin 4\theta_{12}\sin\varepsilon\sin\theta_{13}\cos\delta$$

Summary

Z.Z.X., Z.H. Zhao (1512.04207)

—— A review of mu-tau flavor symmetry in neutrino physics

Report on Progress in Physics

79 (2016) 076201

C.S. Wu: It is easy to do the right thing once you have the right ideas.

I.I. Rabi: Physics needs new ideas. But to have a new idea is a very difficult task.... (Berezhiani's talk)

L.C. Pauling: The best way to have a good idea is to have a lot of ideas.

Lecture A5

- ★ **Lepton number violation**
- ★ **neutrinoless double-beta decays**
- ★ **Possible new physics effects**

1935: 2ν2β decays

2ν2β decay: certain even-even nuclei have an opportunity to decay to the 2nd nearest neighbors via 2 simultaneous β decays (equivalent to the decays of two neutrons).

$$(Z, A) \rightarrow (Z+2, A) + 2e^- + 2\bar{\nu}_e.$$

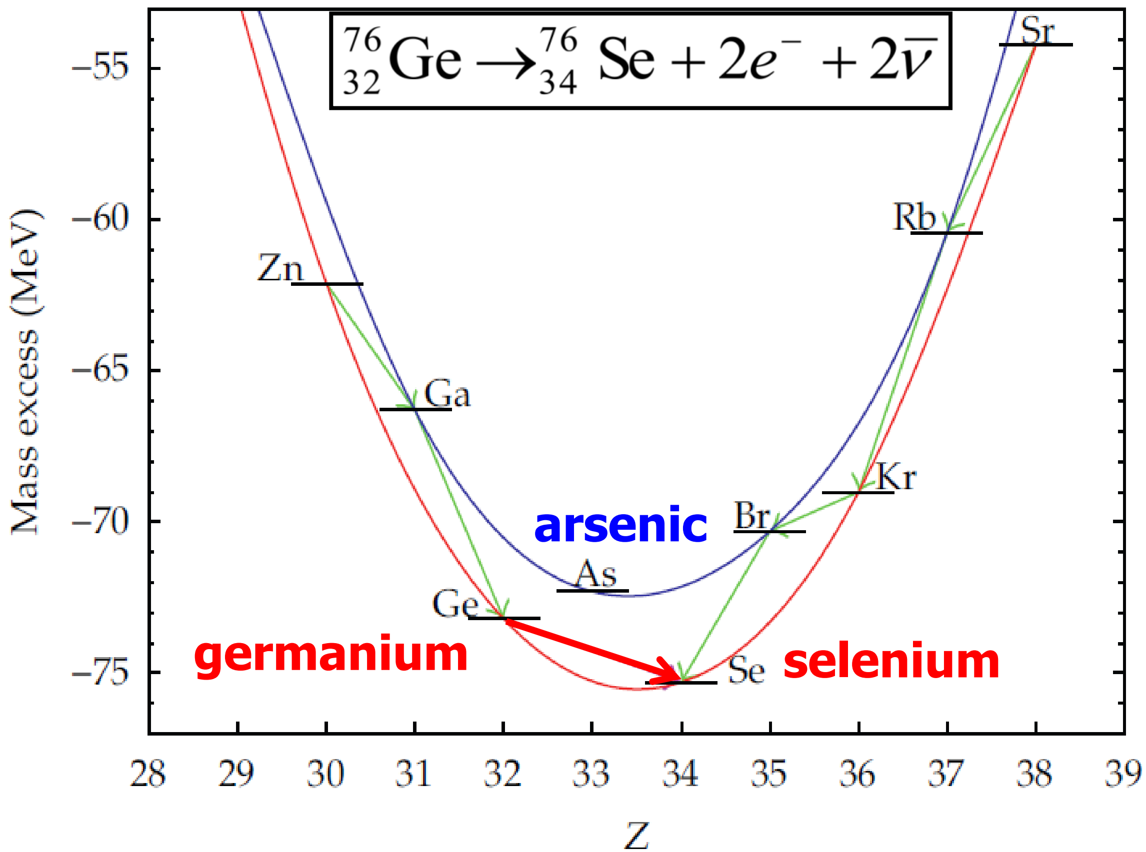
necessary conditions:

$$m(Z,A) > m(Z+2,A)$$

$$m(Z,A) < m(Z+1,A)$$

1935

Maria Goeppert Mayer



1937: Majorana

★ Theory of the Symmetry of Electrons and Positrons

Ettore Majorana *Nuovo Cim. 14 (1937) 171*

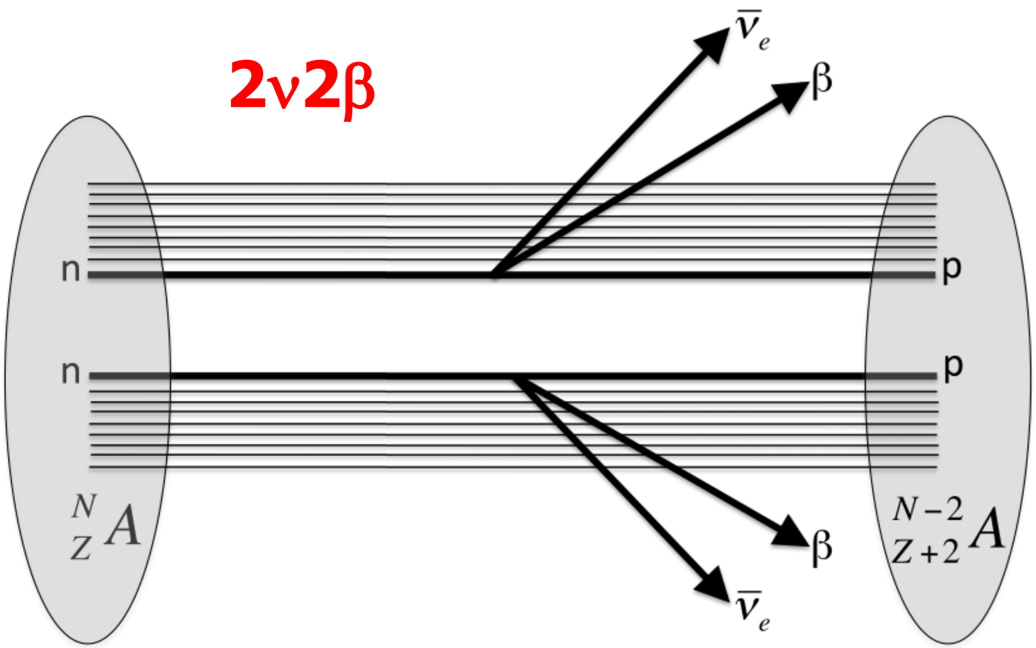
"…there is now no need to assume the existence of antineutron or antineutrinos. The latter particles are indeed introduced in the theory of positive beta-ray emission; the theory, however, can be obviously modified so that the beta-emission, both positive and negative, is always accompanied by the emission of a neutrino."

1938年：自杀；逃往阿根廷，并在那里隐姓埋名地生活了二十几年；遁入空门；遭到绑架或杀害，以阻止他加入制造原子弹的项目；沦为乞丐；…..

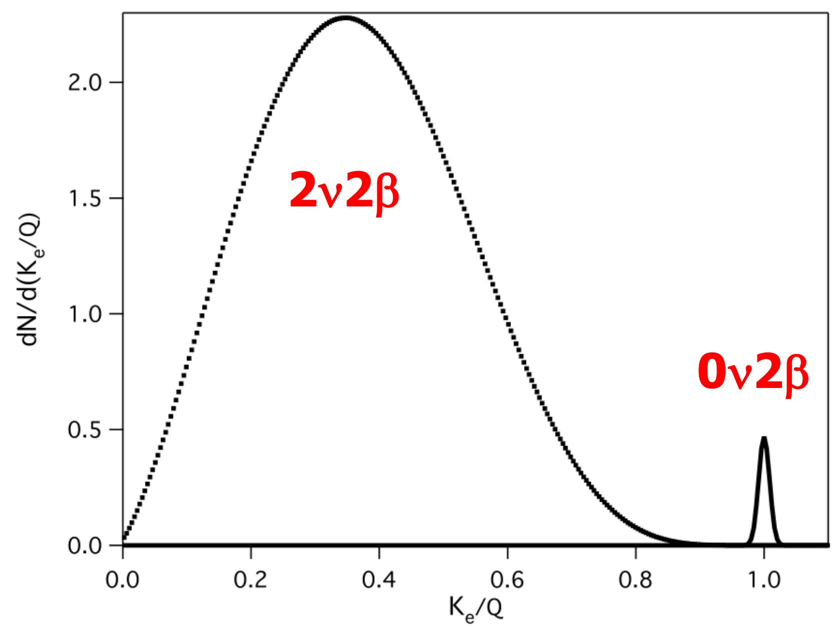
Enrico Fermi (1938):

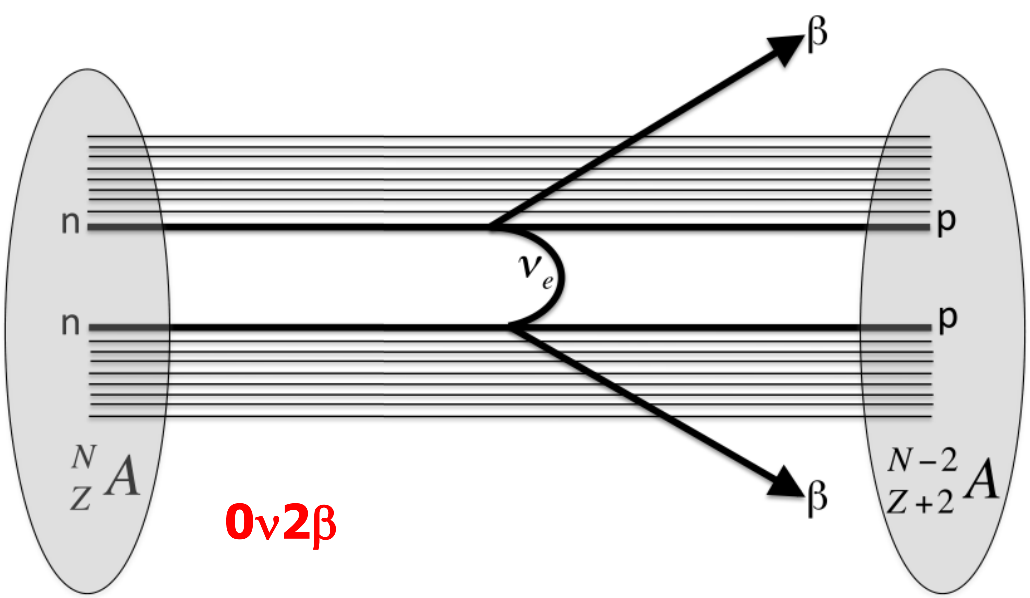
"There are various kind of scientists in the world. The second- and third-rate ones do their best but do not get very far. There are also first-rate people who make very important discoveries which are of capital importance for the development of the science. Then there are genius like Galileo and Newton. Ettore Majorana was one of these. Majorana had greater gifts than anyone else in the world; unfortunately he lacked one quality which other men generally have: plain common sense"

If this is the case, …



$$^{N}_{Z}A \rightarrow ^{N-2}_{Z+2}A + 2\beta^{-} + 2\bar{\nu}_e$$



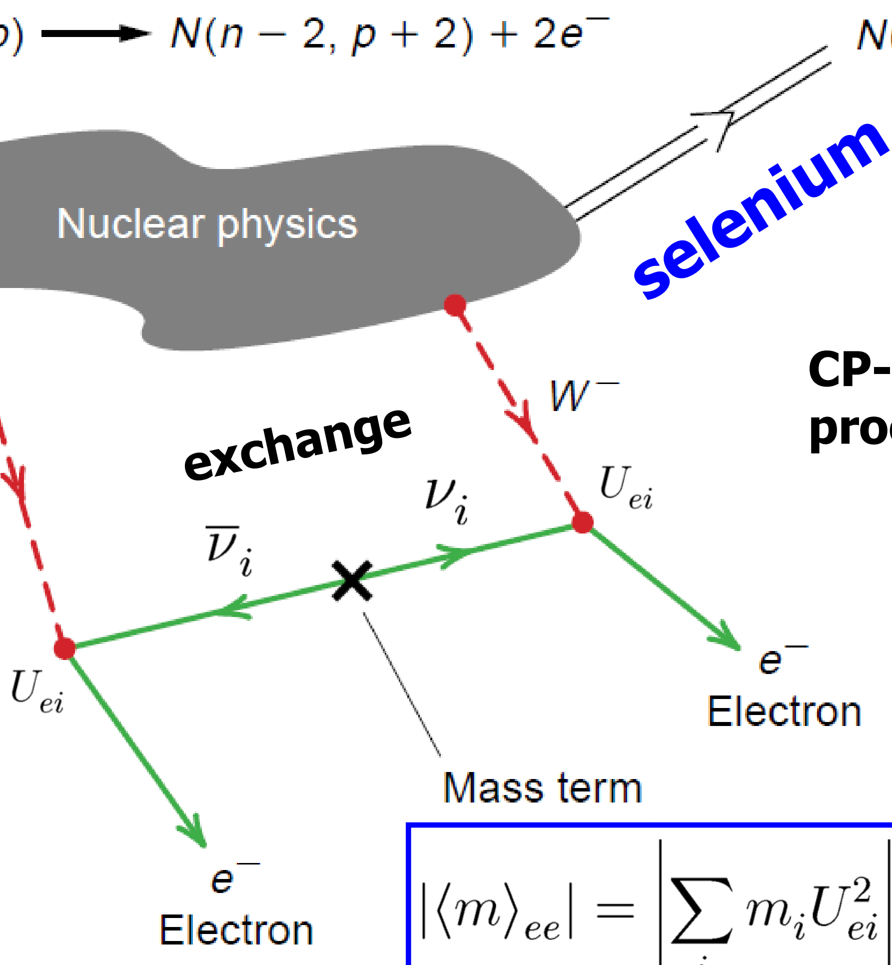


$$^{N}_{Z}A \rightarrow ^{N-2}_{Z+2}A + 2\beta^{-}$$

1939: 0ν2β decays

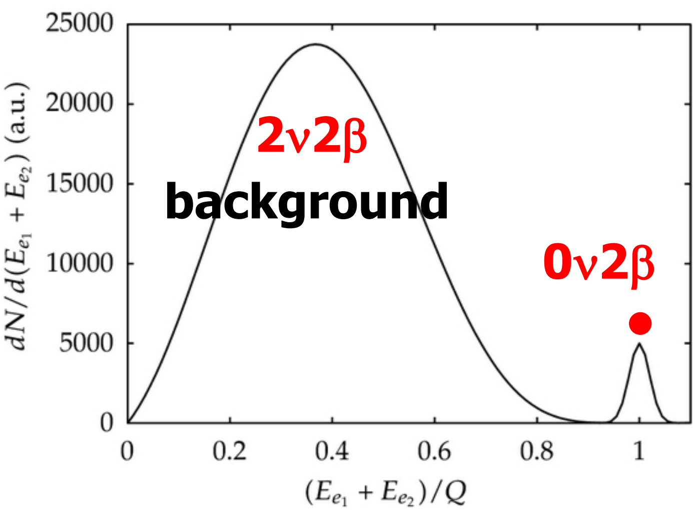
A 0ν2β decay can happen if massive ν's have the Majorana nature (Wendell Furry 1939)

$$T^{0\nu}_{1/2} = \left(G^{0\nu}\right)^{-1} \left|M^{0\nu}\right|^{-2} \left|\langle m \rangle_{ee}\right|^{-2}$$



Lepton number violation

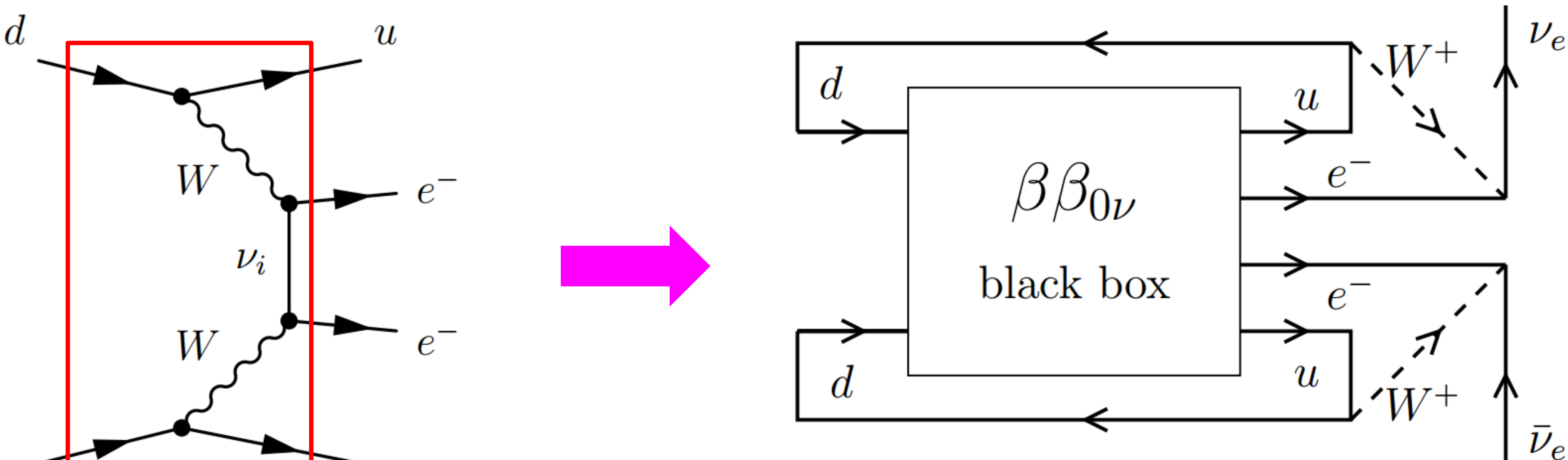
CP-conserving process



$$\left|\langle m \rangle_{ee}\right| = \left|\sum_i m_i U_{ei}^2\right|$$

Schechter-Valle theorem

THEOREM (1982): if a 0ν2β decay happens, there must be an effective Majorana mass term.



Bruno Pontecorvo's Prediction

That is why we want to see 0ν2β

Four-loop ν mass:

$$\delta m_\nu = \mathcal{O}(10^{-24}\,\mathrm{eV})$$

(Duerr, Lindner, Merle, 2011)

Note: The black box can in principle have many different processes (new physics). Only in the simplest case, which is most interesting, it's likely to constrain neutrino masses

Ge + Xe

GERDA has killed the Heidelberg-Moscow's claim on 0ν2β.

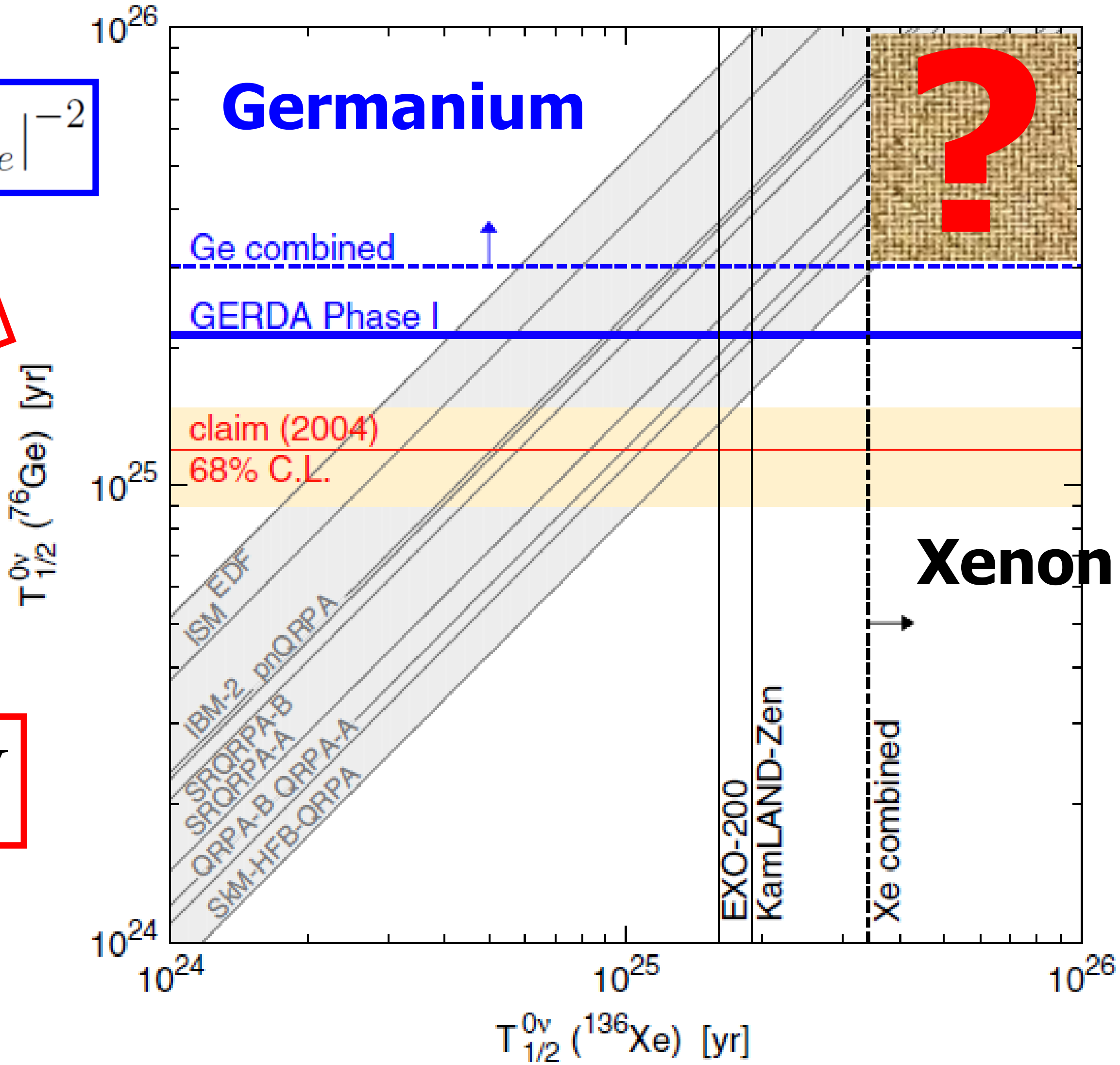
PRL **111**, 122503 (2013)

$$T^{0\nu}_{1/2} = \left(G^{0\nu}\right)^{-1} \left|M^{0\nu}\right|^{-2} \left|\langle m\rangle_{ee}\right|^{-2}$$

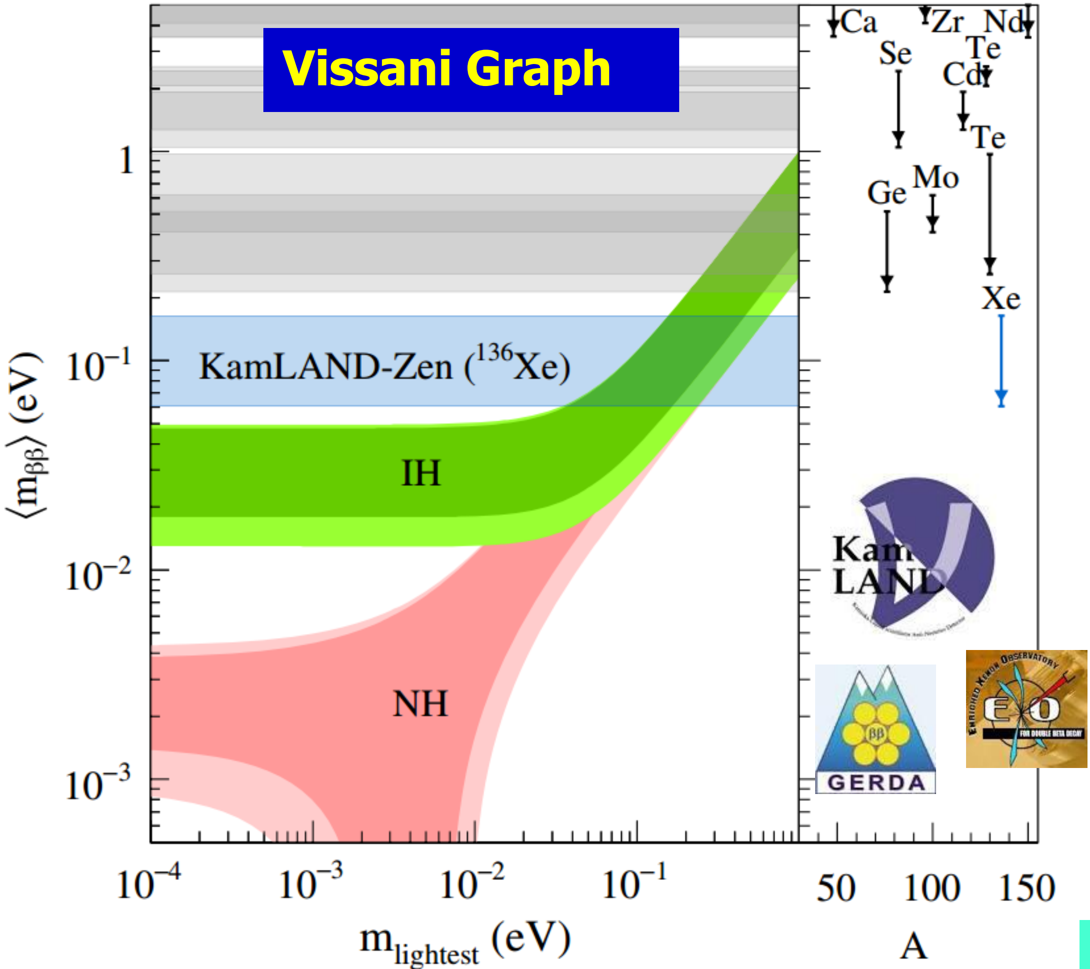
$$T^{0\nu}_{1/2} > 3.0 \times 10^{25}\ \text{yr (90\% C.L.)}.$$

$$\left|\langle m\rangle_{ee}\right| < 0.2 \rightarrow 0.4\ \text{eV}$$

$$\left|\langle m\rangle_{ee}\right| = \left|\sum_i m_i U_{ei}^2\right|$$



Bet on an ordering



The effective mass

$$|\langle m\rangle_{ee}| = \left|\sum_i m_i U_{ei}^2\right|$$

Maury Goodman asks:

Is IH an intelligent design?

If it is inverted, why do not we reorder it?

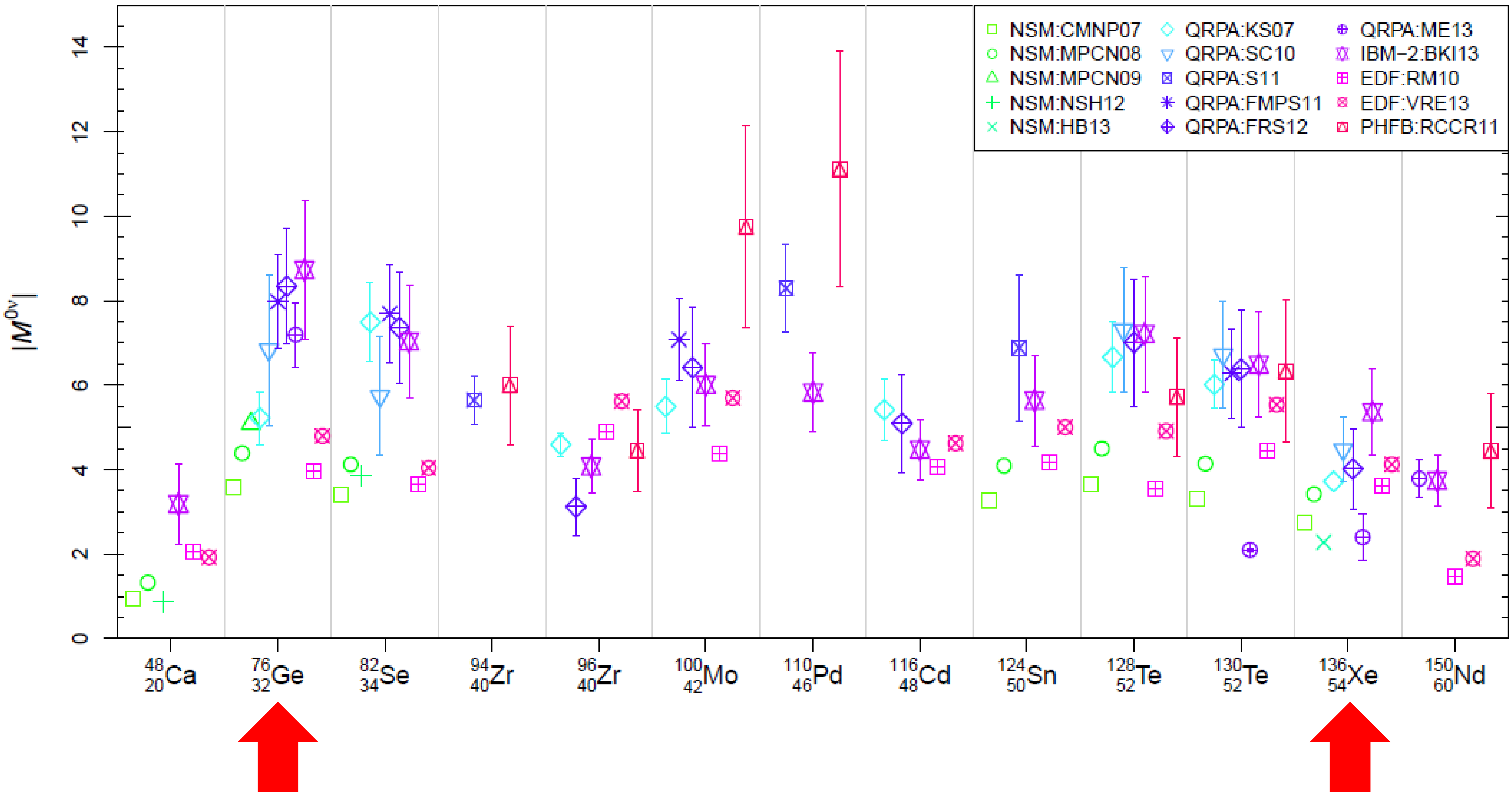
Then how about the PMNS matrix?

If normal, how possible to fall into the well?

1) **The structure of the well?**
2) **Role of Majorana phases?**

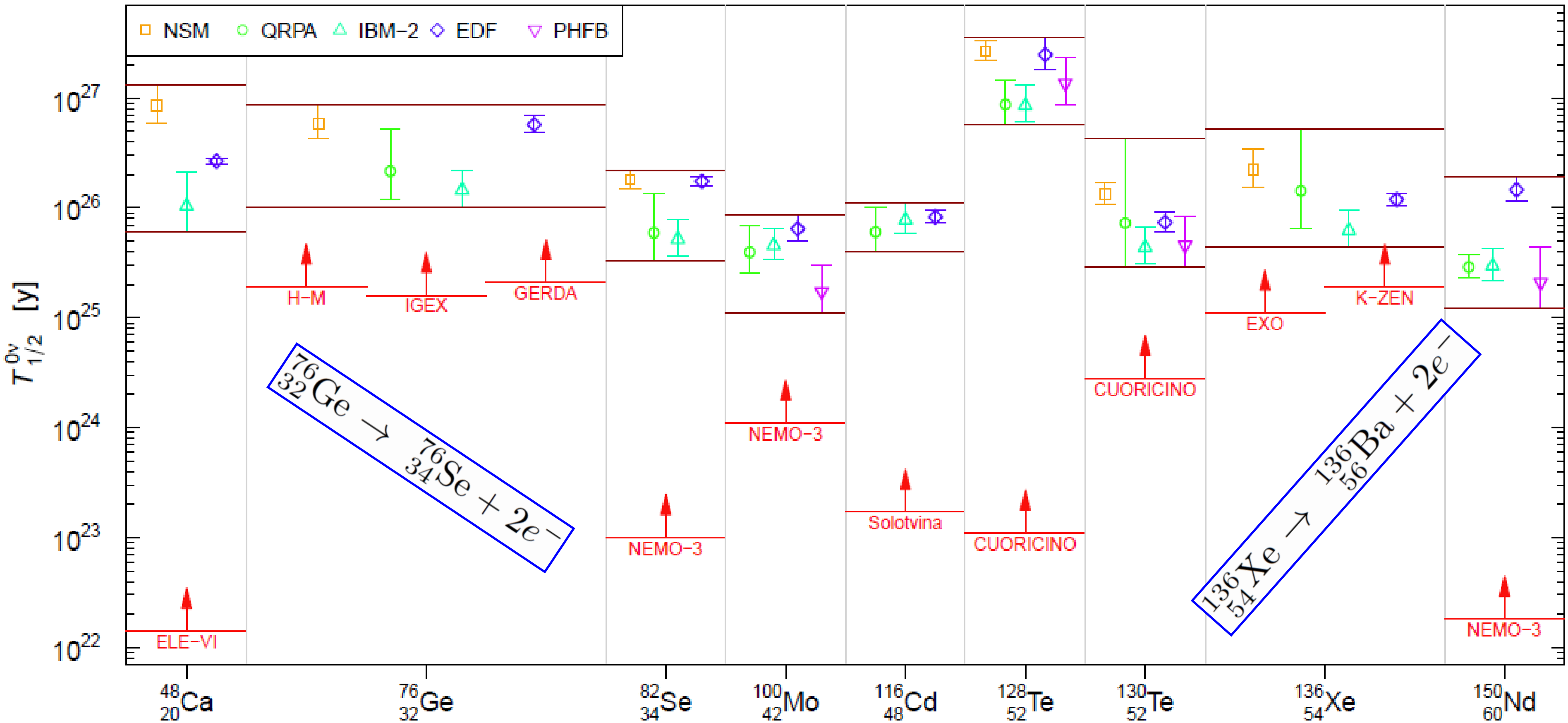
Nuclear matrix elements

Unfortunately, nuclear matrix elements can be calculated only based on some models which describe many-body interactions of nucleons in nuclei. Since different models focus on different aspects of nuclear physics, large uncertainties (a factor of 2 or 3) are unavoidable.

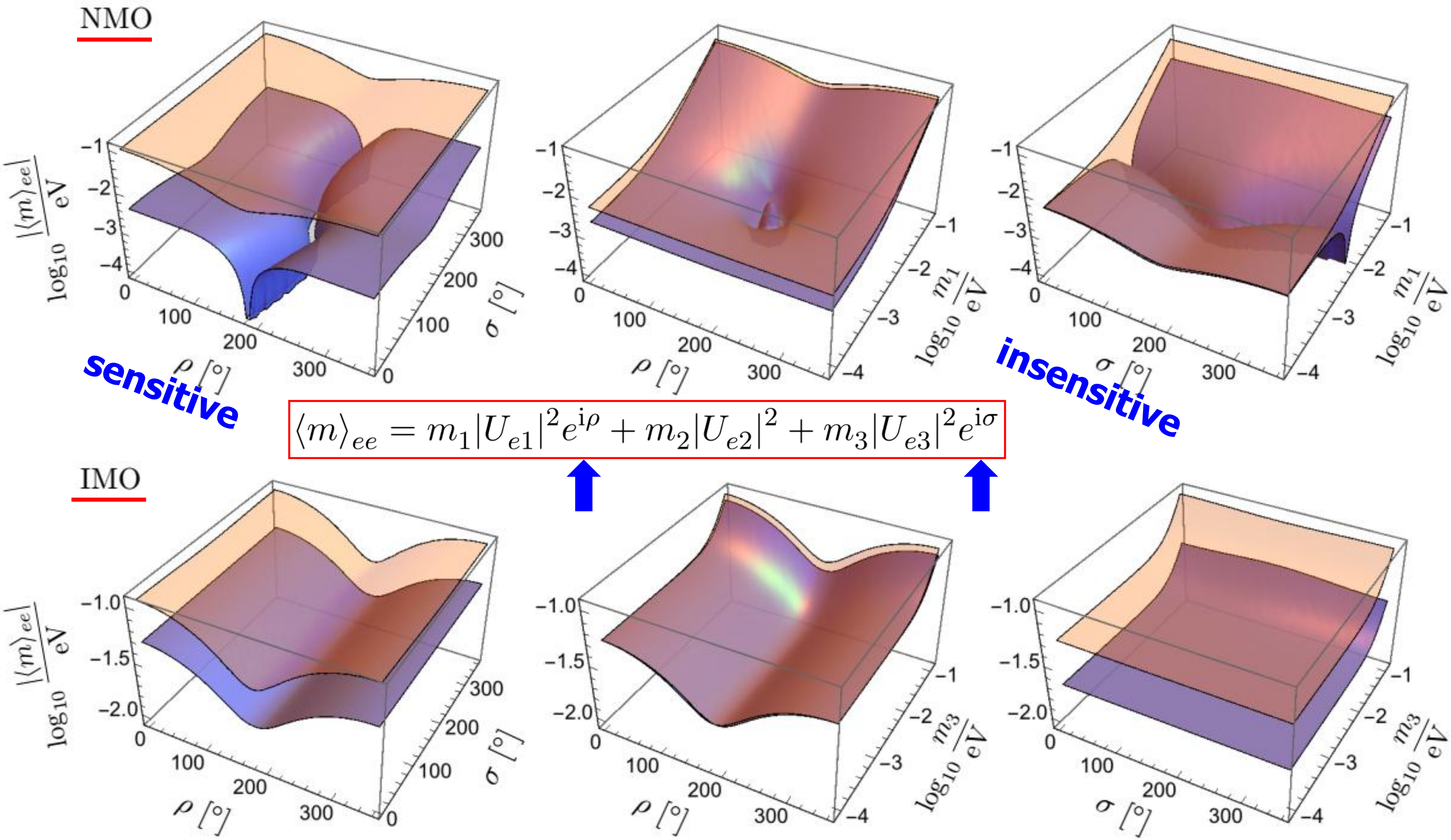


Half-life

Comparing the 90% C.L. experimental lower limits on the half-life of a 0ν2β-decaying nuclide with the corresponding range of theoretical prediction, given a value of 0.1 eV for the effective Majorana neutrino mass term (Bilenky and Giunti, 1411.4791).



Effective mass term



$$\langle m\rangle_{ee} = m_1|U_{e1}|^2 e^{i\rho} + m_2|U_{e2}|^2 + m_3|U_{e3}|^2 e^{i\sigma}$$

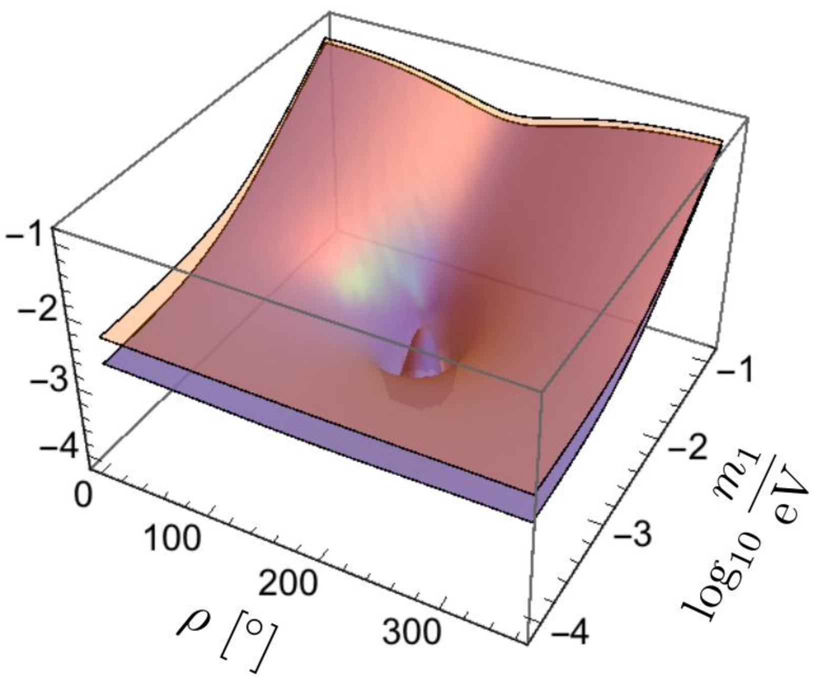
Lower/upper bound: blue/light orange. 3σ inputs of ν-oscillation data with a new phase convention (Xing, Zhao, Zhou, 1504.05820)

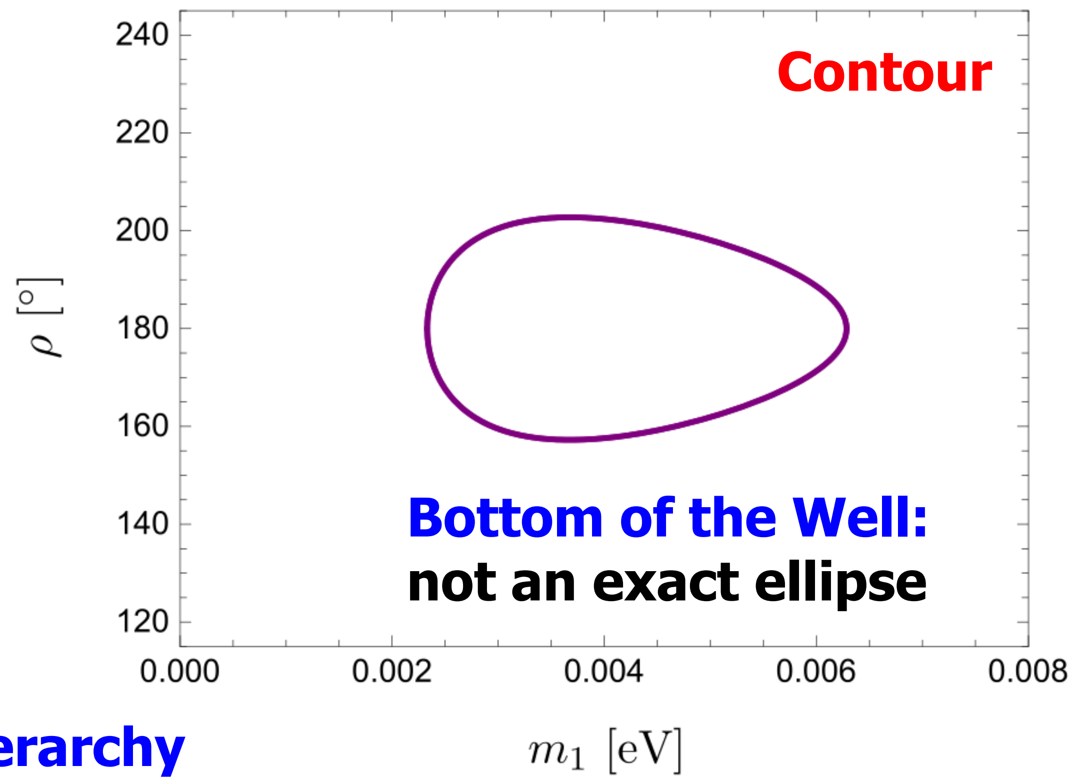
Contour of the bottom

Let us understand the champagne-bottle profile of the effective 0ν2β mass term in the normal hierarchy case:

$$\langle m \rangle_{ee} = \underline{m_1 c_{12}^2 c_{13}^2 e^{i\rho} + m_2 s_{12}^2 c_{13}^2} + m_3 s_{13}^2 e^{i\sigma} = 0$$

$$m_1^2 c_{12}^4 c_{13}^4 + 2 m_1 m_2 c_{12}^2 s_{12}^2 c_{13}^4 \cos\rho + m_2^2 s_{12}^4 c_{13}^4 = m_3^2 s_{13}^4$$





The dark well in the normal hierarchy

A bullet structure

$$|\langle m \rangle_{ee}|_{\rm U,L} = \left|\overline{m}_{12} \cos^2\theta_{13} \pm m_3 \sin^2\theta_{13}\right|$$

$$\overline{m}_{12} \equiv \sqrt{m_1^2 \cos^4\theta_{12} + \frac{1}{2} m_1 m_2 \sin^2 2\theta_{12} \cos\rho + m_2^2 \sin^4\theta_{12}}$$

EXTREMUM

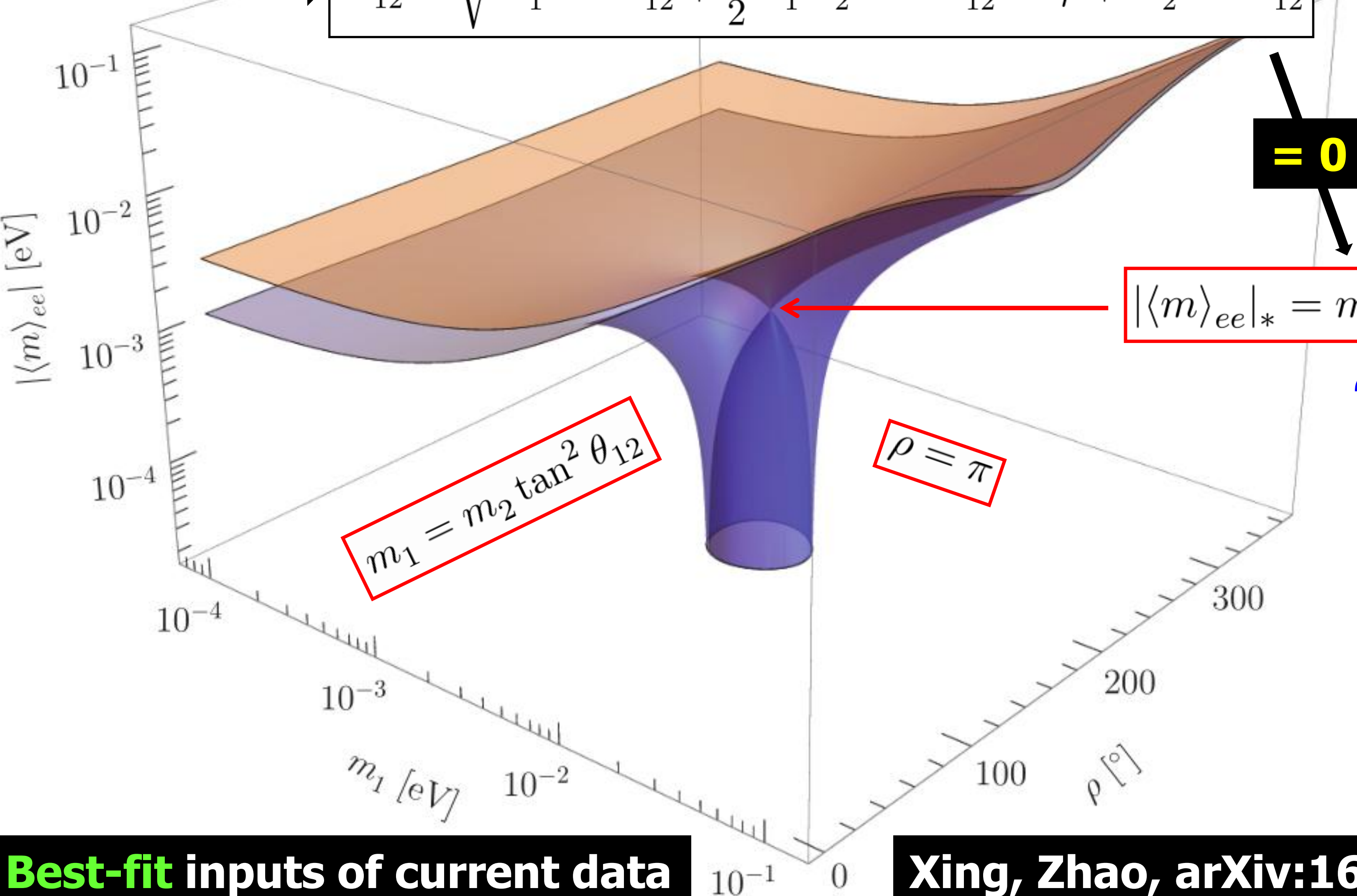
= 0

$$|\langle m \rangle_{ee}|_{*} = m_3 \sin^2\theta_{13}$$

~ 1 meV

$m_1 = m_2 \tan^2\theta_{12}$

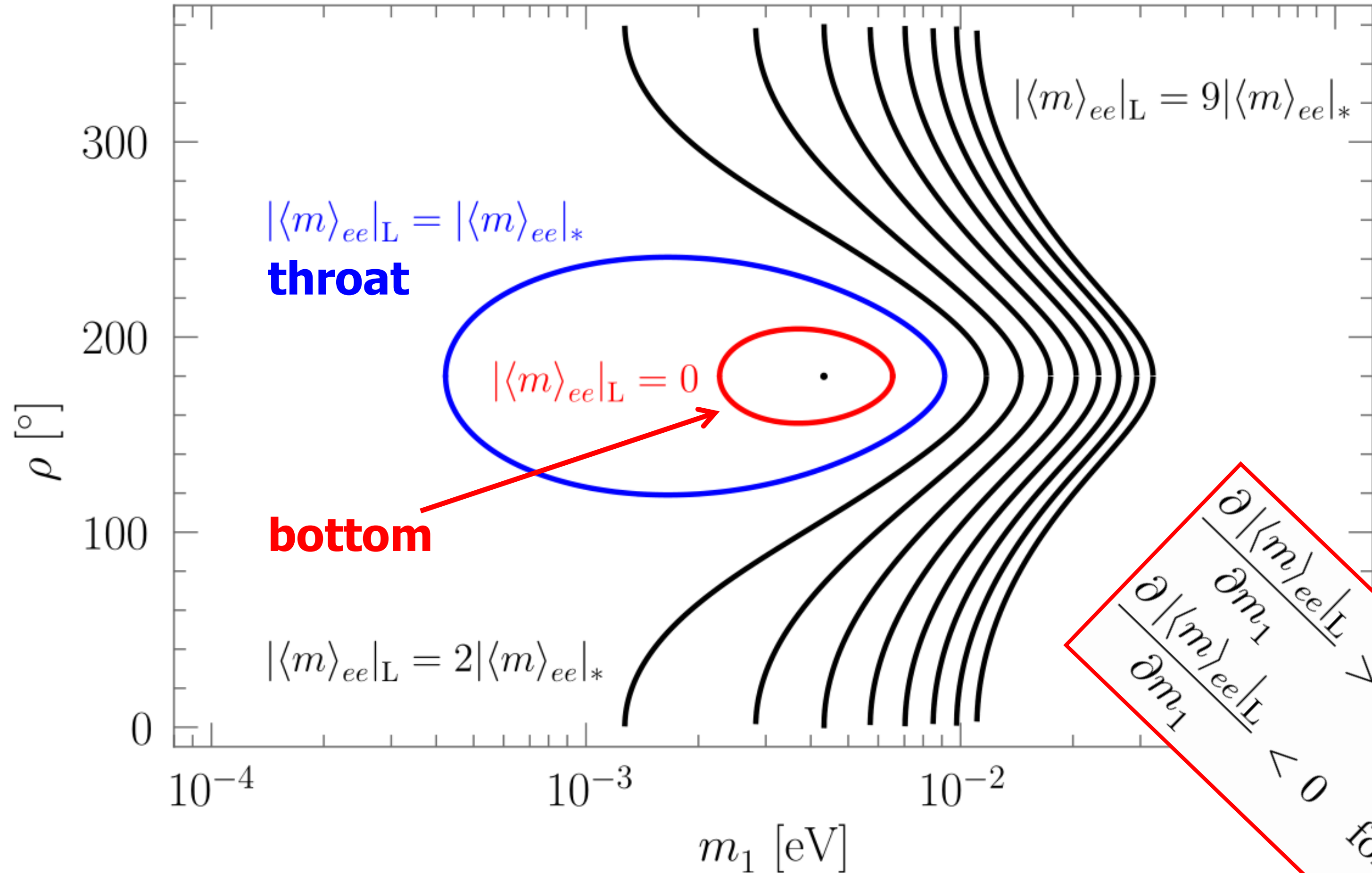
$\rho = \pi$



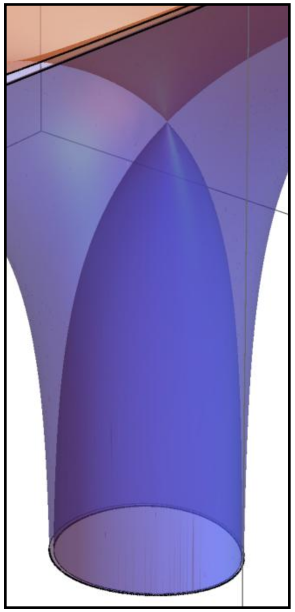
Best-fit inputs of current data

Xing, Zhao, arXiv:1612.08538

The throat



EXTREMUM



$$\frac{\partial |\langle m \rangle_{ee}|_{\rm L}}{\partial m_1} > 0 \quad \text{for} \quad m_1 < m_2 \tan^2\theta_{12}$$

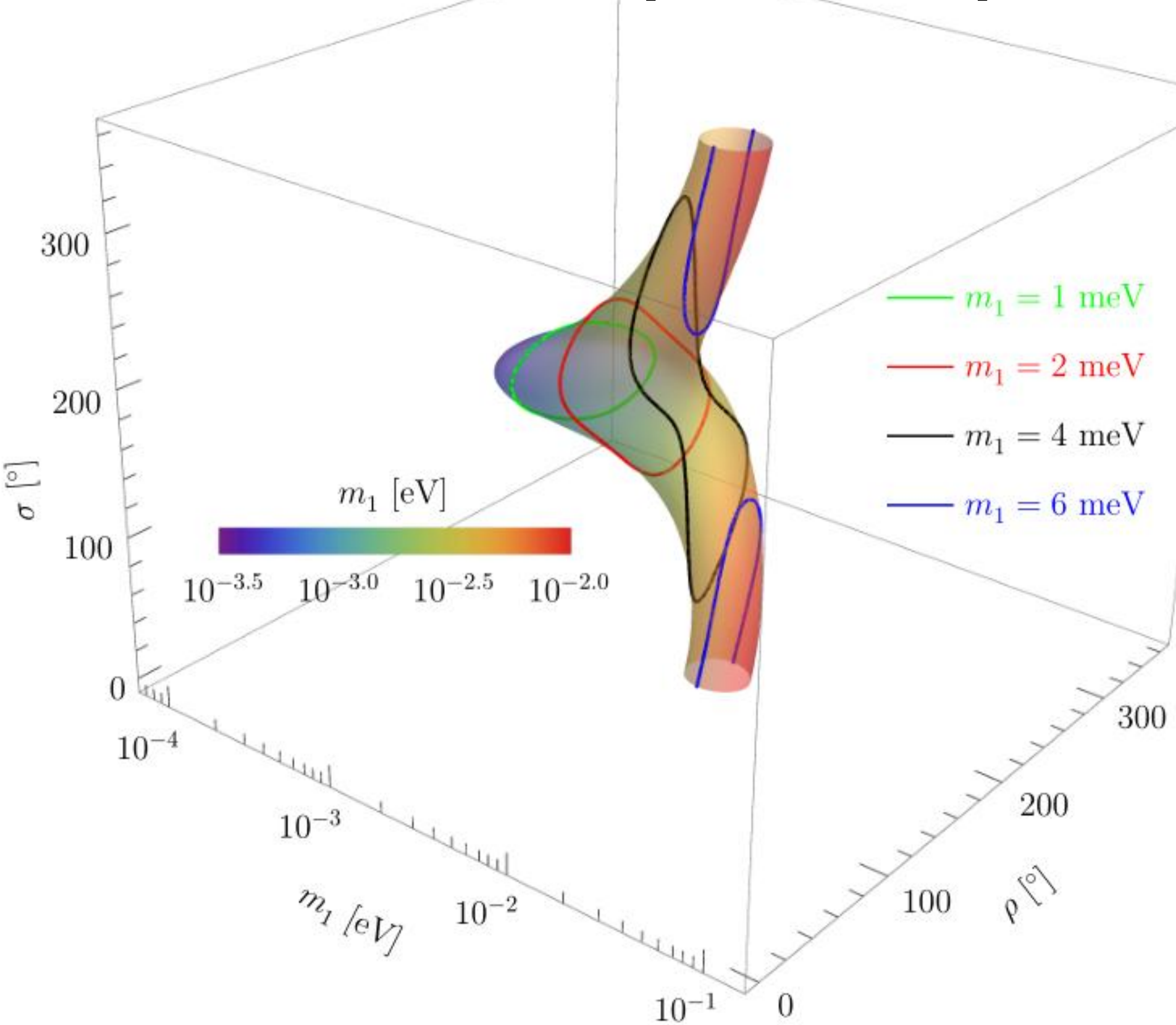
$$\frac{\partial |\langle m \rangle_{ee}|_{\rm L}}{\partial m_1} < 0 \quad \text{for} \quad m_1 > m_2 \tan^2\theta_{12}$$

$$\frac{\partial |\langle m \rangle_{ee}|_{\rm L}}{\partial \rho} = \frac{m_1 m_2 \sin^2 2\theta_{12} \cos^2\theta_{13}}{4\overline{m}_{12}} \sin\rho = 0$$

$$\frac{\partial |\langle m \rangle_{ee}|_{\rm L}}{\partial m_1} = \frac{m_1}{m_3}\sin^2\theta_{13} \pm \left(\cos^2\theta_{12} - \frac{m_1}{m_2}\sin^2\theta_{12}\right)\cos^2\theta_{13} \neq 0$$

To fall into the well

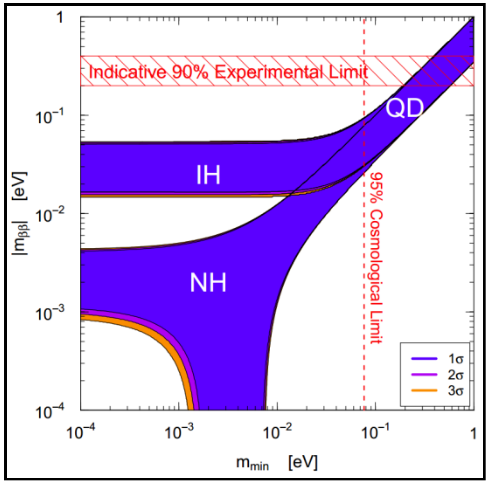
The three-dimensional parameter space of $|\langle m \rangle_{ee}| < |\langle m \rangle_{ee}|_*$



Take it easy!

It is difficult to fall into the well!

Vissani Graph



Model building?

Why the relationship $\tan\theta_{12} = \sqrt{m_1/m_2}$ **is reasonable?**

Remember $\tan\theta_{\rm C} \simeq \sqrt{m_d/m_s}$ **in the quark sector as done by**




S. Weinberg

H. Fritzsch

F. Wilczek + A. Zee

1977

The effective Majorana neutrino mass matrix (Xing, Zhao, 1612.08538)

$$M_\nu = \begin{pmatrix} 0 & A & A \\ A & B & C \\ A & C & B \end{pmatrix} - m_3 \frac{\sin\theta_{13}}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\,\sin\theta_{13} & +{\rm i} & -{\rm i} \\ +{\rm i} & 0 & 0 \\ -{\rm i} & 0 & 0 \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$

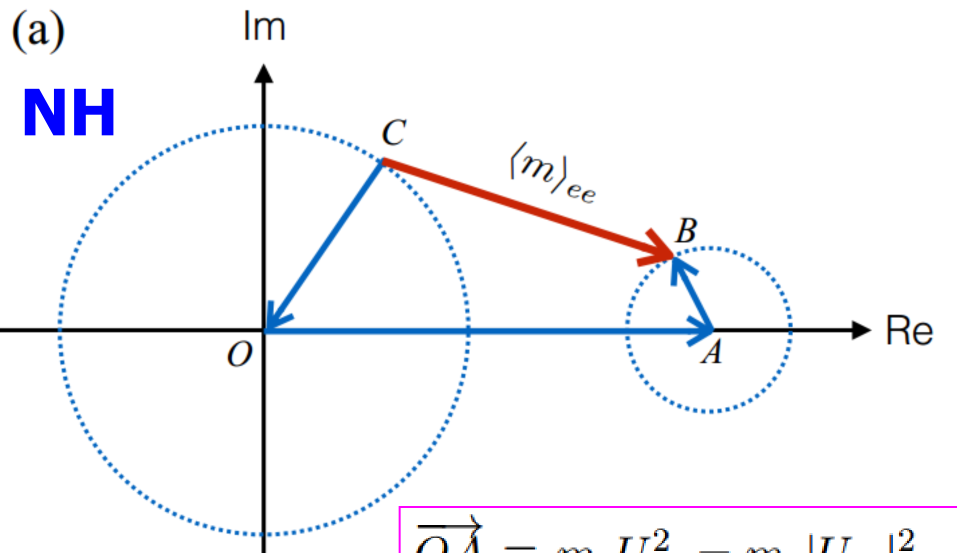
Predictions, thanks to the μ-τ reflection symmetry:

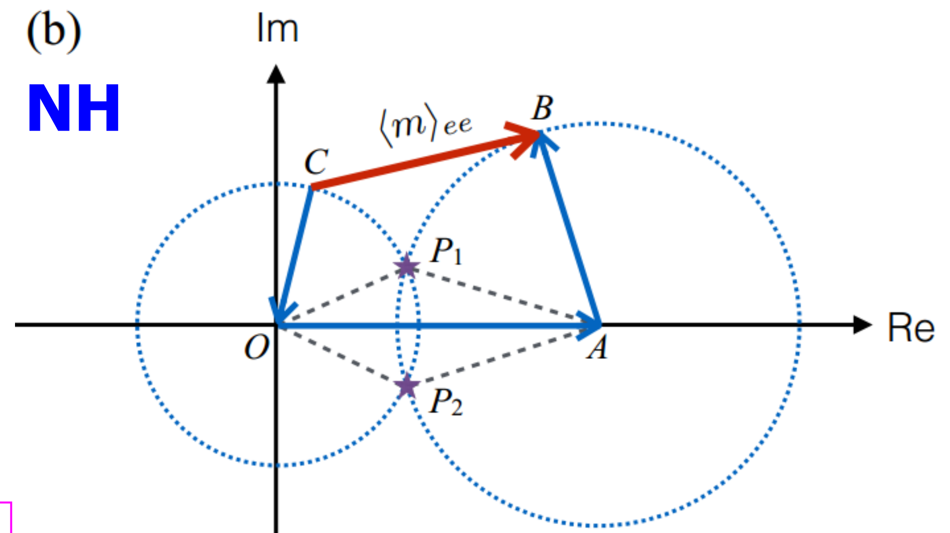
$$|\langle m\rangle_{ee}| = m_3 \sin^2\theta_{13} \text{ and } \tan\theta_{12} = \sqrt{m_1/m_2}$$

$$\theta_{23} = \pi/4,\ \delta = -\pi/2,\ \rho = \pi \text{ and } \sigma = 0$$

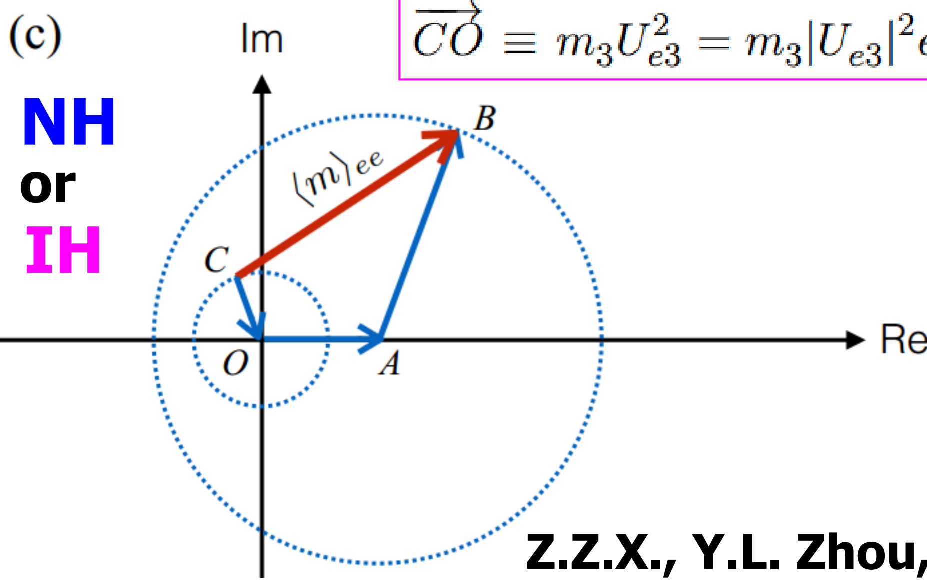
consistent with current data!

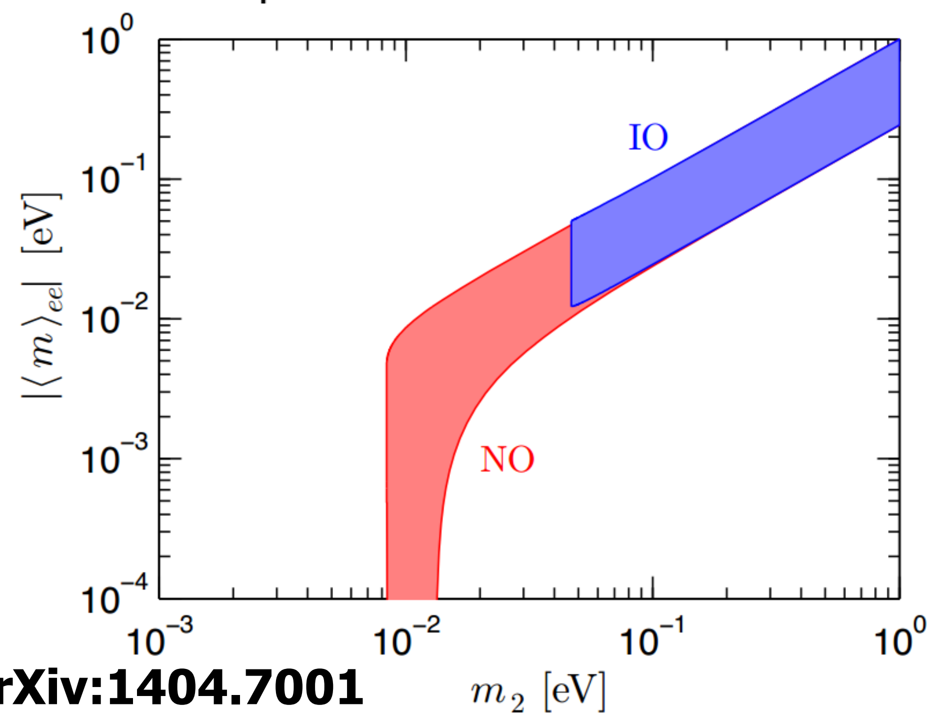
Coupling-rod diagram





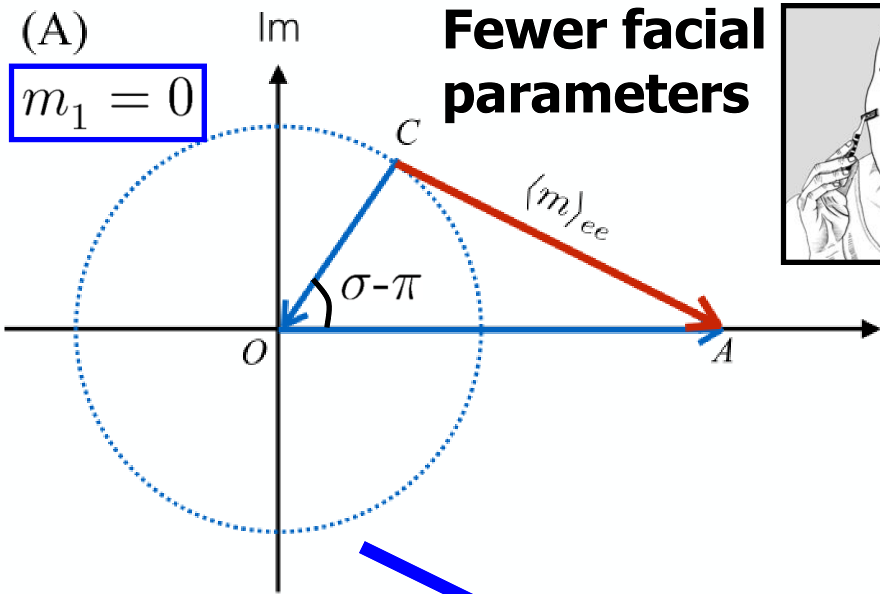
$$\overrightarrow{OA} \equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2 \ ,$$
$$\overrightarrow{AB} \equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{i\rho}$$
$$\overrightarrow{CO} \equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{i\sigma}$$





Z.Z.X., Y.L. Zhou, arXiv:1404.7001

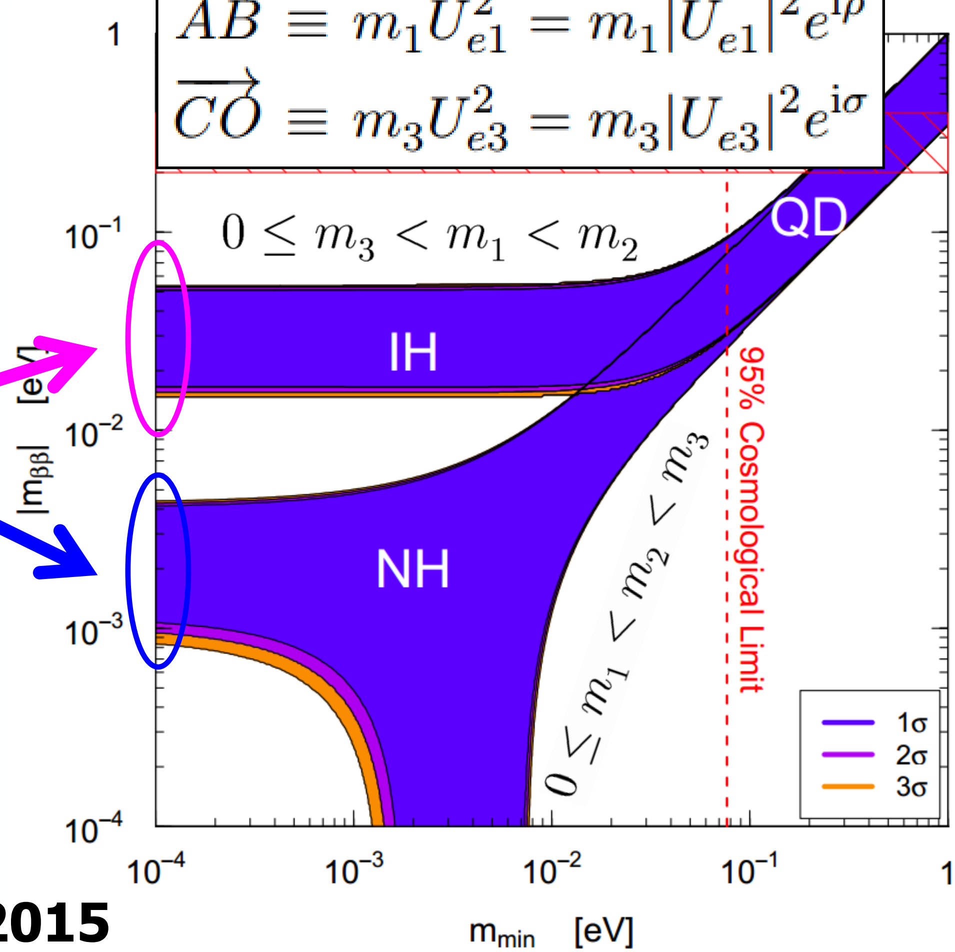
Occam's razor: 0ν2β

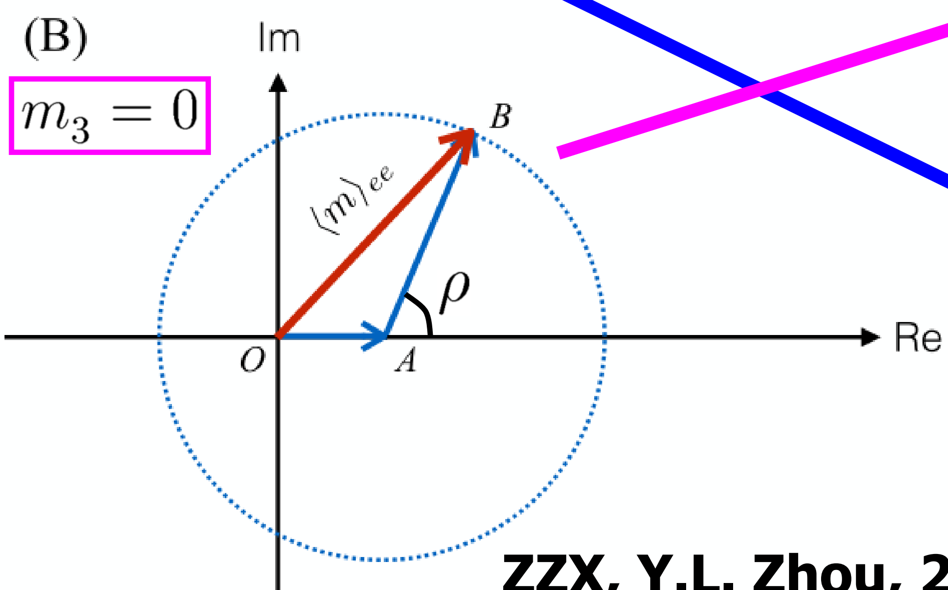


Fewer facial parameters

Entities must not be multiplied beyond necessity.

$$\overrightarrow{OA} \equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2 \; ,$$
$$\overrightarrow{AB} \equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{i\rho}$$
$$\overrightarrow{CO} \equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{i\sigma}$$





ZZX, Y.L. Zhou, 2015

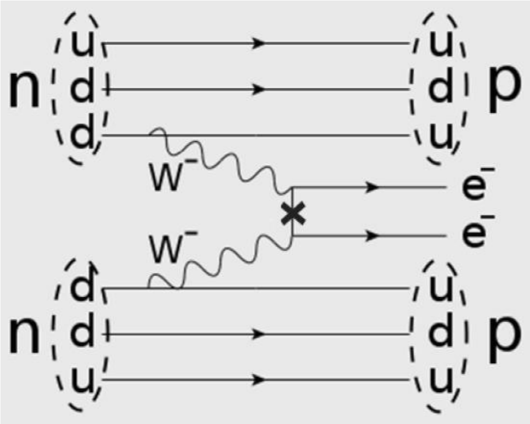
New physics?

Type (A): NP directly related to extra species of neutrinos.

Example 1: heavy Majorana neutrinos from type-I seesaw

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$\Gamma_{0\nu\beta\beta} \propto \left| \sum_{i=1}^{3} m_i U_{ei}^2 \; - \; \sum_{k=1}^{n} \frac{R_{ek}^2}{M_k} M_A^2 \mathcal{F}(A, M_k) \right|^2$$



In most cases the heavy contribution is negligible

Example 2: light sterile neutrinos from LSND etc

$$\langle m \rangle'_{ee} \equiv \sum_{i=1}^{6} m_i U_{ei}^2 = \underline{\langle m \rangle_{ee}} \left(c_{14} c_{15} c_{16}\right)^2 + \underline{m_4 \left(\hat{s}^*_{14} c_{15} c_{16}\right)^2 + m_5 \left(\hat{s}^*_{15} c_{16}\right)^2 + m_6 \left(\hat{s}^*_{16}\right)^2}$$

In this case the new contribution might be constructive or destructive

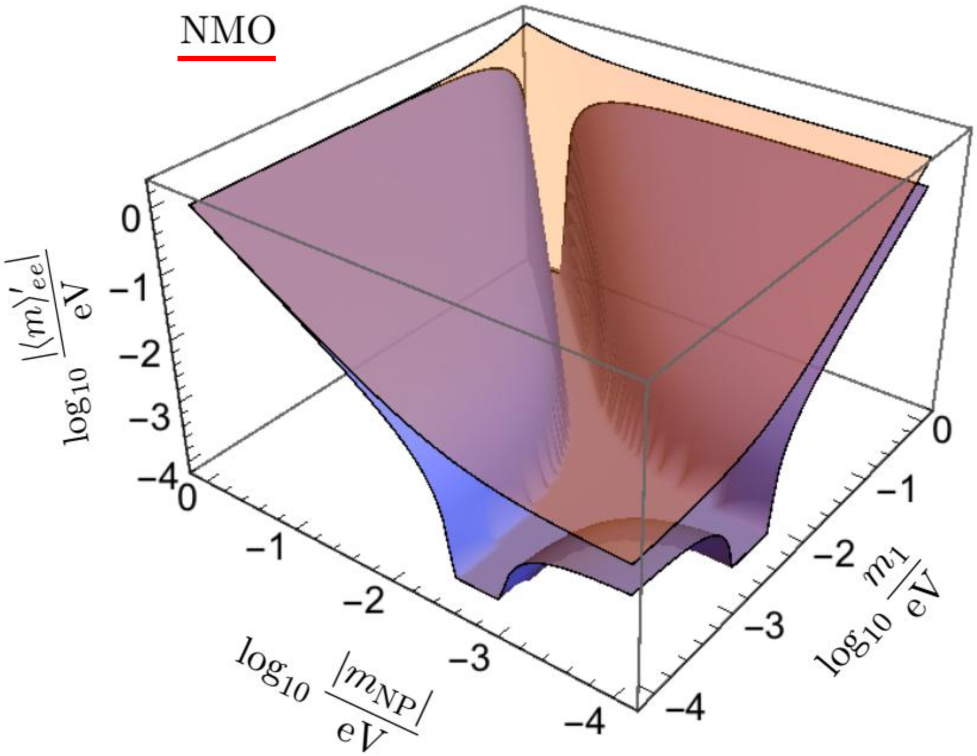
Type (B): NP has little to do with the neutrino mass issue.

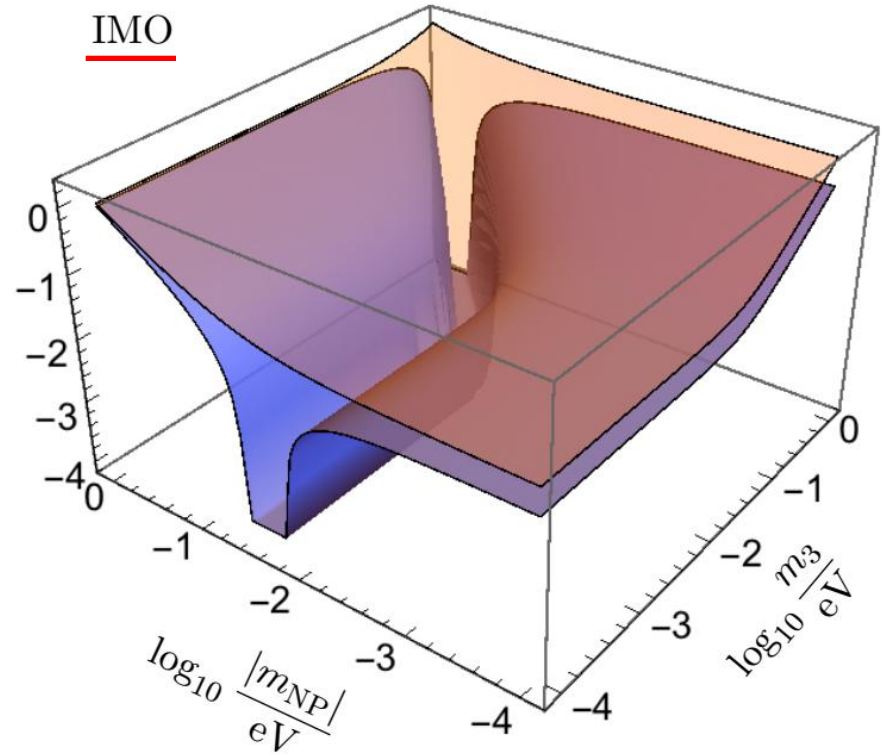
SUSY, Left-right, and some others that I don't understand

Possible effects

New physics effects:

$$\langle m\rangle'_{ee} = m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 + m_{\rm NP}$$





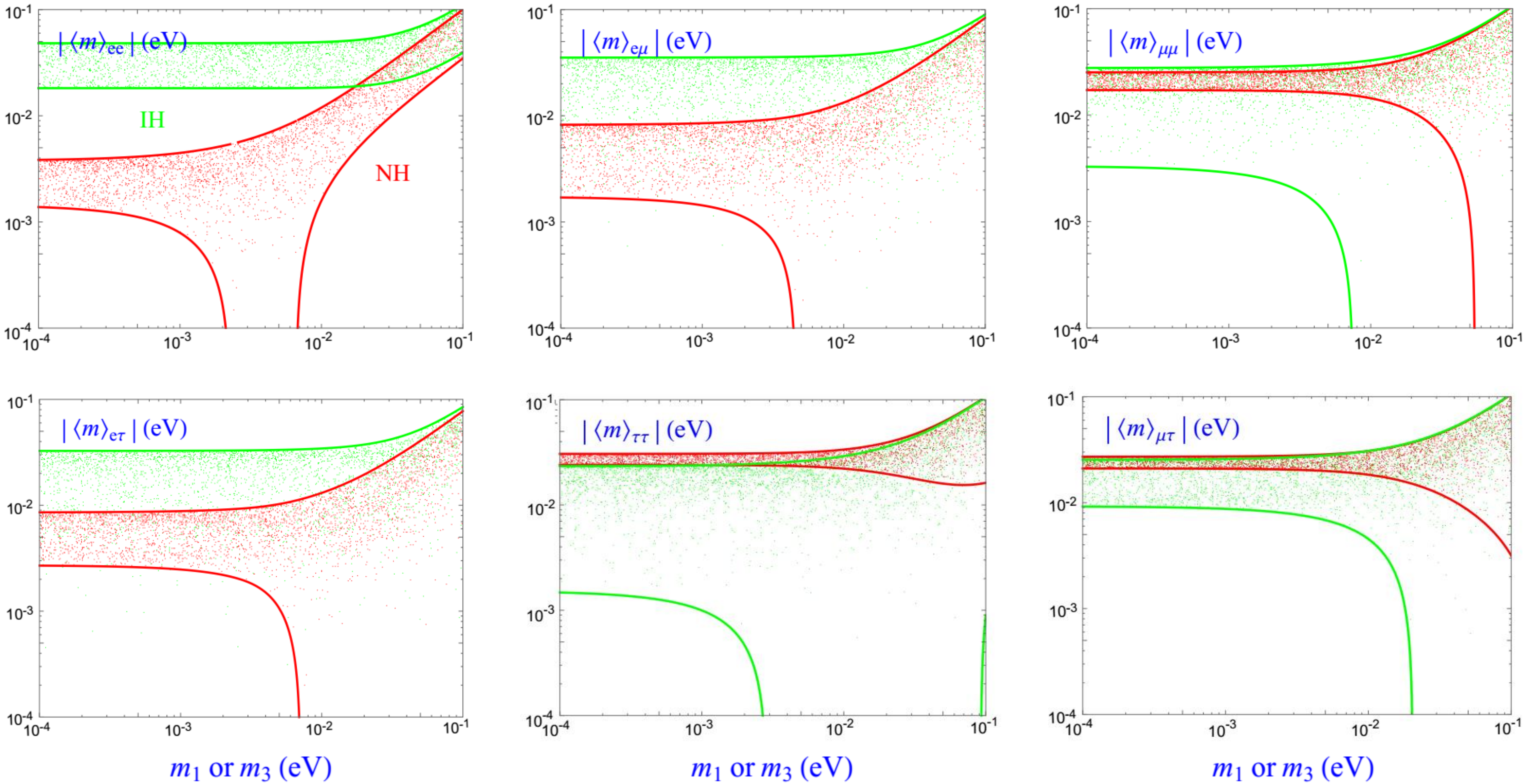
Lower bound: blue; upper bound: light orange. Clearer sensitivities to mass and phase parameters (Xing, Zhao, Zhou, arXiv:1504.05820)

$$\left|\langle m\rangle'_{ee}\right|_{\rm upper} = m_1\left|U_{e1}\right|^2 + m_2\left|U_{e2}\right|^2 + m_3\left|U_{e3}\right|^2 + \left|m_{\rm NP}\right| ,$$

$$\left|\langle m\rangle'_{ee}\right|_{\rm lower} = \max\left\{0,\ 2m_i\left|U_{ei}\right|^2 - \left|\langle m\rangle'_{ee}\right|_{\rm upper} ,\ 2\left|m_{\rm NP}\right| - \left|\langle m\rangle'_{ee}\right|_{\rm upper}\right\}$$

It is hard to tell much

More LNV processes



To identify the Majorana nature, CP-violating phases and new physics it is imperative to observe the 0ν2β decays and other lepton-number-violating processes (e.g., neutrino-antineutrino oscillations, the relic neutrino background, doubly-charged Higgs decays). None is realistic

Lecture A6

- ★ How to Generate Neutrino Mass
- ★ 3 Typical Seesaw Mechanisms
- ★ Active-sterile neutrino mixing

Hybrid mass term (1)

A **hybrid** mass term can be written out in terms of the left- and right-handed neutrino fields and their charge-conjugate counterparts:

$$-\mathcal{L}'_{\rm hybrid} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$
$$= \frac{1}{2}\begin{bmatrix}\overline{\nu_{\rm L}} & \overline{(N_{\rm R})^c}\end{bmatrix}\begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + {\rm h.c.}\ ,$$

type-(I+II) seesaw

Here we have used

$$\overline{(N_{\rm R})^c} M_{\rm D}^T (\nu_{\rm L})^c = \left[(N_{\rm R})^T \mathcal{C} M_{\rm D}^T \mathcal{C} \overline{\nu_{\rm L}}^T\right]^T = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R}$$

Diagonalization by means of a **6×6 unitary** matrix:

$$\begin{pmatrix} V & R \\ S & U\end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{pmatrix} V & R \\ S & U\end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N\end{pmatrix}$$

$$\widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\},\quad \widehat{M}_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$$

Majorana mass states

$$\nu' = \begin{bmatrix}\nu'_{\rm L} \\ (N'_{\rm R})^c\end{bmatrix} + \begin{bmatrix}(\nu'_{\rm L})^c \\ N'_{\rm R}\end{bmatrix} = \begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3 \\ N_1 \\ N_2 \\ N_3\end{pmatrix}$$

$$(\nu')^c = \nu'$$

It is actually a Majorana mass term!

$$-\mathcal{L}'_{\rm hybrid} = \frac{1}{2}\begin{bmatrix}\overline{\nu'_{\rm L}} & \overline{(N'_{\rm R})^c}\end{bmatrix}\begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N\end{pmatrix}\begin{bmatrix}(\nu'_{\rm L})^c \\ N'_{\rm R}\end{bmatrix} + {\rm h.c.}$$

$$\nu'_{\rm L} = V^\dagger \nu_{\rm L} + S^\dagger (N_{\rm R})^c$$

$$N'_{\rm R} = R^T (\nu_{\rm L})^c + U^T N_{\rm R}$$

Hybrid mass term (2)

Physical mass term:

$$-\mathcal{L}'_{\text{hybrid}} = \frac{1}{2}\overline{\nu'}\begin{pmatrix}\widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N\end{pmatrix}\nu' = \frac{1}{2}\sum_{i=1}^{3}\left(m_i\overline{\nu_i}\nu_i + M_i\overline{N_i}N_i\right)$$

Kinetic term:

$$\begin{aligned}
\mathcal{L}_{\text{kinetic}} &= i\overline{\nu_{\mathrm{L}}}\gamma_\mu\partial^\mu\nu_{\mathrm{L}} + i\overline{N_{\mathrm{R}}}\gamma_\mu\partial^\mu N_{\mathrm{R}} \\
&= \frac{i}{2}\left[\overline{\nu_{\mathrm{L}}}\ \ \overline{(N_{\mathrm{R}})^c}\right]\gamma_\mu\partial^\mu\begin{bmatrix}\nu_{\mathrm{L}} \\ (N_{\mathrm{R}})^c\end{bmatrix} + \frac{i}{2}\left[\overline{(\nu_{\mathrm{L}})^c}\ \ \overline{N_{\mathrm{R}}}\right]\gamma_\mu\partial^\mu\begin{bmatrix}(\nu_{\mathrm{L}})^c \\ N_{\mathrm{R}}\end{bmatrix} \\
&= \frac{i}{2}\left[\overline{\nu'_{\mathrm{L}}}\ \ \overline{(N'_{\mathrm{R}})^c}\right]\gamma_\mu\partial^\mu\begin{pmatrix}V & R \\ S & U\end{pmatrix}^\dagger\begin{pmatrix}V & R \\ S & U\end{pmatrix}\begin{bmatrix}\nu'_{\mathrm{L}} \\ (N'_{\mathrm{R}})^c\end{bmatrix} \\
&\quad + \frac{i}{2}\left[\overline{(\nu'_{\mathrm{L}})^c}\ \ \overline{N'_{\mathrm{R}}}\right]\gamma_\mu\partial^\mu\begin{pmatrix}V & R \\ S & U\end{pmatrix}^T\begin{pmatrix}V & R \\ S & U\end{pmatrix}^*\begin{bmatrix}(\nu'_{\mathrm{L}})^c \\ N'_{\mathrm{R}}\end{bmatrix} \\
&= \frac{i}{2}\left[\overline{\nu'_{\mathrm{L}}}\ \ \overline{(N'_{\mathrm{R}})^c}\right]\gamma_\mu\partial^\mu\begin{bmatrix}\nu'_{\mathrm{L}} \\ (N'_{\mathrm{R}})^c\end{bmatrix} + \frac{i}{2}\left[\overline{(\nu'_{\mathrm{L}})^c}\ \ \overline{N'_{\mathrm{R}}}\right]\gamma_\mu\partial^\mu\begin{bmatrix}(\nu'_{\mathrm{L}})^c \\ N'_{\mathrm{R}}\end{bmatrix} \\
&= i\overline{\nu'_{\mathrm{L}}}\gamma_\mu\partial^\mu\nu'_{\mathrm{L}} + i\overline{N'_{\mathrm{R}}}\gamma_\mu\partial^\mu N'_{\mathrm{R}} \\
&= \frac{i}{2}\overline{\nu'}\gamma_\mu\partial^\mu\nu' = \frac{i}{2}\sum_{k=1}^{3}\left(\overline{\nu_k}\gamma_\mu\partial^\mu\nu_k + \overline{N_k}\gamma_\mu\partial^\mu N_k\right)
\end{aligned}$$

Non-unitary flavor mixing

Weak charged-current interactions of leptons:

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

$$\nu_{\rm L} = V\nu'_{\rm L} + R(N'_{\rm R})^c$$

In the mass basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu \left[V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

V = **non-unitary light neutrino mixing (PMNS) matrix** $VV^\dagger + RR^\dagger = 1$

R = **light-heavy neutrino mixing (CC interactions of heavy neutrinos)**

Neutrino oscillations ***V*** ⟺ ***R*** **Collider signatures**

TeV seesaws may bridge the gap between neutrino & collider physics.

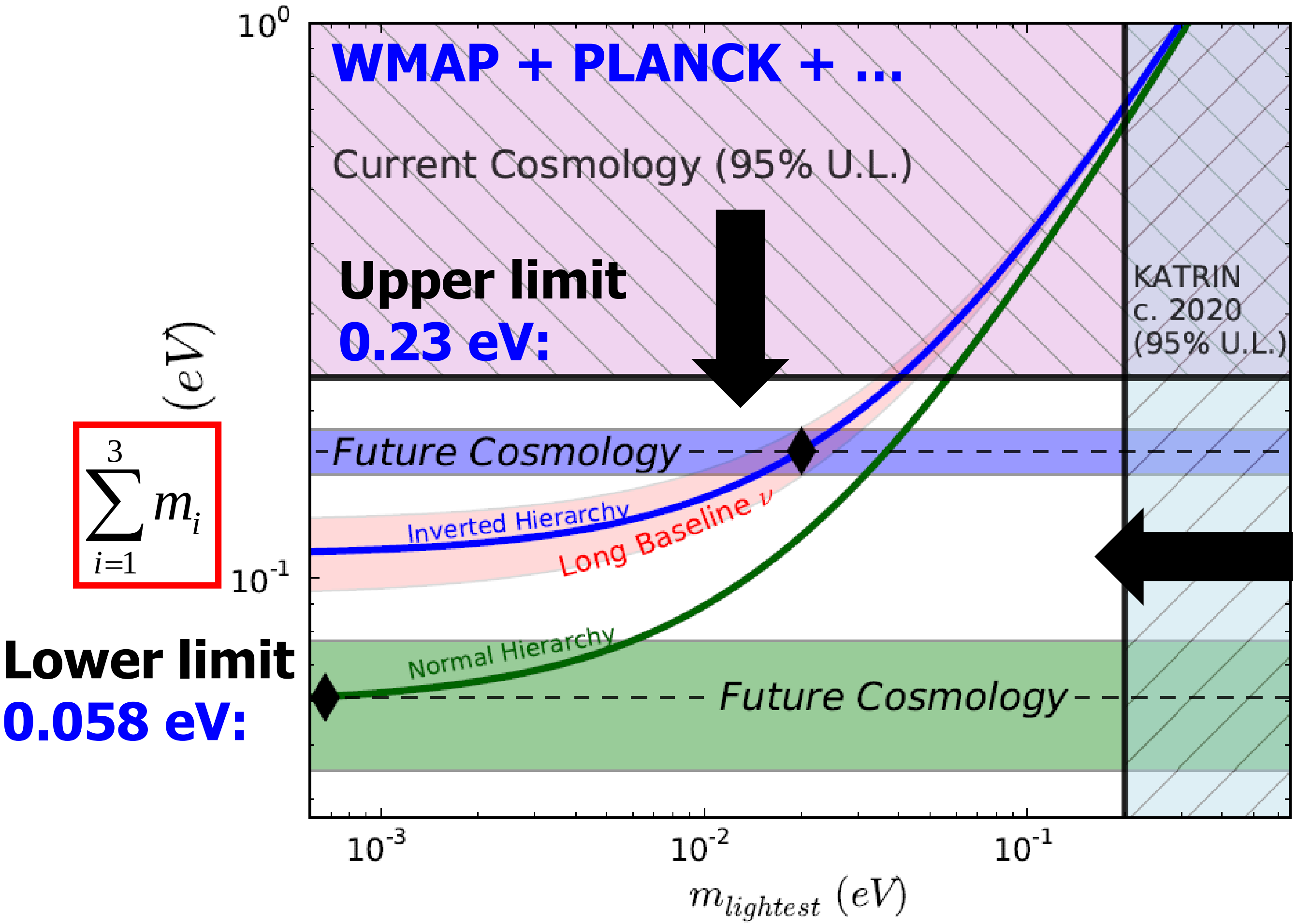
Neutrino mass scale

Three ways: the β decay, the 0νββ decay, and cosmology (CMB + LSS).

$$\langle m\rangle_e^2 = \sum_{i=1}^{3} m_i^2 |U_{ei}|^2$$

$$|\langle m\rangle_{ee}| = \left|\sum_{i=1}^{3} m_i U_{ei}^2\right|$$

$$\sum_{i=1}^{3} m_i$$



mass scale ≤ O(0.1) eV

Why so tiny?

arXiv:1309.5383

Stage-4 CMB

$$\sigma\left(\sum m_\nu\right) = 16\ \text{meV}$$

$$\sigma(N_{\text{eff}}) = 0.020\,.$$

Seesaw mechanisms (1)

A hybrid mass term may have three distinct components:

$$-\mathcal{L}'_{\rm hybrid} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$
$$= \frac{1}{2}\begin{bmatrix}\overline{\nu_{\rm L}} & \overline{(N_{\rm R})^c}\end{bmatrix}\begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + {\rm h.c.}\ ,$$

- ♣ **Normal Dirac mass term**, proportional to the scale of electroweak symmetry breaking (~ **174 GeV**);
- ♣ **Light Majorana mass term**, violating the SM gauge symmetry and much lower than **174 GeV** ('t Hooft's naturalness criterion);
- ♣ **Heavy Majorana mass term**, originating from the SU(2)_L singlet and having a scale much higher than **174 GeV**.

A strong hierarchy of 3 mass scales allows us to make approximation

$$\begin{pmatrix} V & R \\ S & U\end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{pmatrix} V & R \\ S & U\end{pmatrix}^* = \begin{pmatrix}\widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N\end{pmatrix}$$

Seesaw mechanisms (2)

The above unitary transformation leads to the following relationships:

$$R\widehat{M}_N = M_{\rm L} R^* + M_{\rm D} U^*$$
$$S\widehat{M}_\nu = M_{\rm D}^T V^* + M_{\rm R} S^*$$

$$M_{\rm R} \gg M_{\rm D} \gg M_{\rm L}$$
$$R \sim S \sim \mathcal{O}(M_{\rm D}/M_{\rm R})$$

$$U\widehat{M}_N = M_{\rm R} U^* + M_{\rm D}^T R^*$$
$$V\widehat{M}_\nu = M_{\rm L} V^* + M_{\rm D} S^*$$

$$U\widehat{M}_N U^T = M_{\rm R}(UU^\dagger)^T + M_{\rm D}^T(R^* U^T) \approx M_{\rm R} \; ,$$
$$V\widehat{M}_\nu V^T = M_{\rm L}(VV^\dagger)^T + M_{\rm D}(S^* V^T) \approx M_{\rm L} + M_{\rm D}(S^* V^T)$$

$$S^* V^T = M_{\rm R}^{-1} S\widehat{M}_\nu V^T - M_{\rm R}^{-1} M_{\rm D}^T (VV^\dagger)^T \approx -M_{\rm R}^{-1} M_{\rm D}^T$$

Then we arrive at the type-(I+II) seesaw formula:

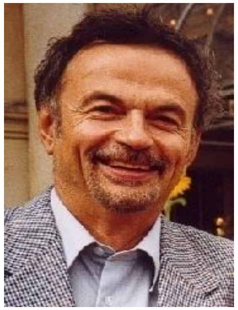
$$M_\nu \equiv V\widehat{M}_\nu V^T \approx M_{\rm L} - M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$

Type-I seesaw limit: $M_\nu \approx -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$ **(Fritzsch, Gell-Mann, Minkowski, 1975; Minkowski, 1977; ...)**

Type-II seesaw limit: $M_\nu = M_{\rm L}$ **(Konetschny, Kummer, 1977; ...)**

History of type-I seesaw

The seesaw idea originally appeared in a paper's footnote.



Seesaw—A Footnote Idea:

H. Fritzsch, M. Gell-Mann,

P. Minkowski, PLB 59 (1975) 256

This idea was very clearly elaborated by Minkowski in Phys. Lett. B 67 (1977) 421 ---- but it was unjustly forgotten until 2004.

The idea was later on embedded into the GUT frameworks in 1979 and 1980:

- **T. Yanagida 1979**
- **M. Gell-Mann, P. Ramond, R. Slansky 1979**
- **S. Glashow 1979**
- **R. Mohapatra, G. Senjanovic 1980**

It was Yanagida who named this mechanism as "seesaw".

What is History?

History is a set of lies agreed upon

Napoleon Bonaparte

Summary of 3 seesaws

Type-I seesaw: SM + right-handed neutrinos + L violation
(Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slansky 79; Mohapatra, Senjanovic 80)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

Type-II seesaw: SM + 1 Higgs triplet + L violation
(Konetschny, Kummer 77; Magg, Wetterich 80; Schechter, Valle 80; Cheng, Li 80; Lazarides et al 80)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2}\overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} - \lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + {\rm h.c.}$$

Type-III seesaw: SM + 3 triplet fermions + L violation
(Foot, Lew, He, Joshi 1989)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2}{\rm Tr}\left(\overline{\Sigma} M_\Sigma \Sigma^c\right) + {\rm h.c.}$$

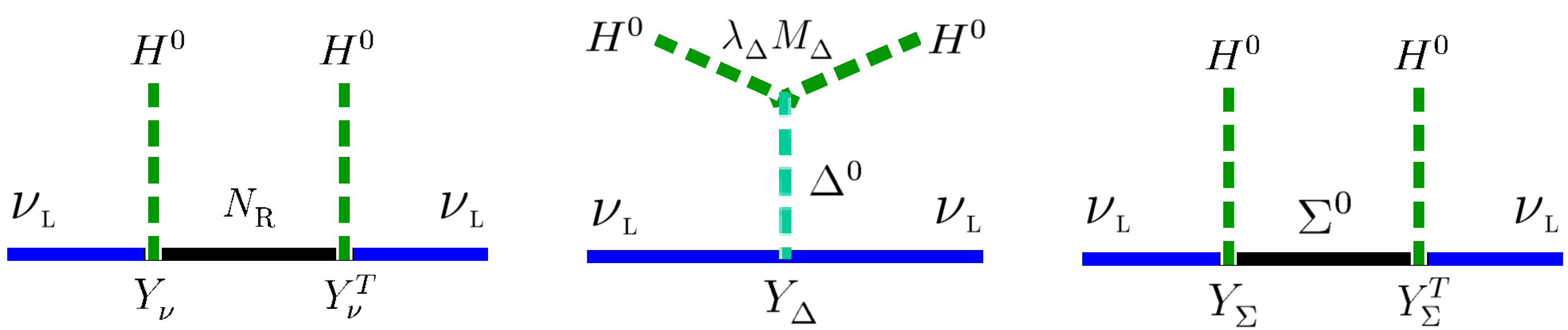
Effective mass term

Weinberg (1979): the unique dimension-five operator of ν-masses after integrating out heavy degrees of freedom.

$$\frac{\mathcal{L}_{d=5}}{\Lambda} = \begin{cases} \frac{1}{2}\left(Y_\nu M_R^{-1} Y_\nu^T\right)_{\alpha\beta} \overline{l_{\alpha L}} \tilde{H} \tilde{H}^T l^c_{\beta L} + \text{h.c.} & \text{(Type 1)} \\ -\frac{\lambda_\Delta}{M_\Delta} \left(Y_\Delta\right)_{\alpha\beta} \overline{l_{\alpha L}} \tilde{H} \tilde{H}^T l^c_{\beta L} + \text{h.c.} & \text{(Type 2)} \\ \frac{1}{2}\left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T\right)_{\alpha\beta} \overline{l_{\alpha L}} \tilde{H} \tilde{H}^T l^c_{\beta L} + \text{h.c.} & \text{(Type 3)} \end{cases}$$

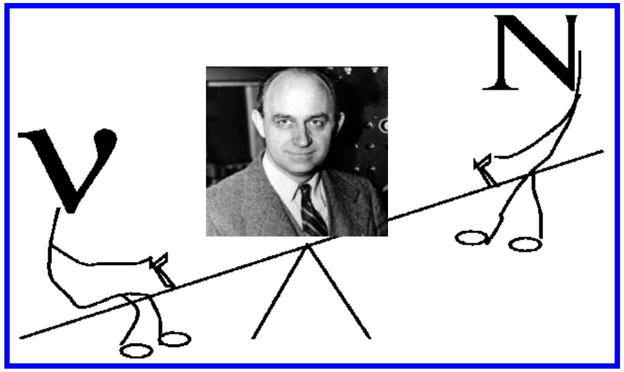
$$M_\nu = \begin{cases} -\frac{1}{2} Y_\nu \frac{v^2}{M_R} Y_\nu^T & \text{(Type 1)} \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & \text{(Type 2)} \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & \text{(Type 3)} \end{cases}$$

After SSB, a Majorana mass term is $-\mathcal{L}_{\text{mass}} = \frac{1}{2} \overline{\nu_L} M_\nu \nu^c_L + \text{h.c.}$ $\qquad \langle \tilde{H} \rangle = v/\sqrt{2}$



Seesaw scale?

What is the scale at which the seesaw mechanism works?



Planck

GUT **to unify strong, weak & electromagnetic forces**

Conventional Seesaws: heavy degrees of freedom near GUT

This appears to be rather reasonable, since one often expects new physics to appear around a fundamental scale

Naturalness √

Testability ✕

Fermi

Uniqueness ✕

Hierarchy ✕

TeV Neutrino Physics?

to discover the SM Higgs boson √

to verify Yukawa interactions √

to pin down heavy seesaw particles

to test seesaw mechanism(s)

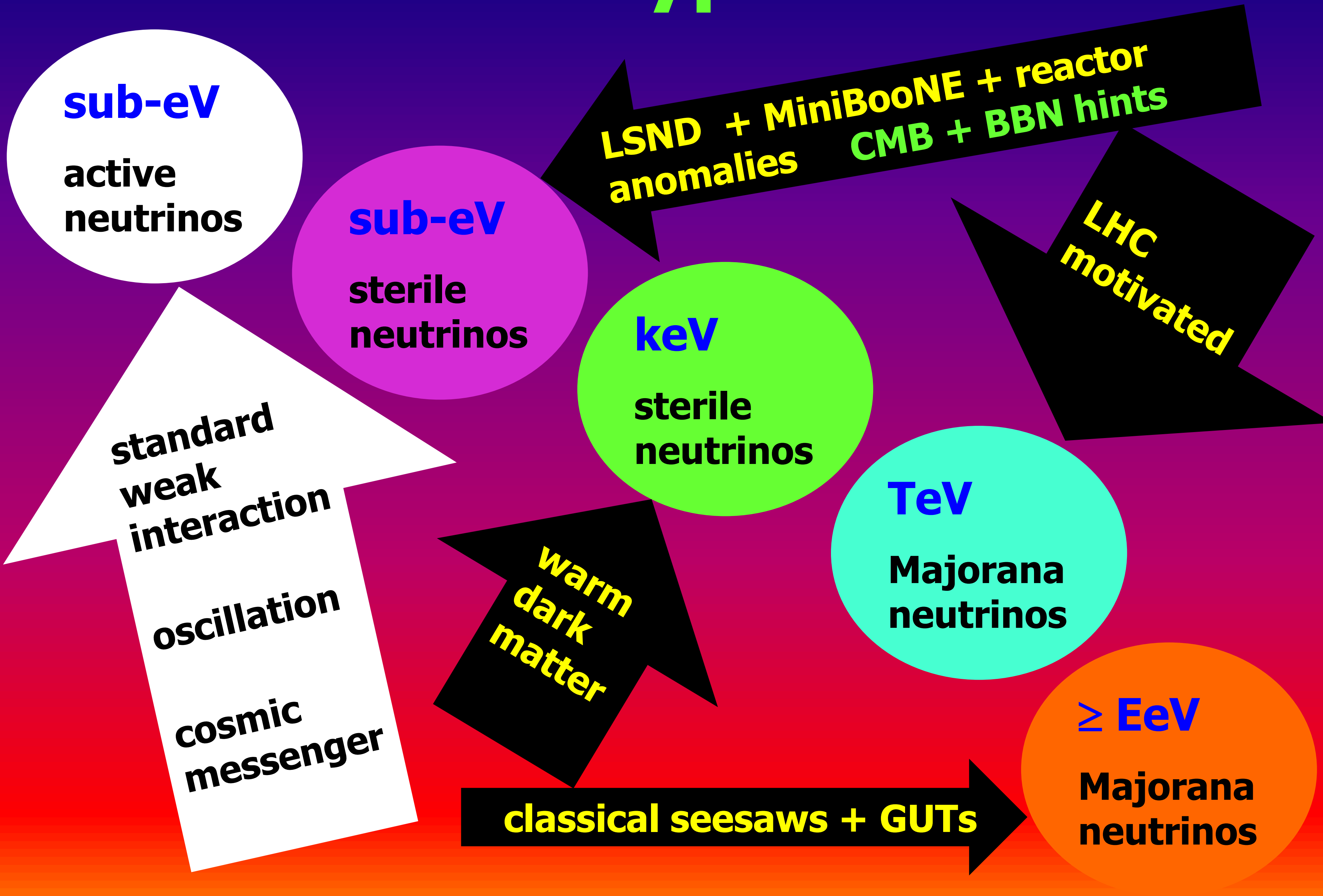
to measure low-energy effects

Why

Not

Try

Real + Hypothetical ν's



(3+3) flavor mixing

active flavor

sterile flavor

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_x \\ \nu_y \\ \nu_z \end{pmatrix} = \mathcal{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix}$$

mass state

A full parametrization

$$\mathcal{U} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

sterile part **interplay** **active part**

$$\begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} = O_{23} O_{13} O_{12} \; ,$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} = O_{56} O_{46} O_{45} \; ,$$

$$\begin{pmatrix} A & R \\ S & B \end{pmatrix} = O_{36} O_{26} O_{16} O_{35} O_{25} O_{15} O_{34} O_{24} O_{14}$$

Full parametrization:

15 rotation angles

15 phase phases

Xing, arXiv:1110.0083

Questions

1) Do you feel happy / painful / sorry to introduce sterile neutrinos into the SM (remember Weinberg's theorem)?

2) How many species of sterile neutrinos should be taken into account for your this or that purpose? 1? 2? 3? ....?

3) If all the current experimental and observational hints disappear, will the sterile neutrino physics still survive?

4) Do you like to throw many stones to only kill few birds or just the opposite? And is this a very stupid question?

Weinberg's *3rd law of progress in theoretical physics* (83):

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry .... What could be better?