Neutrinos——The Basics & Hot Topics

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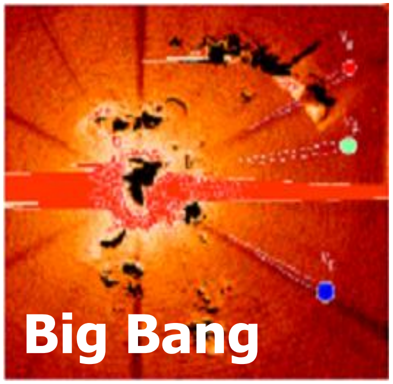
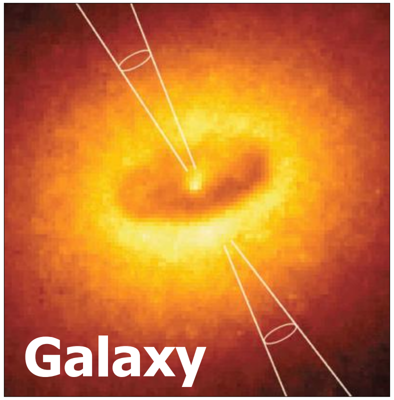
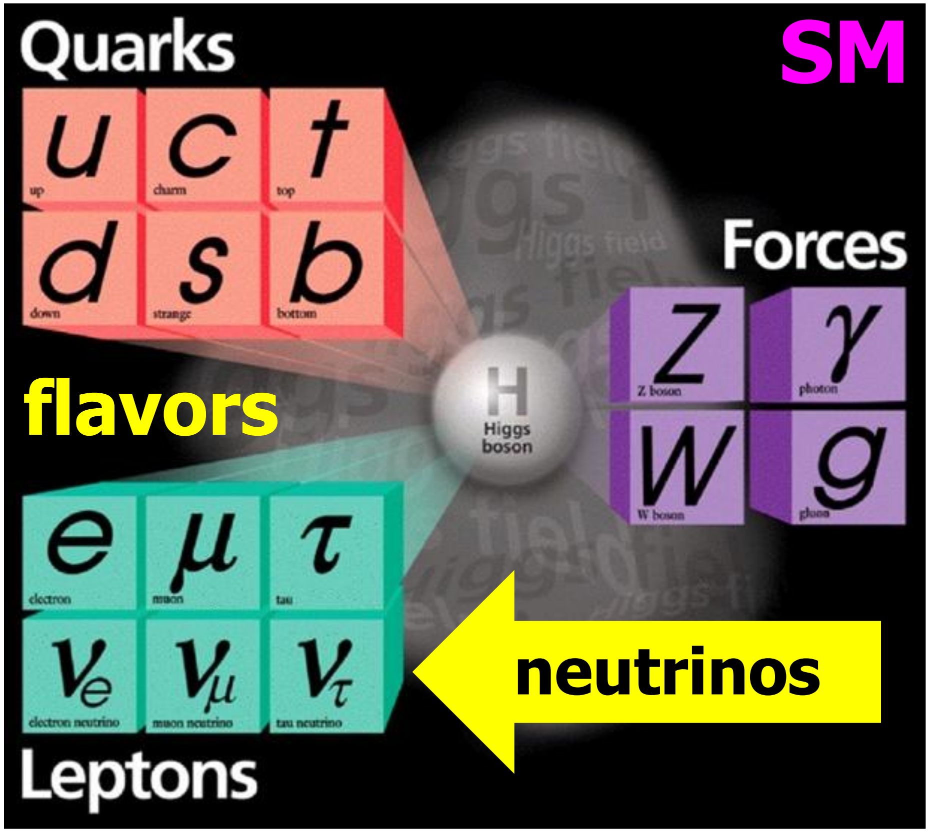
- ★ A brief history of neutrinos
- ★ Basic neutrino interactions
- ★ Dirac and Majorana masses
- ★ Flavor mixing & CP violation
- ★ Oscillation phenomenology
- ★ Neutrinoless double-β decay
- ★ Typical seesaw mechanisms
- ★ Two types of cosmic neutrinos
- ★ Matter-antimatter asymmetry

Neutrinos: sooooooo special?







Properties:

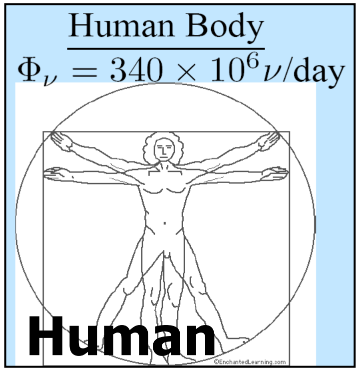
charge = 0

spin = ½

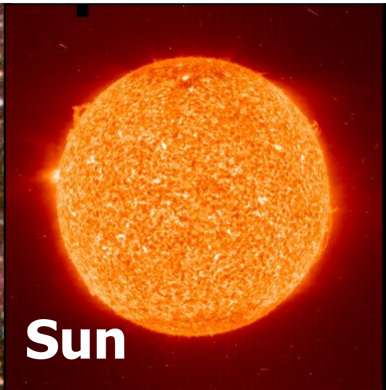
mass = 0

speed = c

Left-handed


Human Body

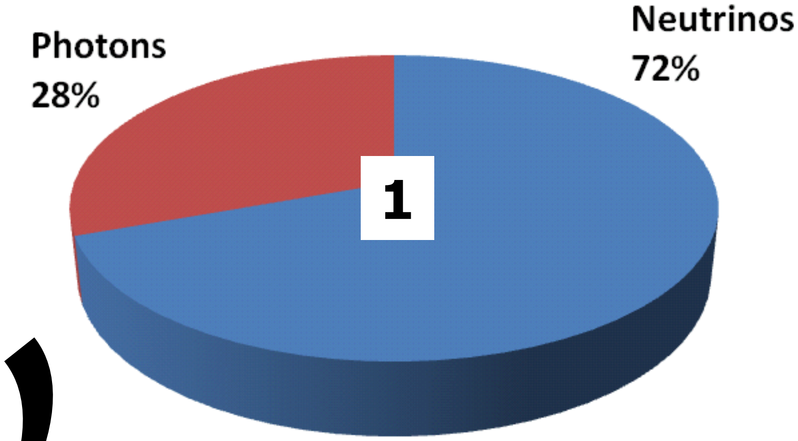
$\Phi_\nu = 340 \times 10^6 \nu/\text{day}$



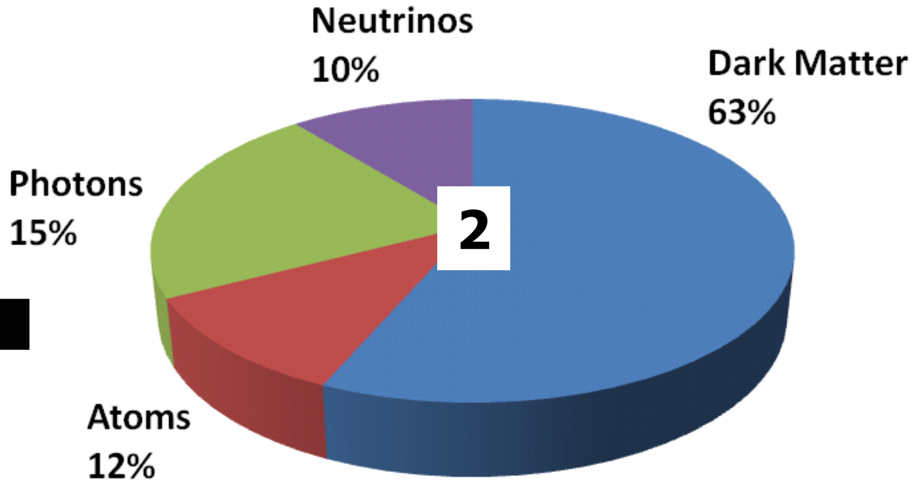
Neutrinos:

witness and participant in the evolution of the Universe

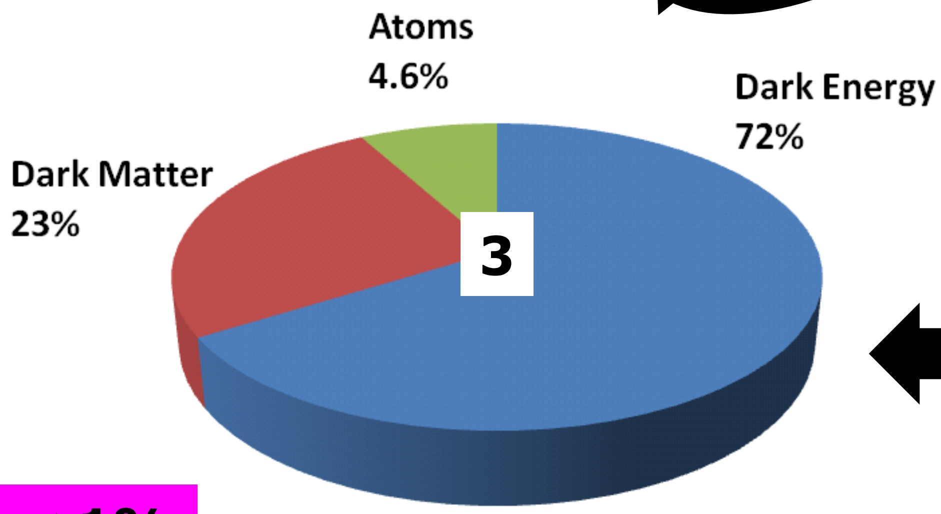
ν ν ν



neutrino decoupling
t = 1 second



photon decoupling
t = 380 000 years



Today
t = 13.7 billion years

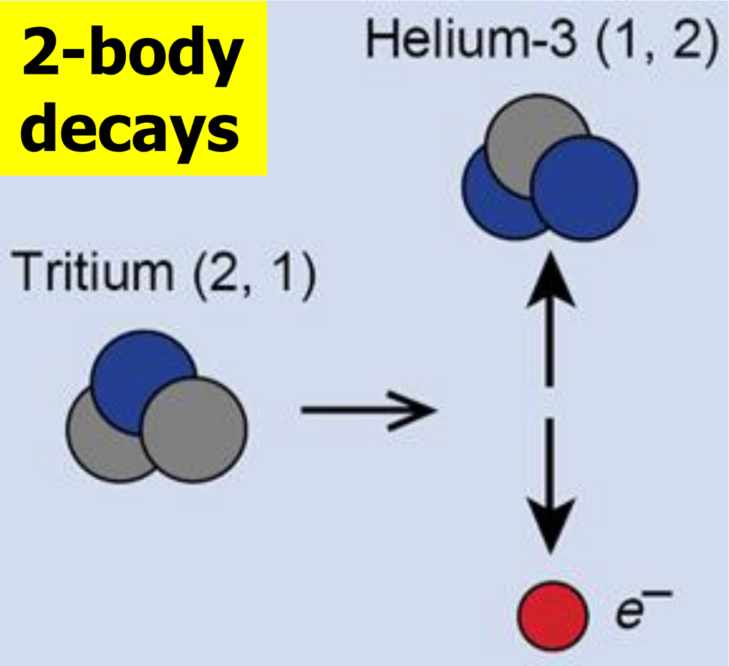
< 1%

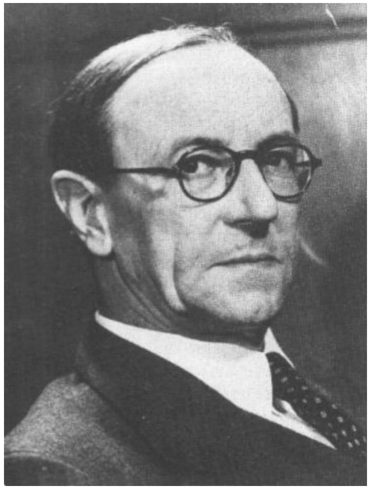
Lecture A1

- ★ Neutrinos from new physics
- ★ Interactions and discoveries
- ★ Three families of the leptons

Beta decays in 1930

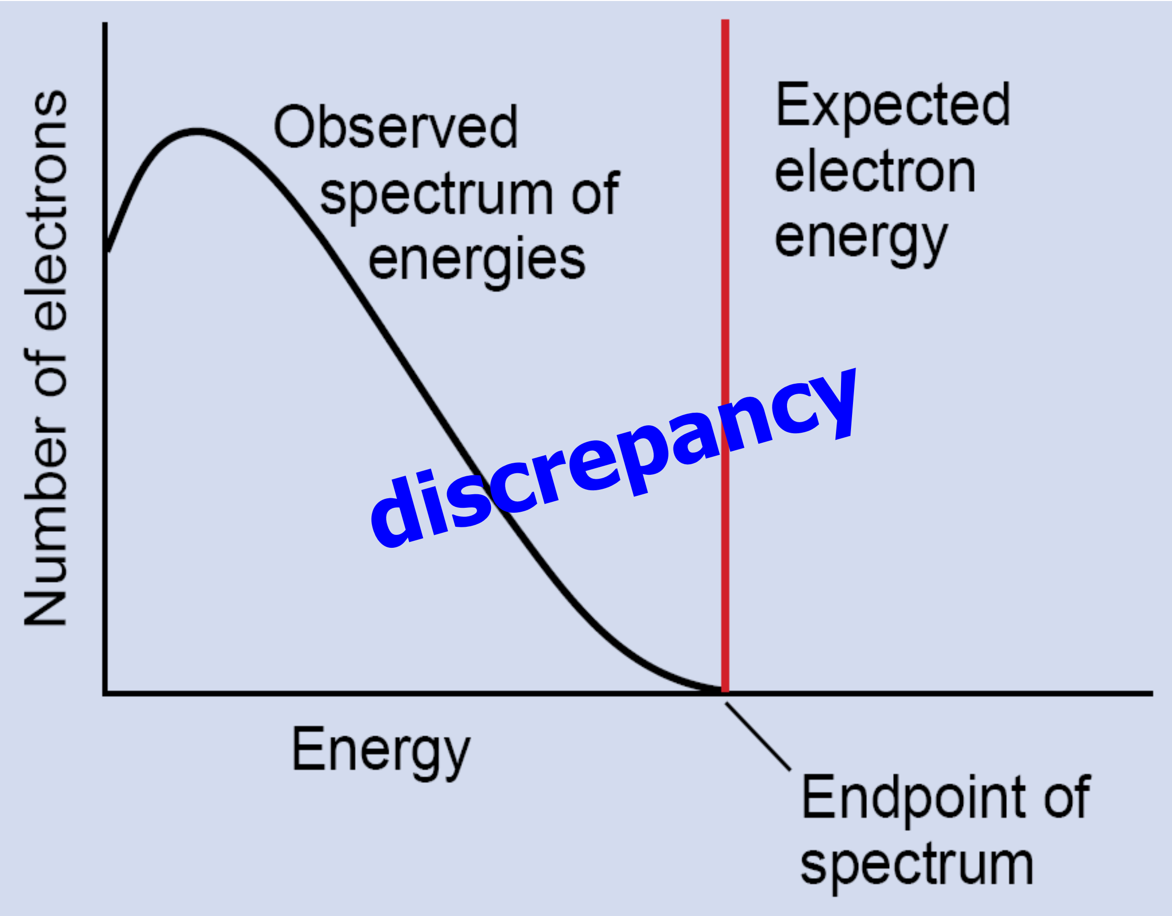
2-body decays





J. Chadwick 1914/C. Ellis 1920-1927

Energy crisis = New physics ?



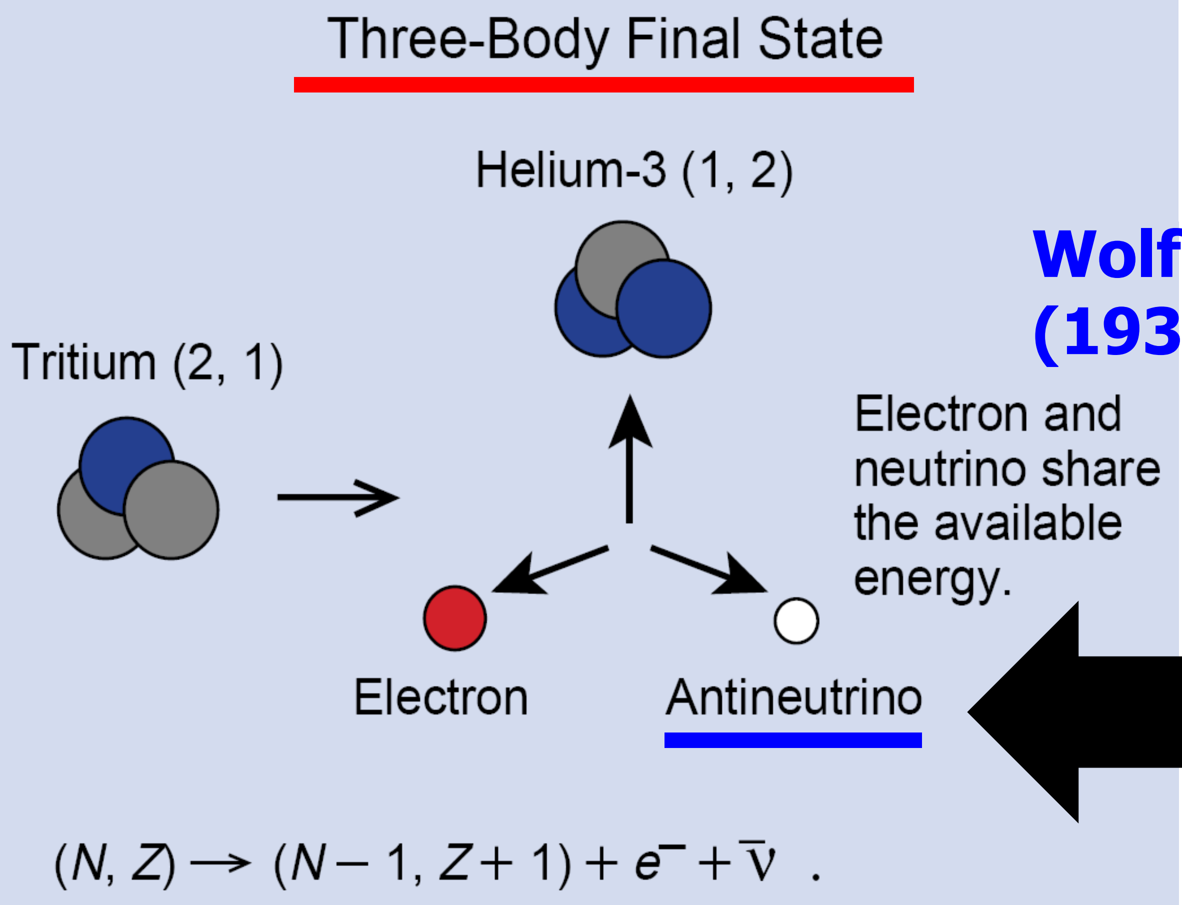
What to do?

Two ways out?

Niels Bohr

- ♣ **giving up sth**
- ♣ **adding in sth**

Wolfgang Pauli (1930)



Three-Body Final State

$(N, Z) \rightarrow (N-1, Z+1) + e^{-} + \bar{\nu}$.

Pauli put forward this idea in a letter instead of a paper.....

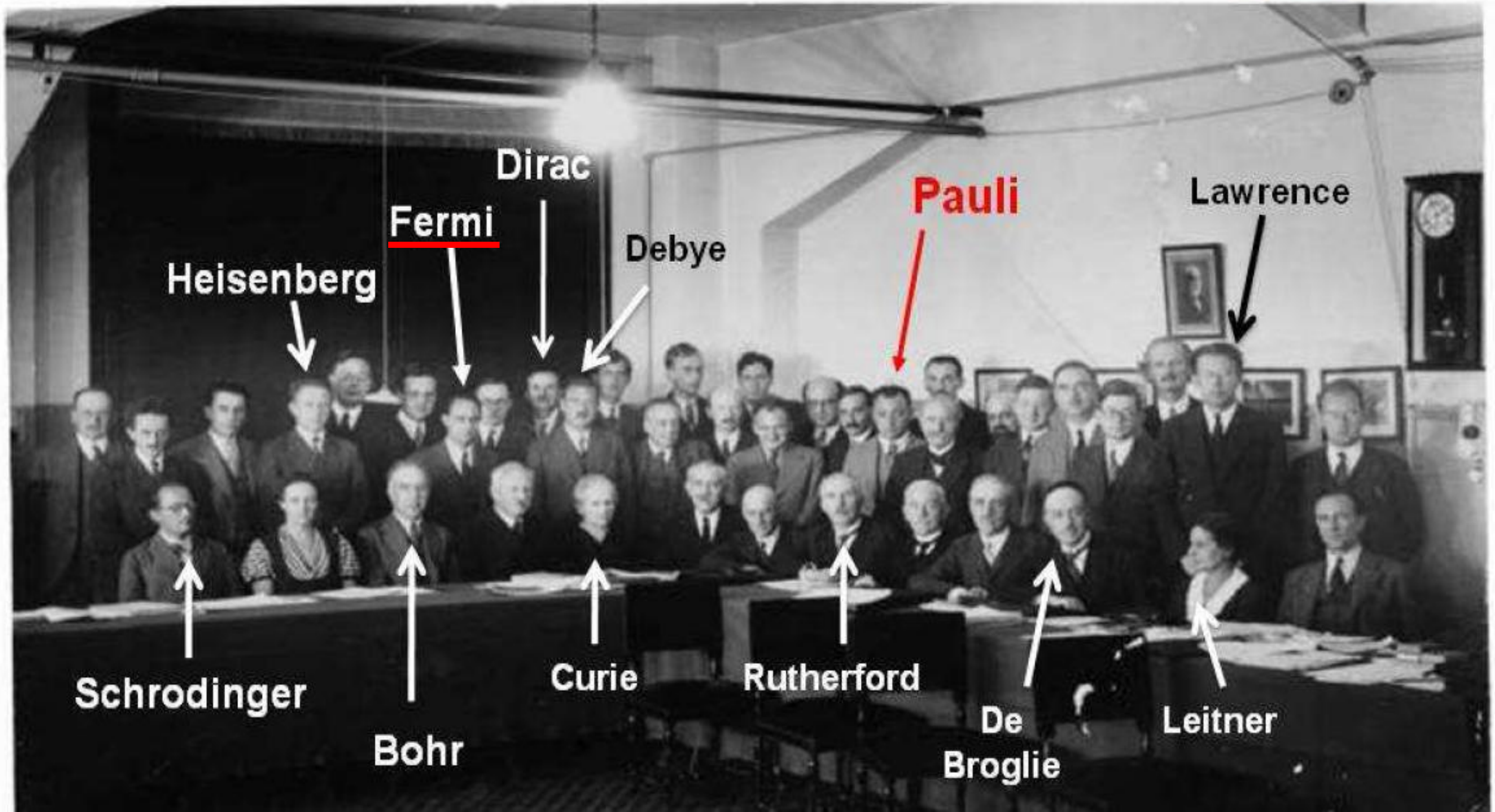
Solvay 1933

Pauli participated + sold his neutrino idea in this congress

INSTITUT INTERNATIONAL DE PHYSIQUE SOLVAY

SEPTIÈME CONSEIL DE PHYSIQUE -- BRUXELLES, 22-29 OCTOBRE 1933

22 – 29 Octobre 1933



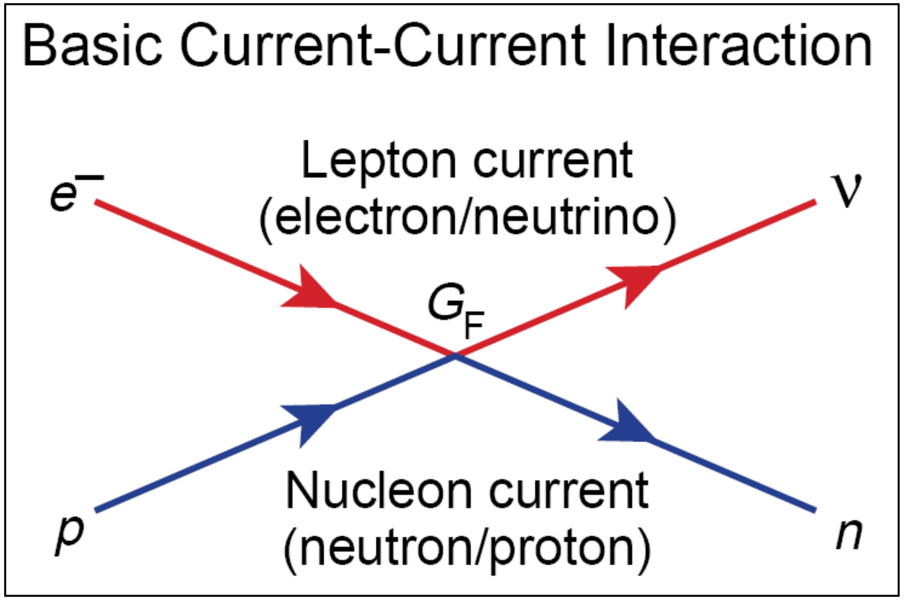
Fermi's theory

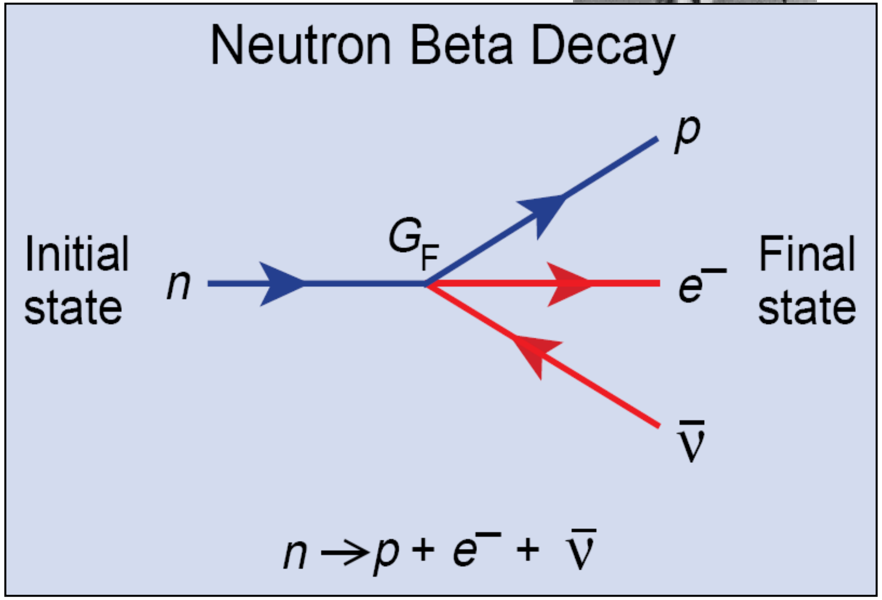
Enrico Fermi assumed a new force for β decay by combining 3 new concepts:

I will be remembered for this paper.

------ Fermi in Italian Alps, Christmas 1933

- ★ **Pauli's idea: neutrinos**
- ★ **Dirac's idea: creation of particles**
- ★ **Heisenberg's idea: isospin symmetry**





Fermi’s paper

E. Fermi's publications on the Weak Interaction

REJECTED

E. Fermi, “Tentative Theory of Beta Rays”
Letter Submitted to Nature (1933)

31 Dec, 1933

ANNO IV - VOL. II - N. 12 QUINDICINALE 31 DICEMBRE 1933 - XII

LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL' ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi “beta”

Nota del prof. ENRICO FERMI

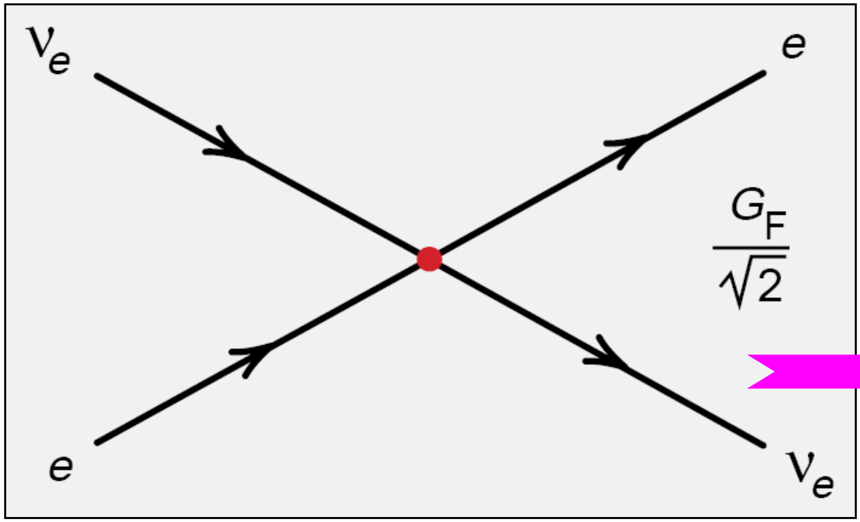
Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

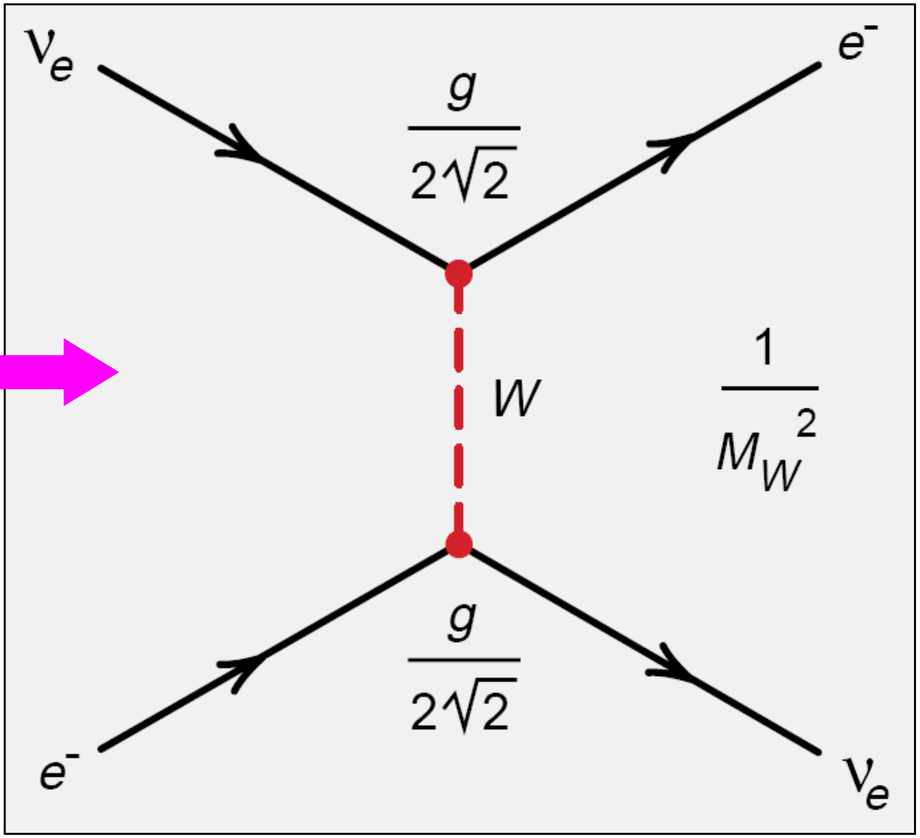
This is Fermi’s best theoretical work!
---- C.N. Yang

Published first in this journal and later in Z. Phys. in 1934

Weak interactions

From Fermi's current-current interaction to weak charged-current gauge interactions (exercise: g).





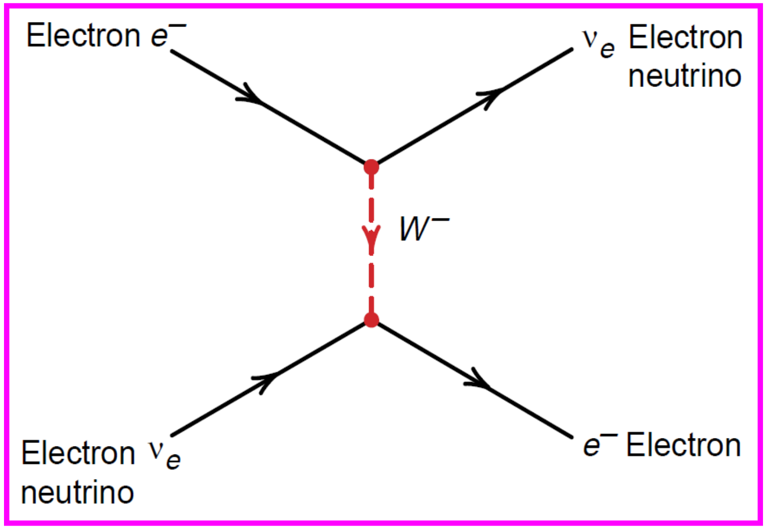
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

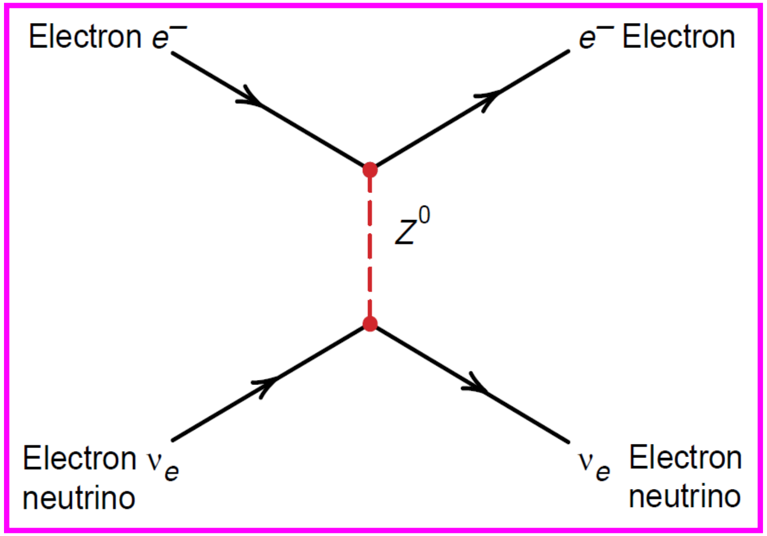
$$M_W = 80.4\,\text{GeV}$$

$$G_F = 1.66 \times 10^{-5}\ \text{GeV}^{-2}$$

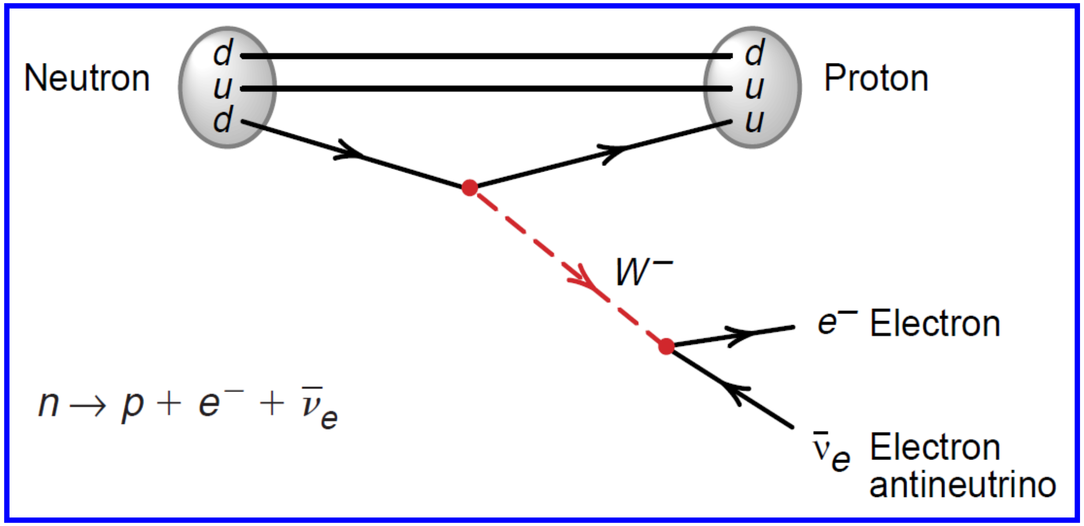
Weak interactions

Electron-neutrino scattering

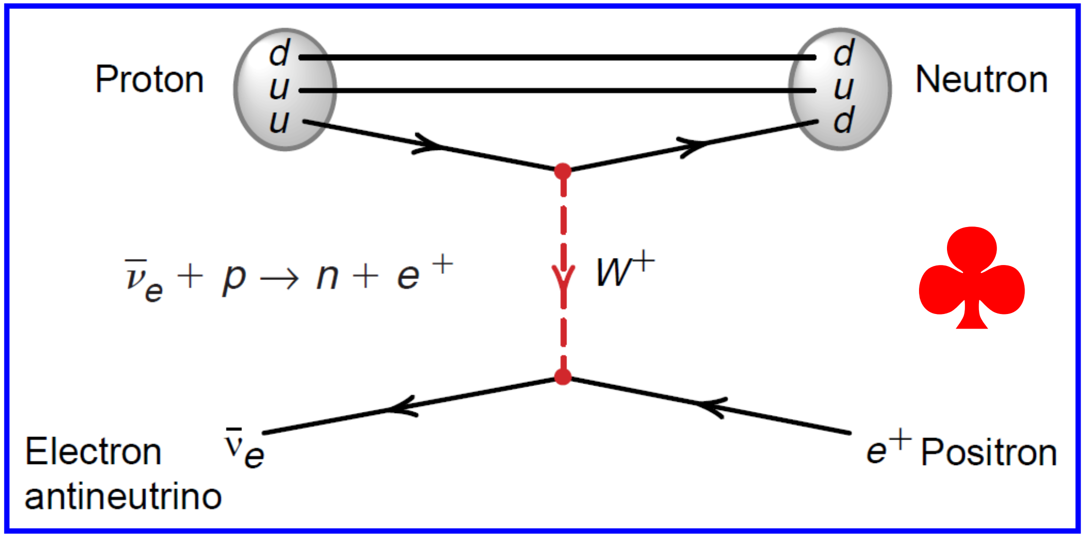




Neutron β decay / inverse β decay



$$n \rightarrow p + e^- + \bar{\nu}_e$$



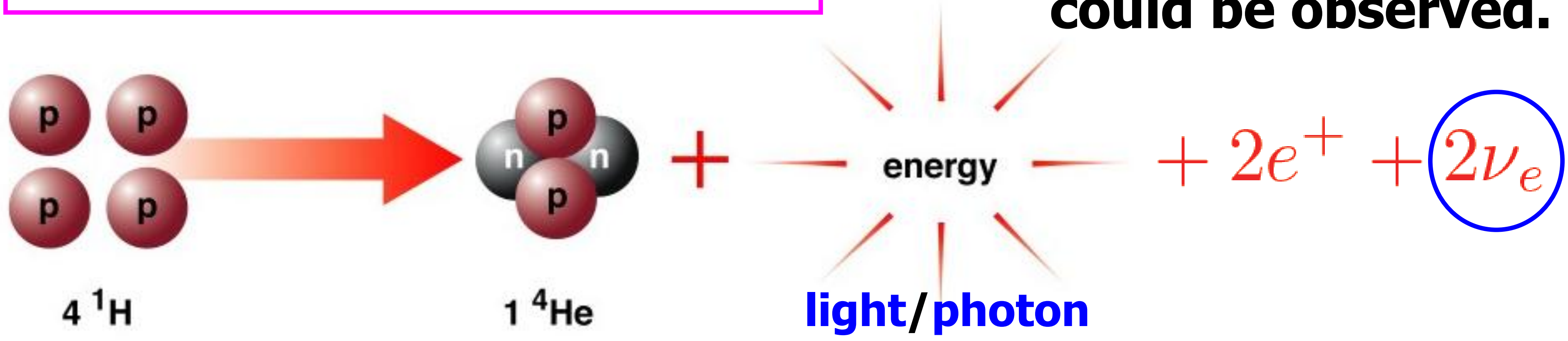
$$\bar{\nu}_e + p \rightarrow n + e^+$$

Exercise: **draw an electron-antineutrino scattering Feynman diagram.**

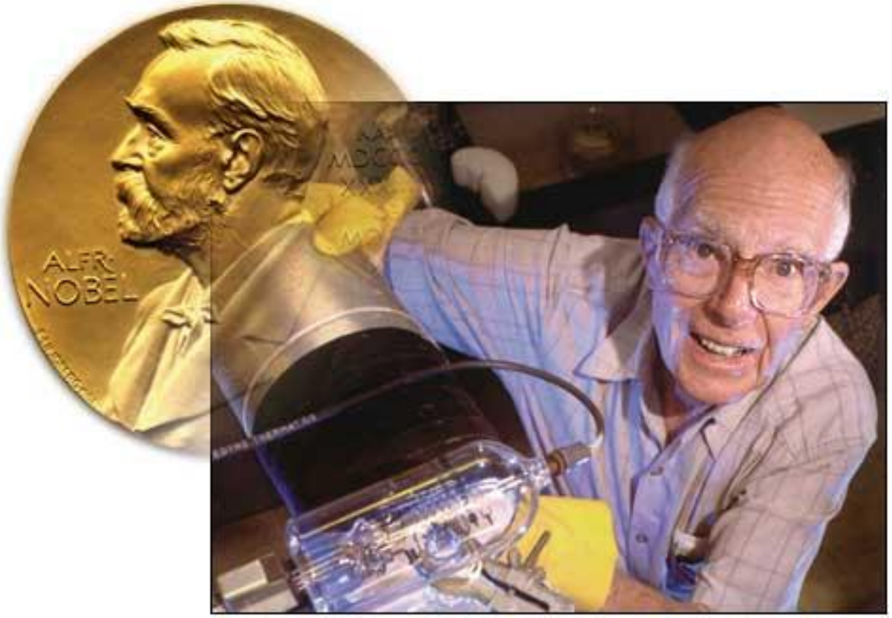
Why the sun shines?

$$\boxed{4p \to {}^{4}\mathrm{He} + 2\mathrm{e}^{+} + 2\nu_e + 26.73\ \mathrm{MeV}}$$

Only the neutrinos could be observed.



Hans Bethe (1939), George Gamow & Mario Schoenberg (1940, 1941)

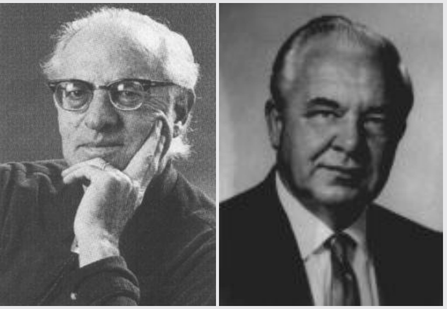


Raymond Davis: born in 1914, discovery in 1968 and Nobel Prize in 2002

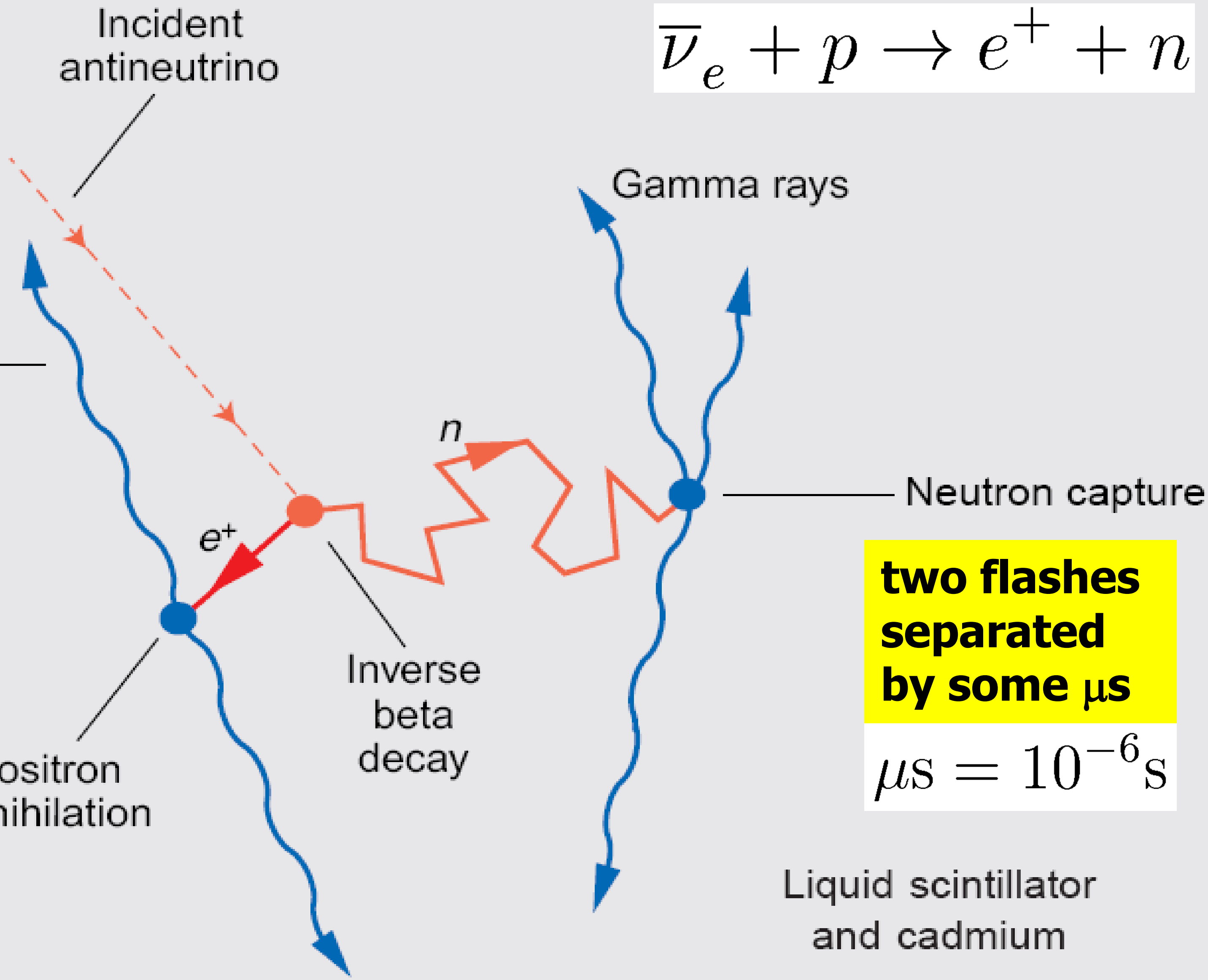
Observed the solar neutrino and its anomaly in 1968

Neutrinos in 1956

F. Reines and C. Cowan detected reactor antineutrinos via



$$\overline{\nu}_e + p \to e^+ + n$$



two flashes separated by some μs

$$\mu\text{s} = 10^{-6}\text{s}$$

Exercise: **请上网调研，什么是液体闪烁体？**

Positive result?

Reines and Cowan's telegram to Pauli on 14/06/1956:

We're happy to inform you that we've definitely detected neutrinos from fission fragments by observing inverse β decay of protons. Observed cross section agrees well with expected $6\times10^{-44}\,\mathrm{cm}^2$. **(Pauli didn't reply, a case of champagne)**

Such a theoretical value was based on a parity-conserving formulation of the β decay with 4 independent degrees of freedom for ν's.

$$\boxed{\sigma(\overline{\nu}_e p) = \sigma(\nu_e n) \approx 9.1\times10^{-44}\left(\frac{E_\nu}{\mathrm{MeV}}\right)^2 \ \mathrm{cm}^2}$$

This value is at least doubled after the discovery of parity violation in 1957, leading to the two-component neutrino theory in 1957 and the V–A weak theory in 1958.

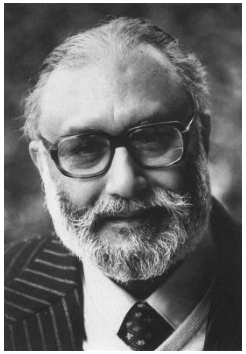
Neutrinos in 1957

The neutrino should have no mass: 2-component ν theory

★ **Abdus Salam**

received 15/11/1956,

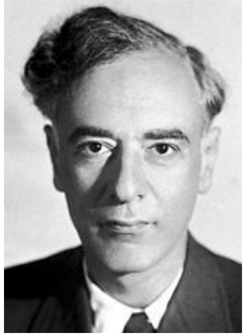
Nuovo Cim. 5 (1957) 299



★ **Lev Landau**

received 9/1/1957,

Nucl. Phys. 3 (1957) 127



★ **T.D. Lee**, **C.N. Yang**

received 10/1/1957,

Phys. Rev. 105 (1957) 1671

John Ward wrote to **Salam**:

So many congratulations and fond hopes for at least one-third of a Nobel Prize.

——— **Norman Bombey** in "Abdus Salam: How to Win the Nobel Prize", Preprint arXiv:1109.1972 (9/2011).

Bruno Pontecorvo challenged the massless ν theory in 1957

Reines’ excuse

A new paper on this experiment published in Phys. Rev. in 1960 reported a cross section twice as large as that given in 1956.

Reines (1979): our initial analysis grossly overestimated the detection efficiency with the result that the measured cross section was at first thought to be in good agreement with [the pre-parity violation] prediction.

Skepticism

{over 39 years}

The Nobel Prize finally came to Frederick Reines in 1995!

Pontecorvo's idea

★ Theory of the Symmetry of Electrons and Positrons

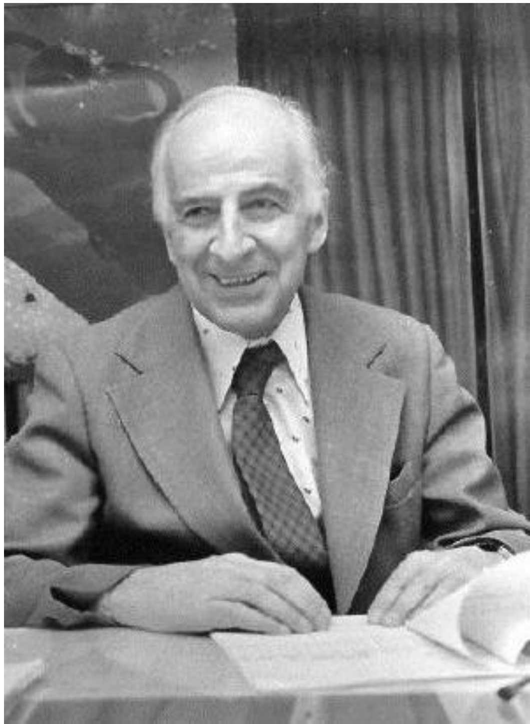
Ettore Majorana

Nuovo Cim. 14 (1937) 171

Are massive neutrinos and antineutrinos identical or different —— a fundamental puzzling question in particle physics.

★ Mesonium and Anti-mesonium

Bruno Pontecorvo



Zh. Eksp. Teor. Fiz. 33 (1957) 549
Sov. Phys. JETP 6 (1957) 429

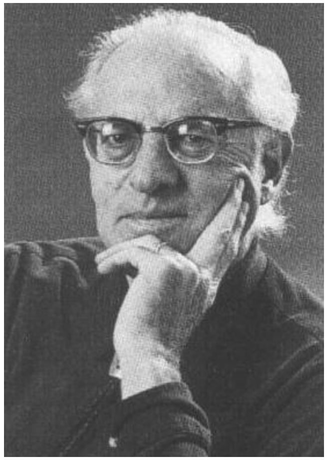
If the two-component neutrino theory turned out to be incorrect and if the conservation law of neutrino charge didn't apply, then neutrino-antineutrino transitions would in principle be possible to take place in vacuum.

Electron and its neutrino

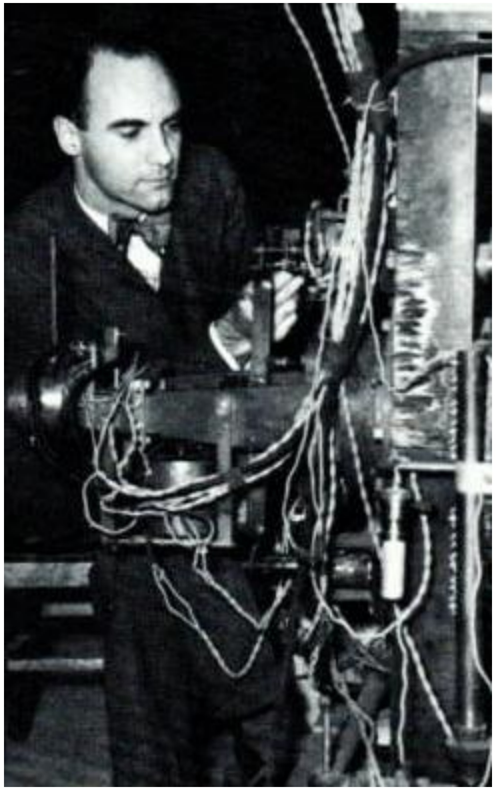
The **electron** was discovered in 1897, by **Joseph Thomson**.

The electron's anti-particle, **positron**, was predicted by **Paul Dirac** in 1928, and discovered by **Carl Anderson** in 1932.

In 1956 **Clyde Cowan** and **Frederick Reines** discovered the positron's partner, **electron antineutrino**.



Muon



The **muon** particle, a sister of the electron, was discovered in 1936 by **Carl Anderson** and his first student **S. Neddermeyer**; and independently by **J. Street** *et al*.

It was not the "**pion**" particle predicted by **Hideki Yukawa** in 1935. And this marked the first flavor puzzle.

Isidor Rabi famously asked:

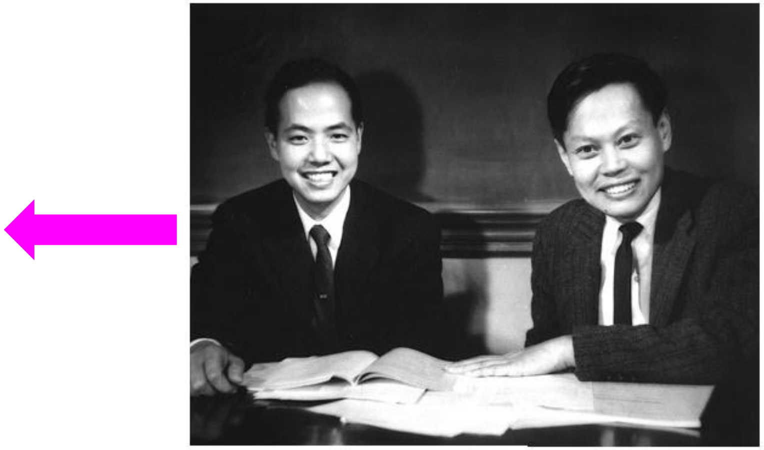
Who ordered that?

Isidor Isaac Rabi

FAMILY

Muon neutrino

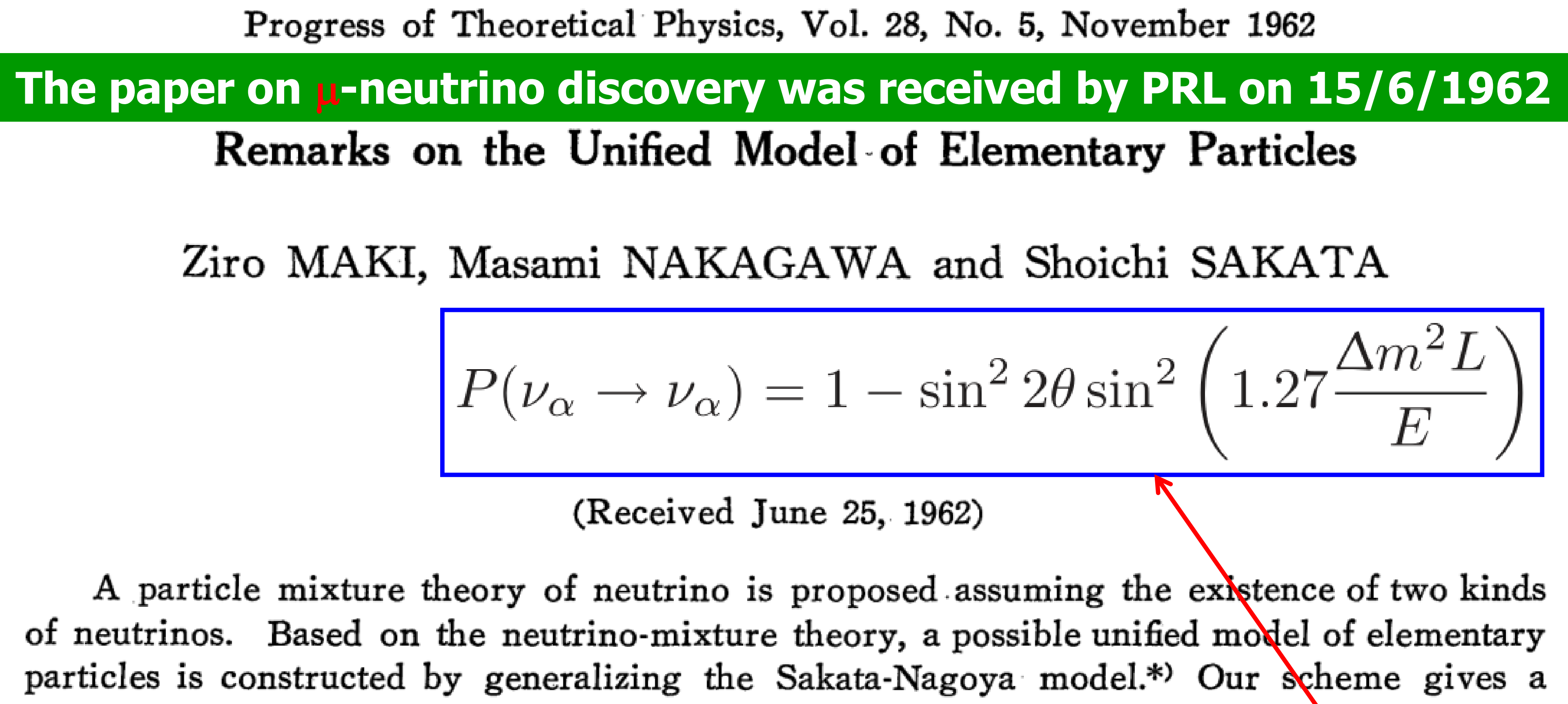
The muon neutrino, the muon's neutral counterpart, was discovered by Leon Lederman, Melvin Schwartz and Jack Steinberger in 1962.



Neutrino flavor conversion was proposed by Z. Maki, M. Nakagawa and S. Sakata in 1962.

Neutrinos convert into antineutrinos first proposed by Bruno Pontecorvo in 1957.

Original idea of ν-mixing



Progress of Theoretical Physics, Vol. 28, No. 5, November 1962

The paper on μ-neutrino discovery was received by PRL on 15/6/1962

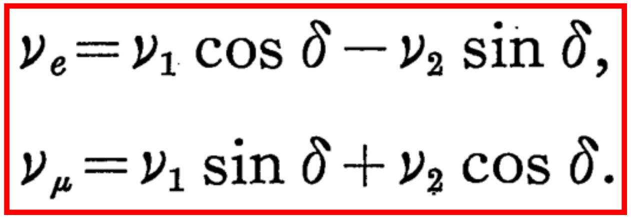
Remarks on the Unified Model of Elementary Particles

Ziro MAKI, Masami NAKAGAWA and Shoichi SAKATA

$$\boxed{P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2 L}{E}\right)}$$

(Received June 25, 1962)

A particle mixture theory of neutrino is proposed assuming the existence of two kinds of neutrinos. Based on the neutrino-mixture theory, a possible unified model of elementary particles is constructed by generalizing the Sakata-Nagoya model.*) Our scheme gives a



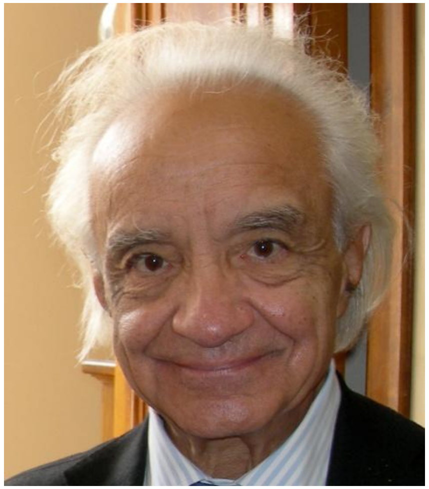
$$\nu_e = \nu_1 \cos\delta - \nu_2 \sin\delta,$$

$$\nu_\mu = \nu_1 \sin\delta + \nu_2 \cos\delta.$$

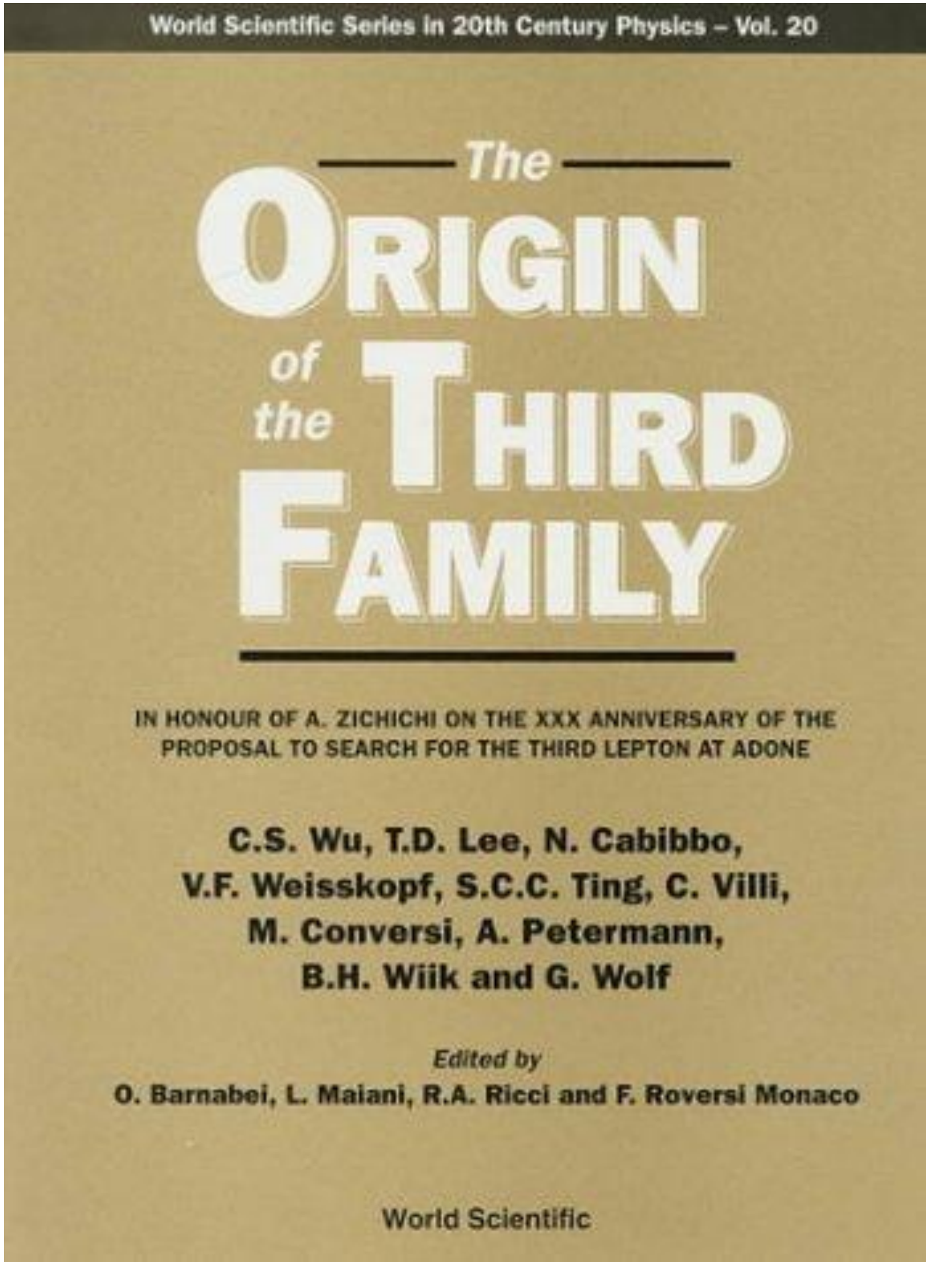
Bruno Pontecorvo formulated neutrino oscillation in 1968.

The 3rd family?

Antonino Zichichi: hunting for heavy leptons in 1960's



Erice school 2016



World Scientific Series in 20th Century Physics – Vol. 20

The
ORIGIN
of the THIRD
FAMILY

IN HONOUR OF A. ZICHICHI ON THE XXX ANNIVERSARY OF THE PROPOSAL TO SEARCH FOR THE THIRD LEPTON AT ADONE

C.S. Wu, T.D. Lee, N. Cabibbo,
V.F. Weisskopf, S.C.C. Ting, C. Villi,
M. Conversi, A. Petermann,
B.H. Wiik and G. Wolf

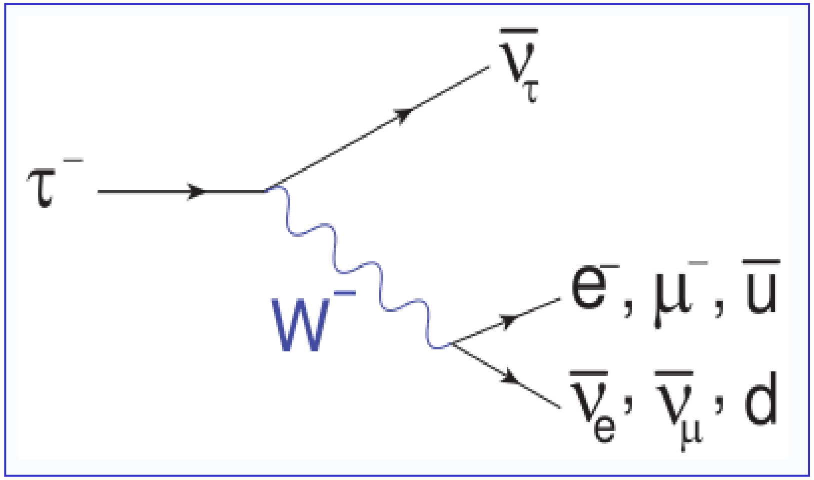
Edited by
O. Barnabei, L. Maiani, R.A. Ricci and F. Roversi Monaco

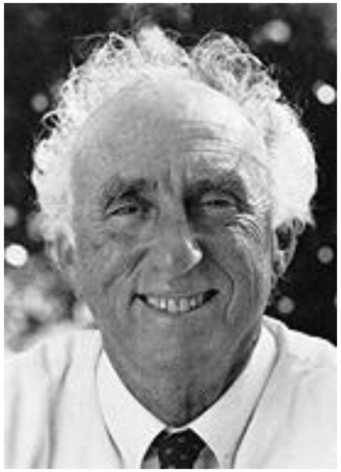
World Scientific

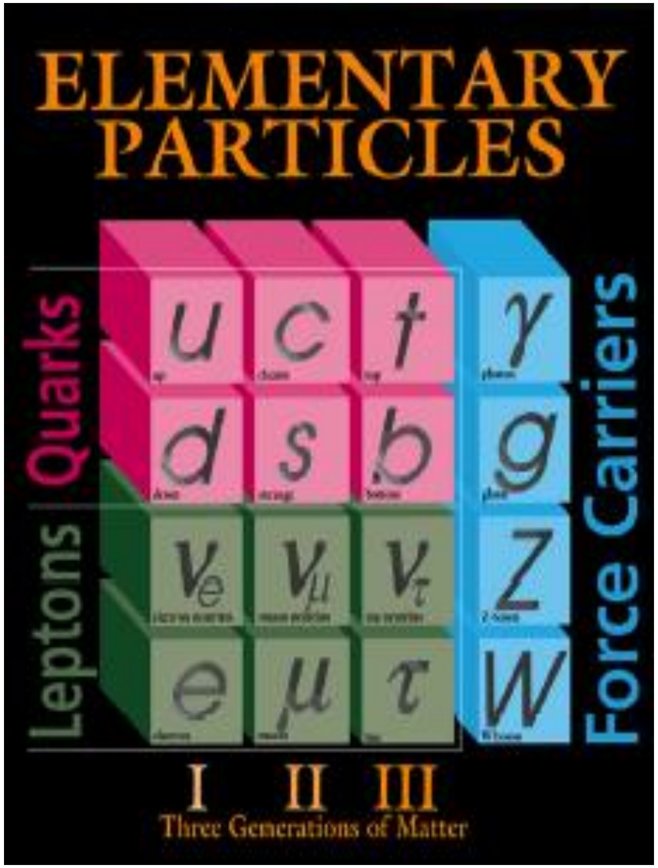
Tau and its neutrino

The tau particle was discovered by Martin Perl in 1975 via:

$$e^{+} + e^{-} \rightarrow e^{\pm} + \mu^{\mp} + \text{undetected particles}$$







In 2000, the tau neutrino was finally discovered at the Fermilab.

The lepton family is complete!

Lecture A2

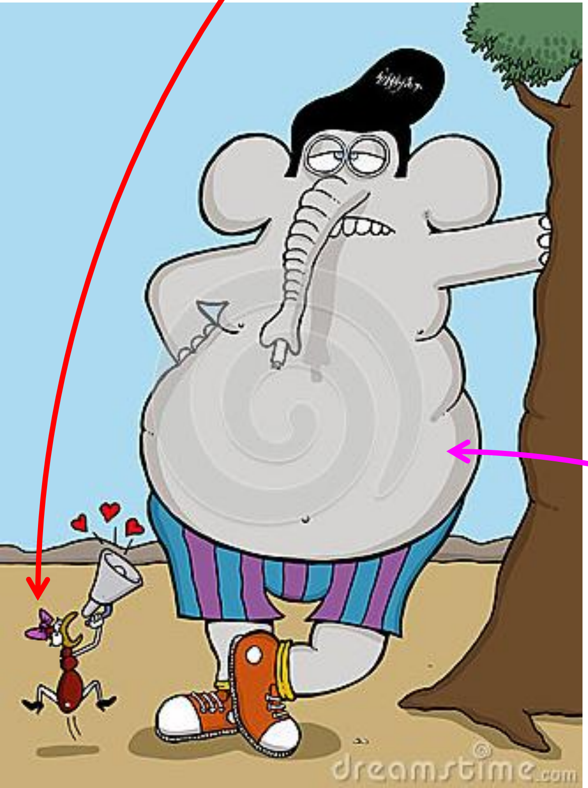
- ★ The standard model
- ★ Lepton number and flavors
- ★ Examples of neutrino interactions

SM particle content

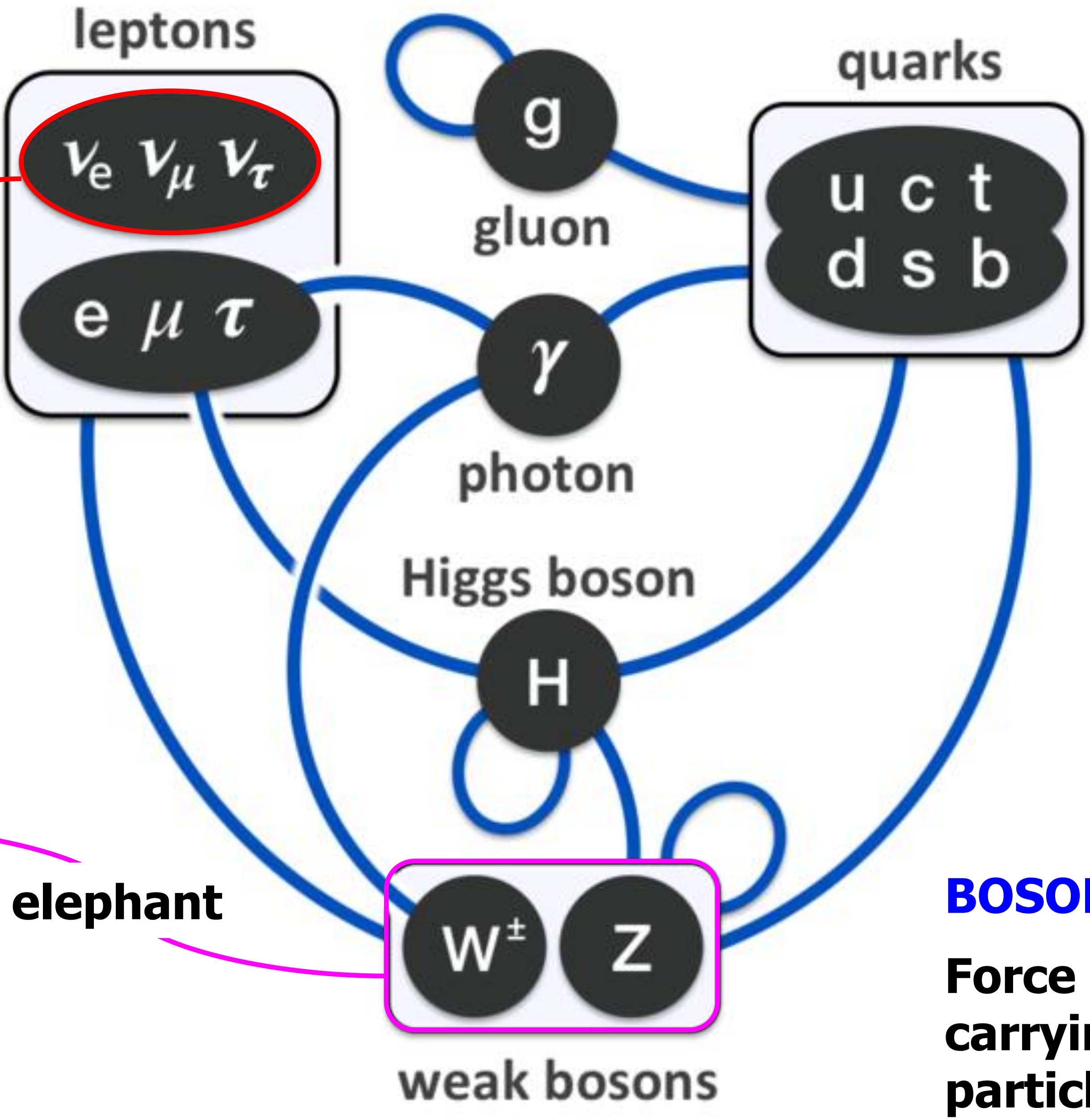
FERMIONS

Matter building block particles

ant



elephant



BOSONS

Force carrying particles

Electroweak Lagrangian

The standard electroweak model's Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{Y}}$$

Sheldon Lee Glashow
Prize share: 1/3

Abdus Salam
Prize share: 1/3

Steven Weinberg
Prize share: 1/3

$$\mathcal{L}_{\mathrm{G}} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right) \ ,$$

$$\mathcal{L}_{\mathrm{H}} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - \mu^{2}H^{\dagger}H - \lambda\left(H^{\dagger}H\right)^{2} \ ,$$

$$\mathcal{L}_{\mathrm{F}} = \overline{Q_{\mathrm{L}}}\,\mathrm{i}\not{D}\,Q_{\mathrm{L}} + \overline{\ell_{\mathrm{L}}}\,\mathrm{i}\not{D}\,\ell_{\mathrm{L}} + \overline{U_{\mathrm{R}}}\,\mathrm{i}\not{\partial}'\,U_{\mathrm{R}} + \overline{D_{\mathrm{R}}}\,\mathrm{i}\not{\partial}'\,D_{\mathrm{R}} + \overline{E_{\mathrm{R}}}\,\mathrm{i}\not{\partial}'\,E_{\mathrm{R}} \ ,$$

$$\mathcal{L}_{\mathrm{Y}} = -\overline{Q_{\mathrm{L}}}\,Y_{\mathrm{u}}\tilde{H}U_{\mathrm{R}} - \overline{Q_{\mathrm{L}}}\,Y_{\mathrm{d}}HD_{\mathrm{R}} - \overline{\ell_{\mathrm{L}}}\,Y_{l}HE_{\mathrm{R}} + \mathrm{h.c.} \ ,$$

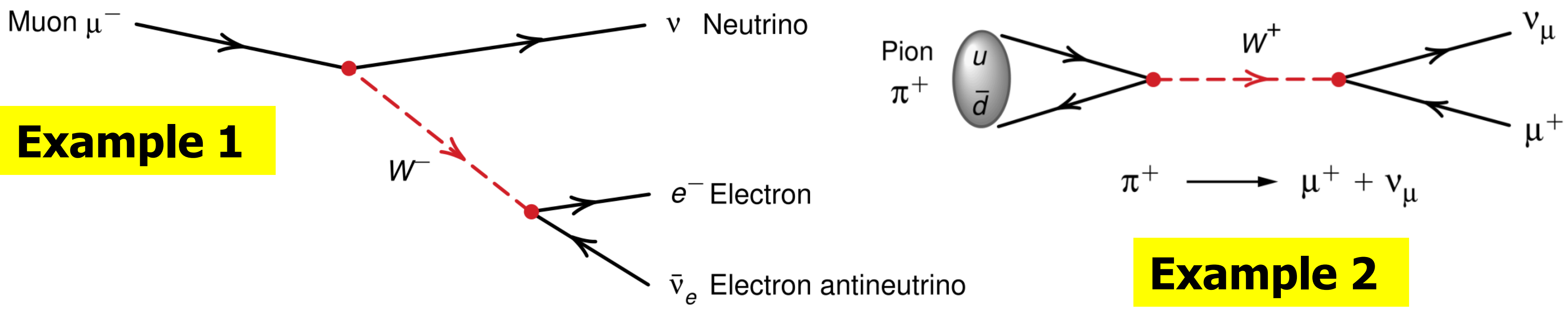
After electroweak symmetry breaking, we are left with weak neutral- and charged-current neutrino interactions:

$$-\mathcal{L}_{\mathrm{cc}} = \frac{g}{2\sqrt{2}}\sum_{\alpha}\left[\overline{\alpha}\,\gamma^{\mu}\left(1-\gamma_{5}\right)\nu_{\alpha}W_{\mu}^{-} + \mathrm{h.c.}\right]$$

$$-\mathcal{L}_{\mathrm{nc}} = \frac{g}{4\cos\theta_{\mathrm{w}}}\sum_{\alpha}\left[\overline{\nu_{\alpha}}\,\gamma^{\mu}\left(1-\gamma_{5}\right)\nu_{\alpha}\right]Z_{\mu}$$

Massive neutrinos obey the same NC or CC interactions

Lepton (flavor) number



$\pi^+ \longrightarrow \mu^+ + \nu_\mu$

Edward Witten (opening talk at Neutrino 2000) ——"Using the fields of the SM, it is impossible at the classical level to violate the baryon and lepton number symmetries by renormalizable interactions."

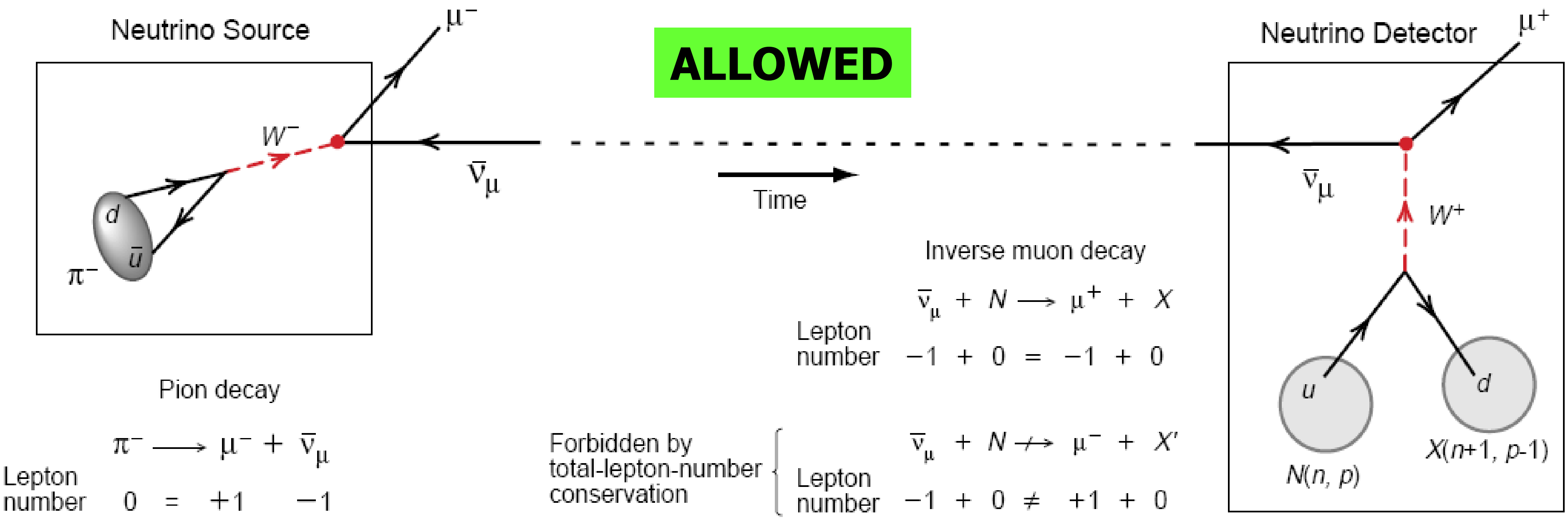
	e^-	ν_e	e^+	$\overline{\nu}_e$	μ^-	ν_μ	μ^+	$\overline{\nu}_\mu$	τ^-	ν_τ	τ^+	$\overline{\nu}_\tau$
L	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1
L_e	$+1$	$+1$	-1	-1	0	0	0	0	0	0	0	0
L_μ	0	0	0	0	$+1$	$+1$	-1	-1	0	0	0	0
L_τ	0	0	0	0	0	0	0	0	$+1$	$+1$	-1	-1

E. Witten

In the SM: both the lepton number and flavor numbers are conserved.

Some processes

ALLOWED



Pion decay

$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$

Lepton number $0 = +1 \quad -1$

Inverse muon decay

$$\bar{\nu}_\mu + N \longrightarrow \mu^+ + X$$

Lepton number $-1 + 0 = -1 + 0$

Forbidden by total-lepton-number conservation

$$\bar{\nu}_\mu + N \not\longrightarrow \mu^- + X'$$

Lepton number $-1 + 0 \neq +1 + 0$

$$\mu^+ \to e^+ + \gamma$$

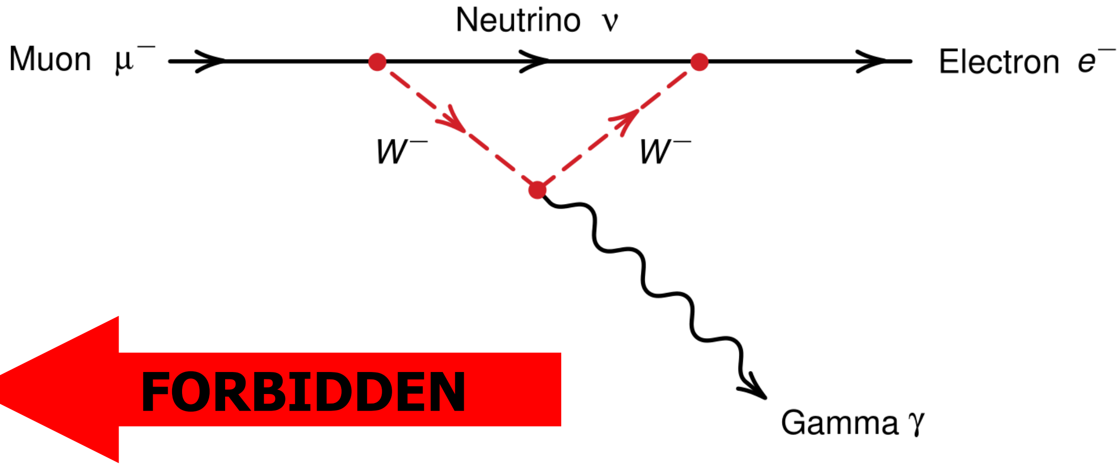
$$\mu^+ \to e^+ + e^- + e^+$$

$$\mu^- + N(n, p) \to e^- + N(n, p)$$

$$\mu^- + N(n, p) \to e^+ + N(n+2, p-2)$$

$$\mu^+ \to e^+ + \bar{\nu}_e + \nu_\mu$$

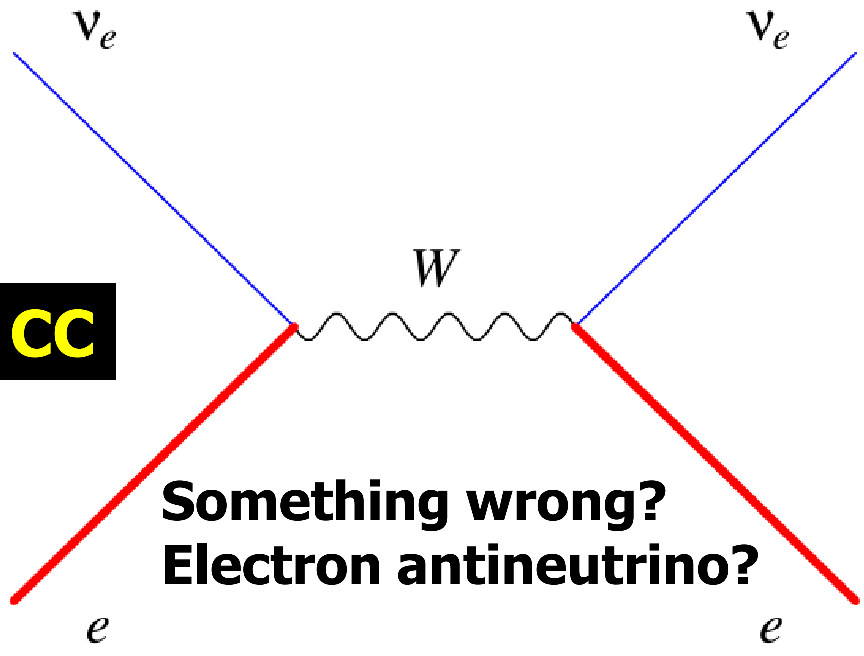
FORBIDDEN



Exercise: 检验每个过程是轻子数/轻子味破坏?

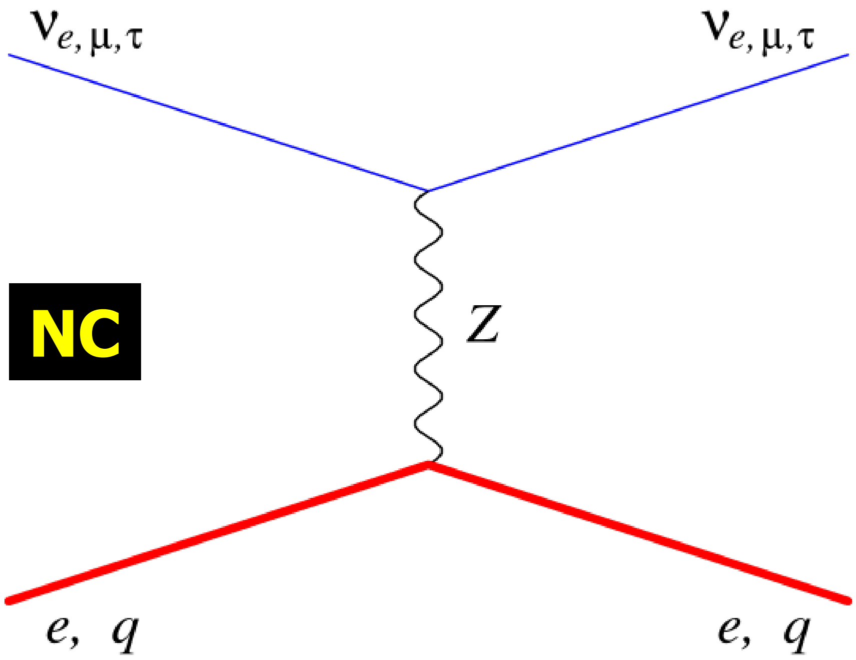
CC + NC

CC



Something wrong?
Electron antineutrino?

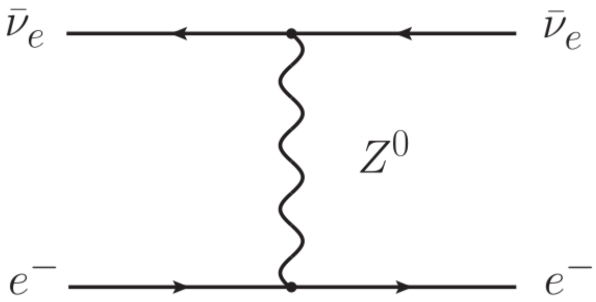
NC

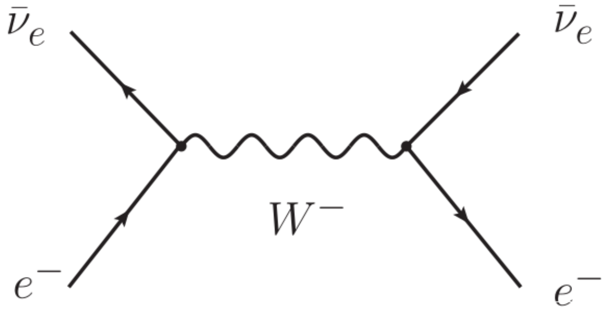


from A. Strumia + F. Vissani, hep-ph/0606054

Matter effects: **Forward scattering of neutrinos interferes with free neutrino propagation, leading to refraction. Scattering of neutrinos on electrons and quarks mediated by the Z boson is the same for all the 3 flavors, and that is why it does not affect flavor conversions between the active neutrinos. While scattering of the electron (anti)neutrinos and the electrons mediated by the W boson can change the behaviors of flavor oscillation of massive neutrinos.**

Neutrino-electron scattering



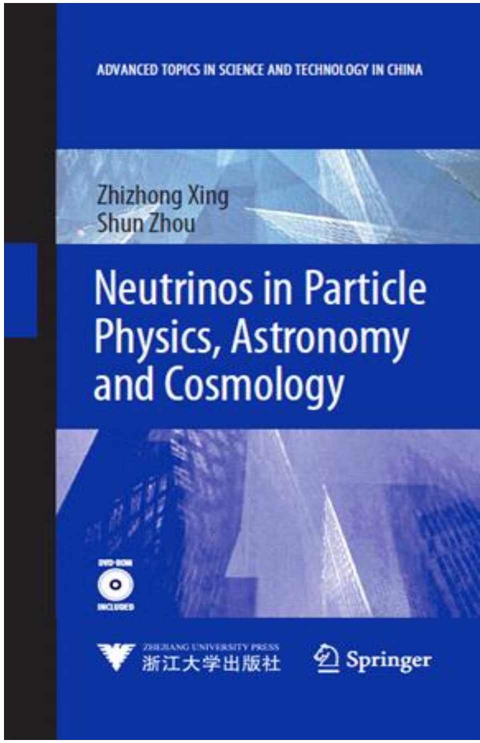


$$\sigma^{\rm ES}(\nu_e e^-) = \frac{2G_{\rm F}^2}{\pi} m_e E_\nu \left[\left(1 + C_{\rm L}^l\right)^2 + \frac{1}{3}\left(C_{\rm R}^l\right)^2\right]$$

$$\sigma^{\rm ES}(\overline{\nu}_e e^-) = \frac{2G_{\rm F}^2}{\pi} m_e E_\nu \left[\frac{1}{3}\left(1 + C_{\rm L}^l\right)^2 + \left(C_{\rm R}^l\right)^2\right]$$

$$\sigma^{\rm ES}(\nu_x e^-) = \frac{2G_{\rm F}^2}{\pi} m_e E_\nu \left[\left(C_{\rm L}^l\right)^2 + \frac{1}{3}\left(C_{\rm R}^l\right)^2\right]$$

$$\sigma^{\rm ES}(\overline{\nu}_x e^-) = \frac{2G_{\rm F}^2}{\pi} m_e E_\nu \left[\frac{1}{3}\left(C_{\rm L}^l\right)^2 + \left(C_{\rm R}^l\right)^2\right]$$



ADVANCED TOPICS IN SCIENCE AND TECHNOLOGY IN CHINA

Zhizhong Xing
Shun Zhou

Neutrinos in Particle Physics, Astronomy and Cosmology

ZHEJIANG UNIVERSITY PRESS
浙江大学出版社
Springer

$$\boxed{\frac{\sigma^{\rm ES}(\nu_x e^-)}{\sigma^{\rm ES}(\nu_e e^-)} = \frac{3\left(C_{\rm L}^l\right)^2 + \left(C_{\rm R}^l\right)^2}{3\left(1 + C_{\rm L}^l\right)^2 + \left(C_{\rm R}^l\right)^2} \approx 0.155}$$

$$\boxed{\begin{aligned} C_{\rm L}^l &= -\frac{1}{2} + \sin^2\theta_{\rm w} \simeq -0.262 \\ C_{\rm R}^l &= \sin^2\theta_{\rm w} \simeq 0.238 \end{aligned}}$$

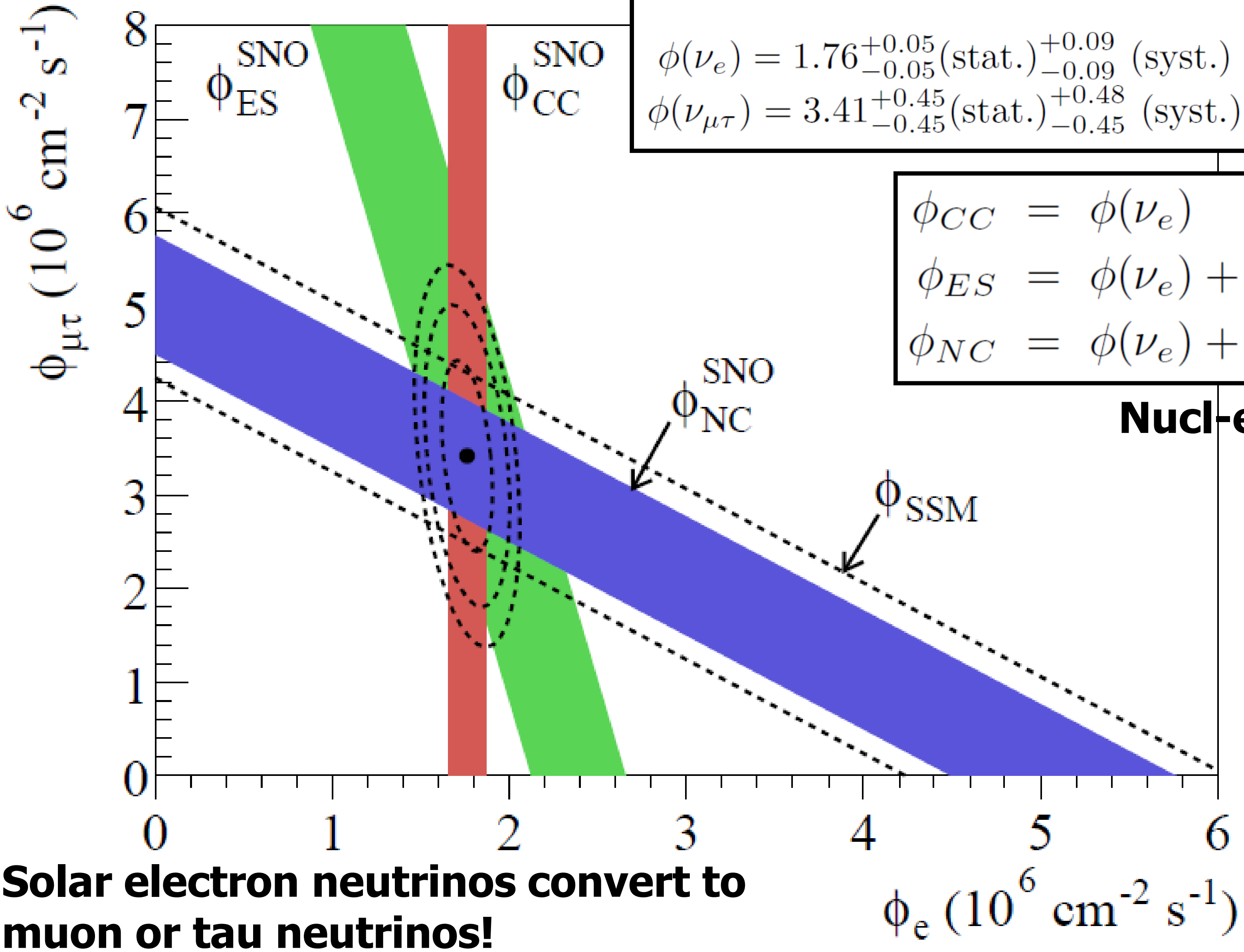
The SNO result

$$\phi_{CC} = 1.76^{+0.06}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{ES} = 2.39^{+0.24}_{-0.23}(\text{stat.})^{+0.12}_{-0.12}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{NC} = 5.09^{+0.44}_{-0.43}(\text{stat.})^{+0.46}_{-0.43}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

$$\phi(\nu_e) = 1.76^{+0.05}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi(\nu_{\mu\tau}) = 3.41^{+0.45}_{-0.45}(\text{stat.})^{+0.48}_{-0.45}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

$$\begin{aligned}\phi_{CC} &= \phi(\nu_e)\\ \phi_{ES} &= \phi(\nu_e) + 0.1559\phi(\nu_{\mu\tau})\\ \phi_{NC} &= \phi(\nu_e) + \phi(\nu_{\mu\tau})\end{aligned}$$

Nucl-ex/0610020



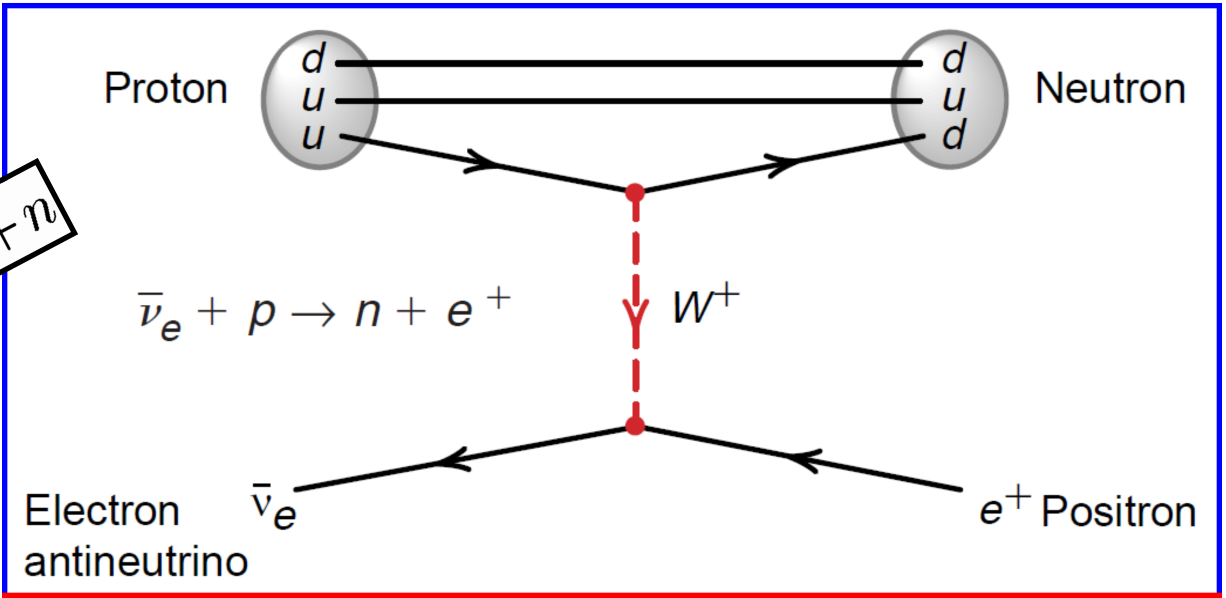
Solar electron neutrinos convert to muon or tau neutrinos!

J. Bahcall

Neutrino-nucleon scattering

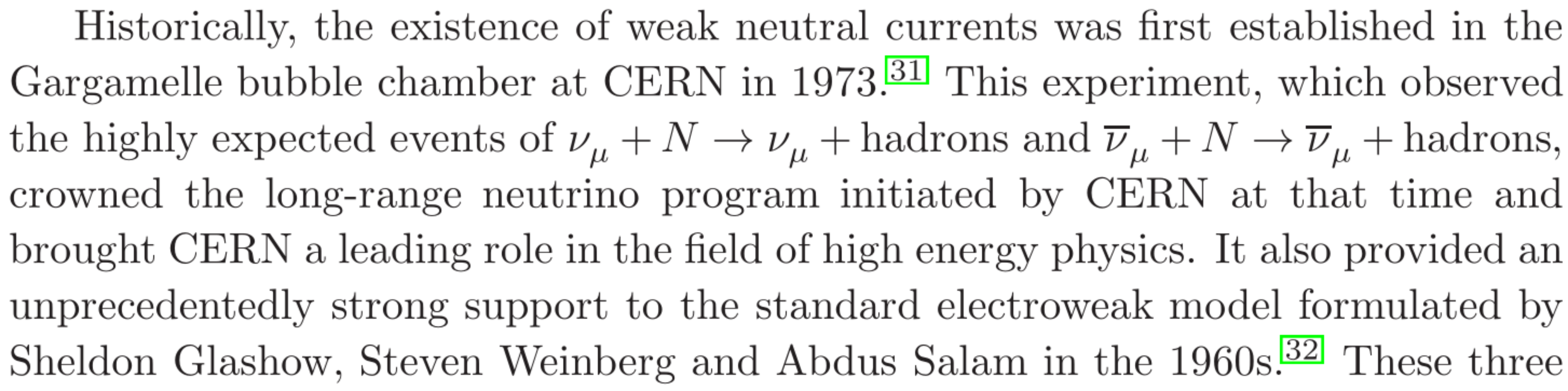
The inverse β decay:

$$\nu_e + n \rightleftharpoons e^- + p, \overline{\nu}_e + p \rightleftharpoons e^+ + n$$



Cross section of scattering:

$$\sigma(\overline{\nu}_e p) = \sigma(\nu_e n) \approx 9.1 \times 10^{-44} \left(\frac{E_\nu}{\text{MeV}} \right)^2 \text{ cm}^2$$



Historically, the existence of weak neutral currents was first established in the Gargamelle bubble chamber at CERN in 1973.[31] This experiment, which observed the highly expected events of $\nu_\mu + N \to \nu_\mu +$ hadrons and $\overline{\nu}_\mu + N \to \overline{\nu}_\mu +$ hadrons, crowned the long-range neutrino program initiated by CERN at that time and brought CERN a leading role in the field of high energy physics. It also provided an unprecedentedly strong support to the standard electroweak model formulated by Sheldon Glashow, Steven Weinberg and Abdus Salam in the 1960s.[32] These three

Detection of CνB

- ★ **CνB-induced mechanical effects on Cavendish-type torsion balance;**
- ★ **Capture of relic ν's on radioactive β-decaying nuclei (Weinberg 62);**
- ★ **Z-resonance annihilation of UHE cosmic ν's and relic ν's (Weiler 82).**

Temperature today

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \simeq 1.945 \text{ K}$$

Mean momentum today

$$\langle p_\nu \rangle \simeq 3.151 T_\nu \simeq 5.281 \times 10^{-4} \text{ eV}$$

At least 2 ν's cold today

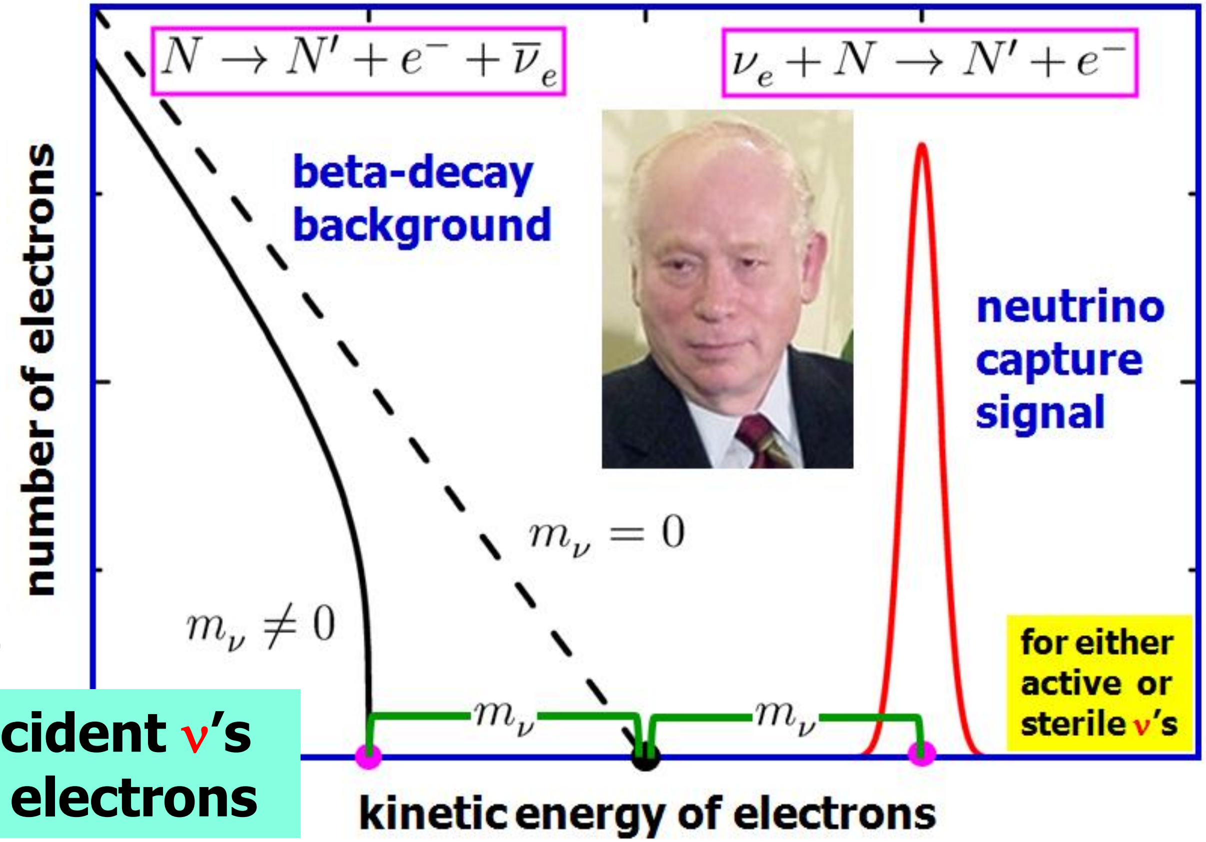
Non-relativistic ν's!

(Irvine & Humphreys, 83)

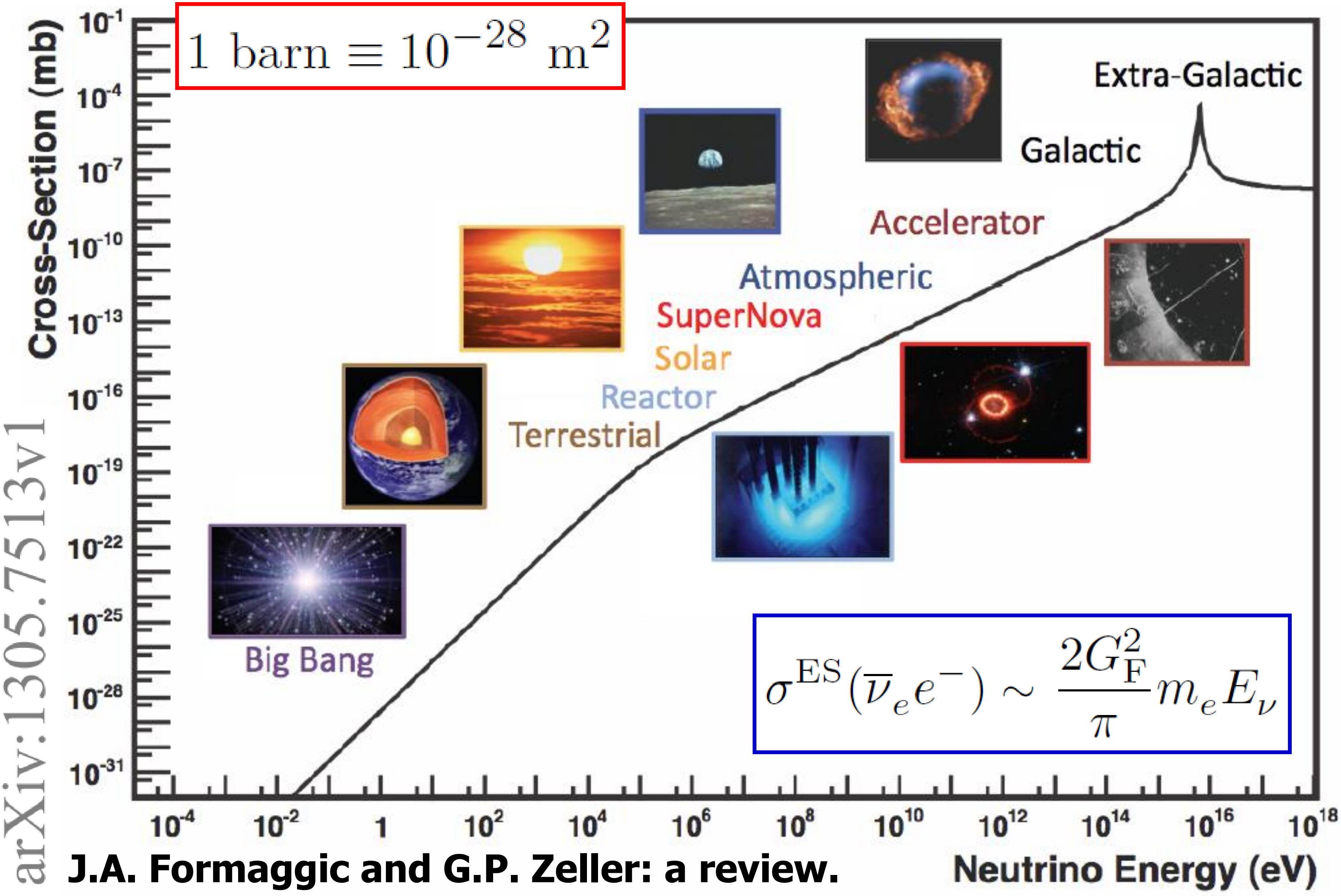
no energy threshold on incident ν's

mono-energetic outgoing electrons

Relic neutrino capture on β-decaying nuclei



A brief summary



$$1\ \mathrm{barn} \equiv 10^{-28}\ \mathrm{m}^2$$

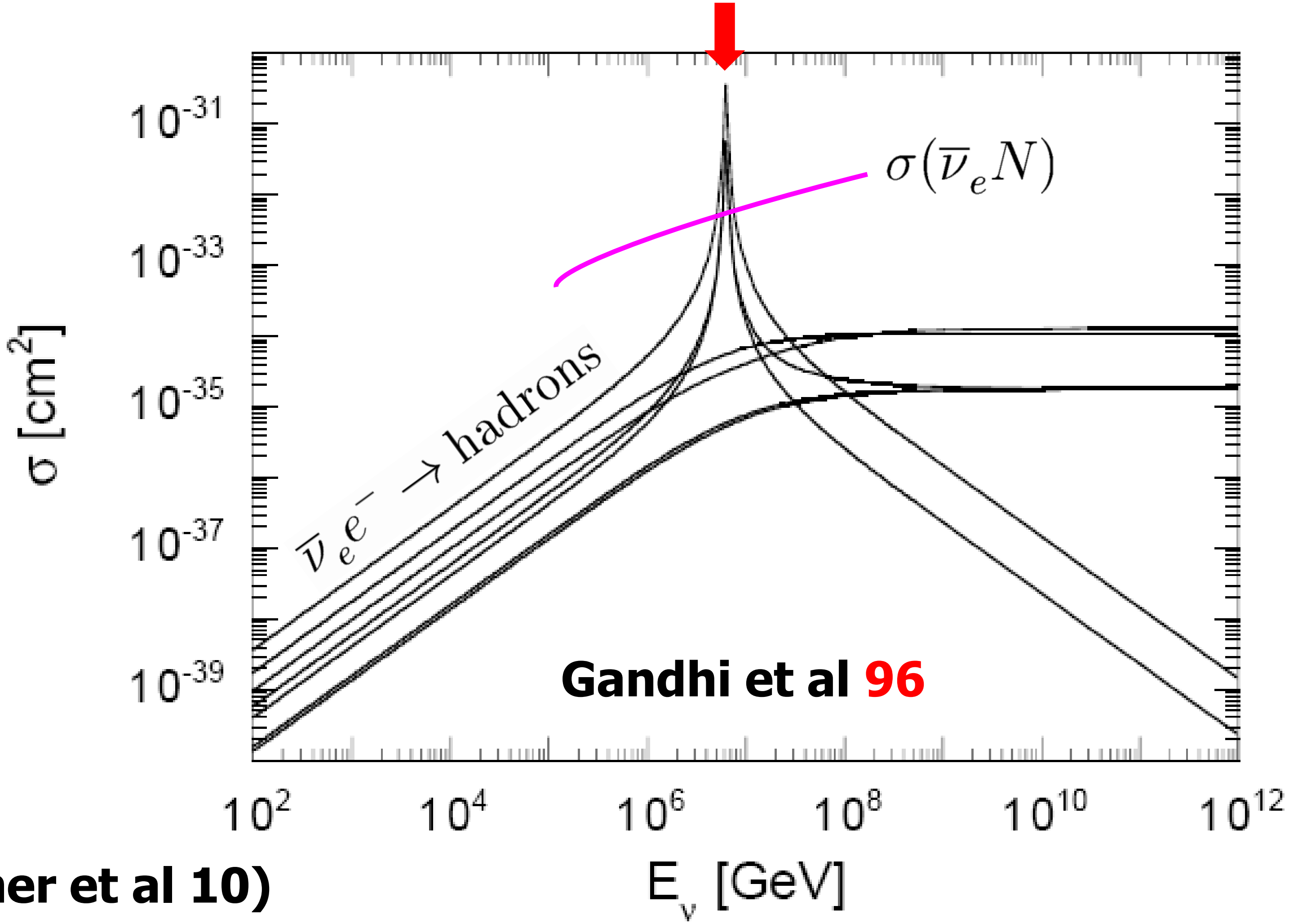
$$\sigma^{\mathrm{ES}}(\overline{\nu}_e e^-) \sim \frac{2G_{\mathrm{F}}^2}{\pi} m_e E_\nu$$

J.A. Formaggic and G.P. Zeller: a review.

Exercise: Glashow resonance

$\overline{\nu}_e + e^- \to W^- \to \text{anything}$ **Unique for electron anti-ν's!**

(S.L. Glashow 60) $E_{\overline{\nu}_e} \simeq M_W^2/(2m_e) \simeq 6.3 \text{ PeV}$



Gandhi et al 96

An interesting discriminator between pγ & pp collisions at an optically thin source of cosmic rays. (Anchordoqui et al 05, Hummer et al 10)

Lecture A3

- ★ **The Dirac mass term**
- ★ **The Majorana mass term**
- ★ **Electromagnetic properties**

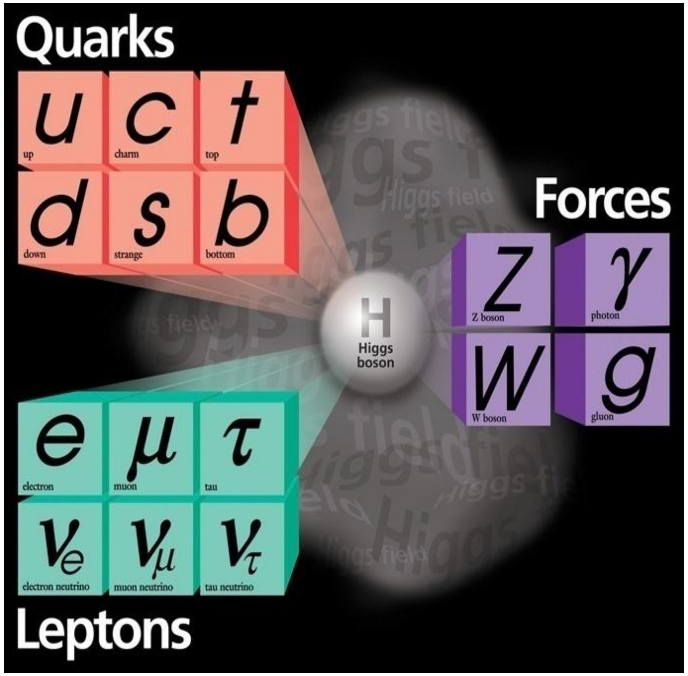
What is mass?

Mass is the inertial energy of a particle existing at rest.

- A **massless** particle has no way to exist at rest. It must always move at the speed of light.
- A **massive** fermion (lepton or quark) must exist in both the left- and right-handed states.

The **Brout-Englert-Higgs** mechanism is responsible for the origin of *W* / *Z* and fermion masses in the SM.

$$L_{\rm SM} = L(f,G) + \underline{L(f,H)} + \underline{L(G,H)} + L(G) - V(H)$$



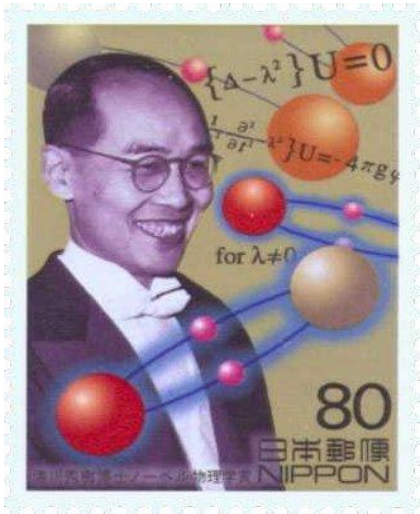
All the **bosons** were discovered in **Europe**, and most of the **fermions** were discovered in **America**.

Masses of force particles

force	strength	range	mediator	mass
strong	1	$10^{-15}\,\mathrm{m}$	gluon/π	$\sim 10^{2}\,\mathrm{MeV}$
EM	$1/137$	∞	photon	$=0$
weak	10^{-12}	$10^{-18}\,\mathrm{m}$	W/Z/H	$\sim 10^{2}\,\mathrm{GeV}$
gravitation	6×10^{-39}	∞	graviton	$=0$

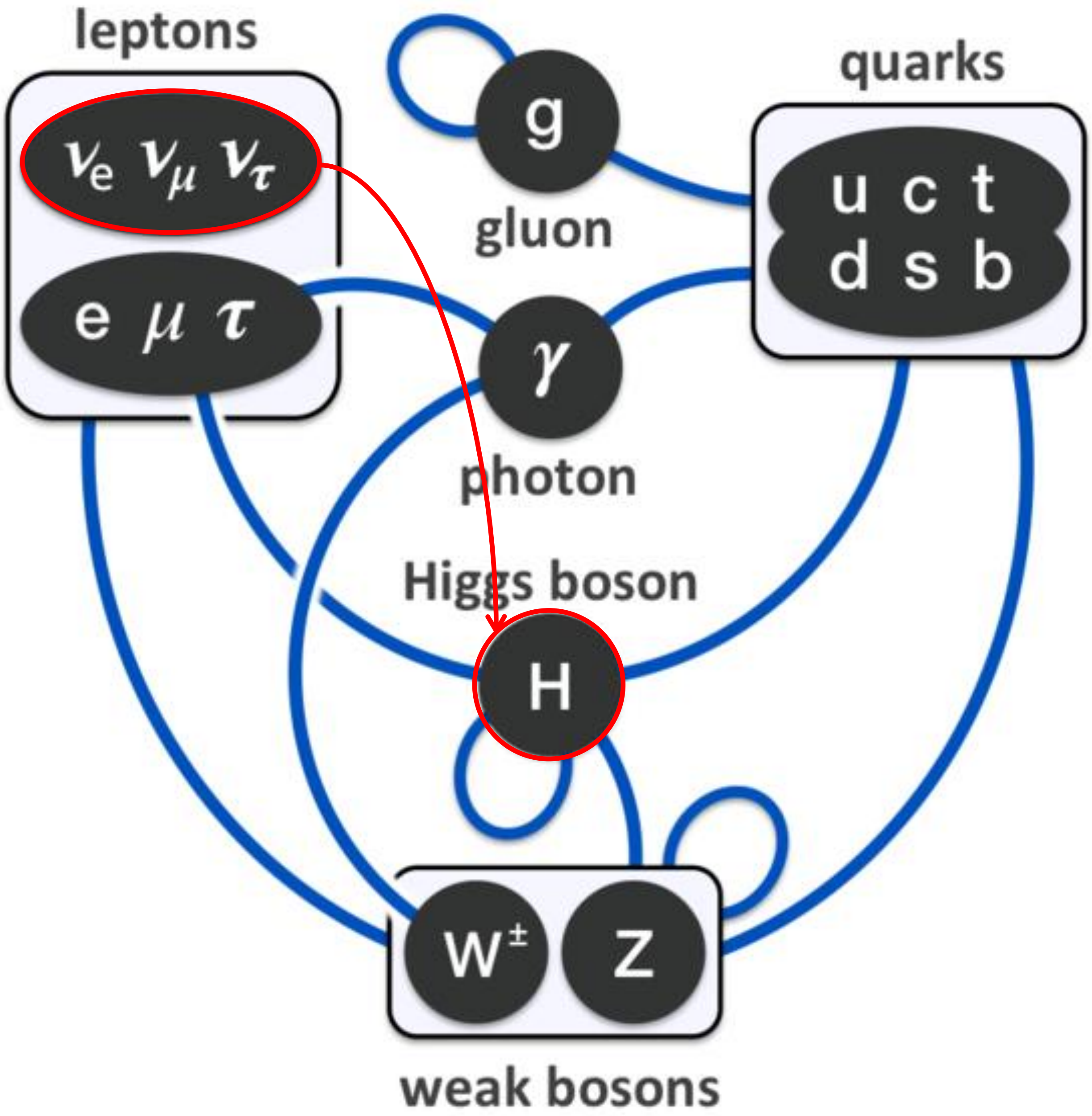
Yukawa relation for the mediator's mass M and the force's range R:

$$\boxed{M \sim \frac{200\mathrm{MeV}\times 10^{-15}\,\mathrm{m}}{R}}$$



汤川秀树 (Hideki Yukawa): His first paper in 1935 made him get the Nobel Prize in 1949.

Masses of matter particles



Dirac mass:
introducing the right-handed neutrino field and allowing Yukawa interactions

Majorana mass:
Using the left-handed neutrino field and its charge-conjugate one

Some notations

Define the left- and right-handed neutrino fields:

$$\nu_{\mathrm{L}} = \begin{pmatrix} \nu_{e\mathrm{L}} \\ \nu_{\mu\mathrm{L}} \\ \nu_{\tau\mathrm{L}} \end{pmatrix}$$

[redacted]

$$N_{\mathrm{R}} = \begin{pmatrix} N_{1\mathrm{R}} \\ N_{2\mathrm{R}} \\ N_{3\mathrm{R}} \end{pmatrix}$$

Extend the SM's particle content

$$\psi_{\mathrm{L}} \equiv \frac{1-\gamma_5}{2}\psi$$
$$\psi_{\mathrm{R}} \equiv \frac{1+\gamma_5}{2}\psi$$

The charge-conjugate counterparts are defined below and transform as right- and left-handed fields, respectively:

$$(\nu_{\mathrm{L}})^c \equiv \mathcal{C}\overline{\nu_{\mathrm{L}}}^T , \qquad (N_{\mathrm{R}})^c \equiv \mathcal{C}\overline{N_{\mathrm{R}}}^T$$

$$\overline{(\nu_{\mathrm{L}})^c} = (\nu_{\mathrm{L}})^T\mathcal{C} , \qquad \overline{(N_{\mathrm{R}})^c} = (N_{\mathrm{R}})^T\mathcal{C}$$

$$(\nu_{\mathrm{L}})^c = (\nu^c)_{\mathrm{R}} \text{ and } (N_{\mathrm{R}})^c = (N^c)_{\mathrm{L}} \text{ hold}$$

Properties of the charge-conjugation matrix:

$$\mathcal{C}\gamma_\mu^T\mathcal{C}^{-1} = -\gamma_\mu , \qquad \mathcal{C}\gamma_5^T\mathcal{C}^{-1} = \gamma_5 , \qquad \mathcal{C}^{-1} = \mathcal{C}^\dagger = \mathcal{C}^T = -\mathcal{C}$$

They are from the requirement that the charge-conjugated field must satisfy the same Dirac equation ($\mathcal{C} = i\gamma^2\gamma^0$ in the Dirac representation)

Dirac mass term

A Dirac neutrino is described by a 4-component spinor:

$$\nu = \nu_{\rm L} + N_{\rm R}$$

Step 1: the gauge-invariant Dirac mass term and SSB:

$$-\mathcal{L}_{\rm Dirac} = \overline{\ell_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.} \quad \Longrightarrow \quad -\mathcal{L}'_{\rm Dirac} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + {\rm h.c.}$$

$$M_{\rm D} = Y_\nu \langle H \rangle \ \text{with} \ \langle H \rangle \simeq 174\ {\rm GeV}$$

Step 2: basis transformation:

$$V^\dagger M_{\rm D} U = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$$

$$-\mathcal{L}'_{\rm Dirac} = \overline{\nu'_{\rm L}} \widehat{M}_\nu N'_{\rm R} + {\rm h.c.}$$

Mass states link to flavor states:

$$\nu'_{\rm L} = V^\dagger \nu_{\rm L} \ \text{ and } \ N'_{\rm R} = U^\dagger N_{\rm R}$$

$$\nu' = \nu'_{\rm L} + N'_{\rm R} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Step 3: physical mass term and kinetic term:

$$-\mathcal{L}'_{\rm Dirac} = \overline{\nu'} \widehat{M}_\nu \nu' = \sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i$$

$$\mathcal{L}_{\rm kinetic} = i \overline{\nu_{\rm L}} \gamma_\mu \partial^\mu \nu_{\rm L} + i \overline{N_{\rm R}} \gamma_\mu \partial^\mu N_{\rm R} = i \overline{\nu'} \gamma_\mu \partial^\mu \nu' = i \sum_{k=1}^{3} \overline{\nu_k} \gamma_\mu \partial^\mu \nu_k$$

Dirac neutrino mixing

Weak charged-current interactions of leptons:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)_{\rm L}} \ \gamma^{\mu} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)_{\rm L}} \ \gamma^{\mu} V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the mass basis

One may take mass states = flavor states for the charged leptons. So *V* is just the PMNS matrix of neutrino mixing.

Both the mass and CC terms are invariant with respect to a global phase transformation, and thus lepton number is conserved. However, lepton flavors are violated.

$$l(x) \longrightarrow e^{i\Phi} l(x)$$

$$\nu'_{\rm L}(x) \rightarrow e^{i\Phi} \nu'_{\rm L}(x)$$

$$N'_{\rm R}(x) \rightarrow e^{i\Phi} N'_{\rm R}(x)$$

√

×

×

×

	e^-	ν_e	e^+	$\overline{\nu}_e$	μ^-	ν_μ	μ^+	$\overline{\nu}_\mu$	τ^-	ν_τ	τ^+	$\overline{\nu}_\tau$
L	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1
L_e	$+1$	$+1$	-1	-1	0	0	0	0	0	0	0	0
L_μ	0	0	0	0	$+1$	$+1$	-1	-1	0	0	0	0
L_τ	0	0	0	0	0	0	0	0	$+1$	$+1$	-1	-1

Majorana mass term (1)

A Majorana mass term can be obtained by introducing the Higgs triplet into the SM, writing out the gauge-invariant Yukawa interactions and Higgs potentials, integrating out heavy degrees of freedom (type-II seesaw mechanism):

$$-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} (\nu_{\rm L})^c + {\rm h.c.}$$

The Majorana mass matrix must be a symmetric matrix. It can be diagonalized by a unitary matrix

$$\overline{\nu_L} M_{\rm L} (\nu_L)^c = [\overline{\nu_L} \underline{M_{\rm L}} (\nu_L)^c]^T = -\overline{\nu_L}\mathcal{C}^T M_{\rm L}^T \overline{\nu_L}^T = \overline{\nu_L} \underline{M_{\rm L}^T} (\nu_L)^c$$

Diagonalization:

$$V^\dagger M_{\rm L} V^* = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$$

$$-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu'_{\rm L}} \widehat{M}_\nu (\nu'_{\rm L})^c + {\rm h.c.}$$

$$\nu'_{\rm L} = V^\dagger \nu_{\rm L} \ \text{and} \ (\nu'_{\rm L})^c = \mathcal{C}\overline{\nu'_{\rm L}}^T$$

Physical mass term:

$$\nu' = \nu'_{\rm L} + (\nu'_{\rm L})^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu'} \widehat{M}_\nu \nu' = \frac{1}{2}\sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i$$

$$\text{Majorana condition } (\nu')^c = \nu'$$

Majorana mass term (2)

Kinetic term (you may prove $\overline{(\psi_{\rm L})^c}\gamma_\mu\partial^\mu(\psi_{\rm L})^c = \overline{\psi_{\rm L}}\gamma_\mu\partial^\mu\psi_{\rm L}$)

$$\boxed{\mathcal{L}_{\rm kinetic} = i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} = i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} = \frac{i}{2}\overline{\nu'}\gamma_\mu\partial^\mu\nu' = \frac{i}{2}\sum_{k=1}^{3}\overline{\nu_k}\gamma_\mu\partial^\mu\nu_k}$$

Question: why is there a factor **1/2** in the **Majorana** mass term?

Answer: it allows us to get the correct **Dirac** equation of motion.

A proof: write out the Lagrangian of free massive **Majorana** neutrinos

$$\begin{aligned}\mathcal{L}_\nu &= i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} - \left[\frac{1}{2}\overline{\nu_{\rm L}}M_{\rm L}(\nu_{\rm L})^c + {\rm h.c.}\right] \\ &= i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} - \left[\frac{1}{2}\overline{\nu'_{\rm L}}\widehat{M}_\nu(\nu'_{\rm L})^c + {\rm h.c.}\right] \\ &= \frac{1}{2}\left(i\overline{\nu'}\gamma_\mu\partial^\mu\nu' - \overline{\nu'}\widehat{M}_\nu\nu'\right) = -\frac{1}{2}\left(i\partial^\mu\overline{\nu'}\gamma_\mu\nu' + \overline{\nu'}\widehat{M}_\nu\nu'\right)\end{aligned}$$

Euler-Lagrange equation:

$$\partial^\mu\frac{\partial\mathcal{L}_\nu}{\partial\left(\partial^\mu\overline{\nu'}\right)} - \frac{\partial\mathcal{L}_\nu}{\partial\overline{\nu'}} = 0 \Longrightarrow \begin{cases} i\gamma_\mu\partial^\mu\nu' - \widehat{M}_\nu\nu' = 0 \\ i\gamma_\mu\partial^\mu\nu_k - m_k\nu_k = 0\end{cases}$$

Majorana neutrino mixing

Weak charged-current interactions of leptons:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^{\mu} \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_{\rm L} W^{-}_{\mu} + {\rm h.c.}$$

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^{\mu} V \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_{\rm L} W^{-}_{\mu} + {\rm h.c.}$$

In the mass basis

The PMNS matrix *V* contains 2 extra CP-violating phases.

Mass and CC terms are not simultaneously invariant under a global phase transformation --- Lepton number violation

$$l(x) \rightarrow e^{i\Phi} l(x)$$

$$\nu^{\prime}_{\rm L}(x) \rightarrow e^{i\Phi} \nu^{\prime}_{\rm L}(x)$$

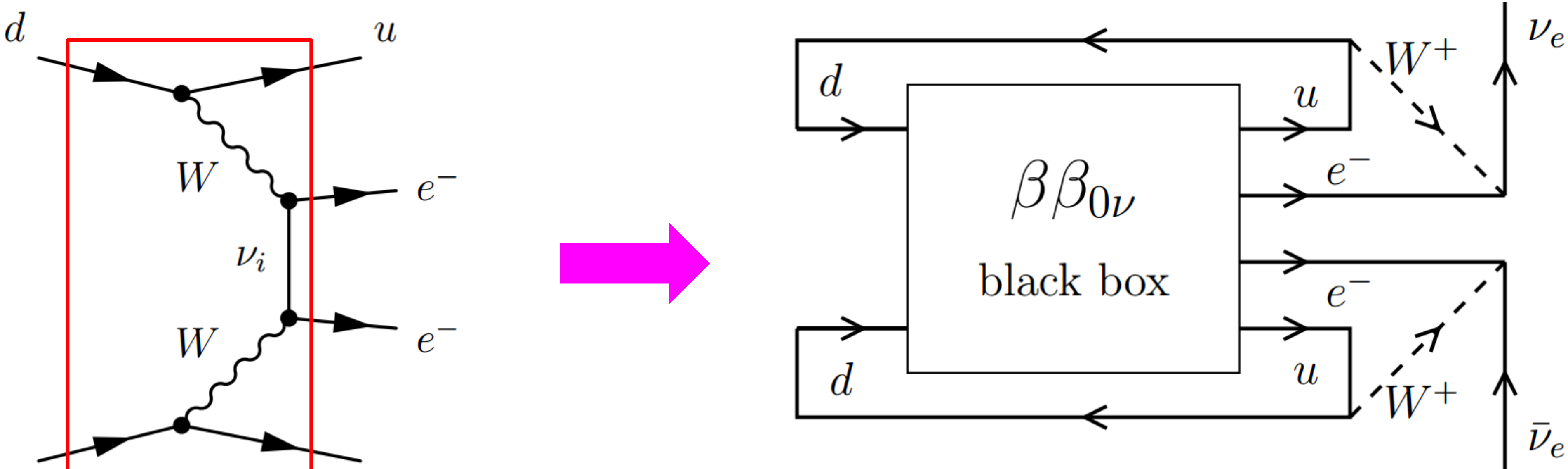
$$-\mathcal{L}^{\prime}_{\rm Majorana} = \frac{1}{2}\overline{\nu^{\prime}_{\rm L}}\widehat{M}_{\nu}(\nu^{\prime}_{\rm L})^{c} + {\rm h.c.}$$

$$\overline{\nu^{\prime}_{\rm L}} \rightarrow e^{-i\Phi}\overline{\nu^{\prime}_{\rm L}} \text{ and } (\nu^{\prime}_{\rm L})^{c} \rightarrow e^{-i\Phi}(\nu^{\prime}_{\rm L})^{c}$$

$$e^{-2i\Phi}$$

Schechter-Valle theorem

THEOREM (1982): if a **0νββ** decay happens, there must be an effective **Majorana** mass term.



Bruno Pontecorvo's Prediction

指导我们实验的理论基础是SV定理

Four-loop ν mass:

$|\delta m_\nu^{ee}| < 7 \times 10^{-29}\ \mathrm{eV}$ **(Duerr, Lindner, Merle, 2011; Liu, Zhang, Zhou, 2016)**

Note: The **black box** can in principle have many different processes (new physics). Only in the simplest case, which is most interesting, it's likely to constrain neutrino masses

YES or NO?

QUESTION: are massive neutrinos the **Majorana** particles?

One might be able to answer **YES** through a measurement of the **$0\nu\beta\beta$** decay or other **LNV** processes someday, but how to answer with **NO**?

YES

or

I don't know!

The same question: how to distinguish between **Dirac** and **Majorana** neutrinos in a realistic experiment?

Answer 1: The **$0\nu\beta\beta$** decay is currently the only possibility.

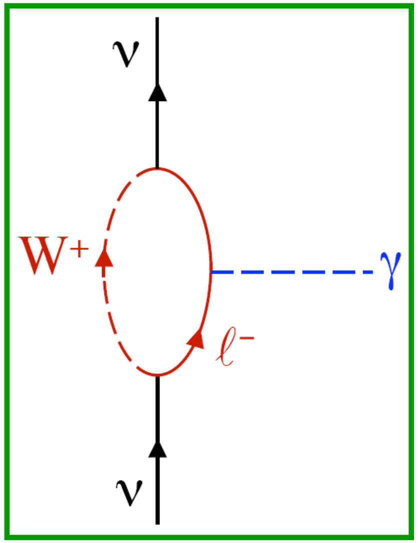
Answer 2: In principle their dipole moments are different.

Answer 3: They show different behavior if nonrelativistic.

Electromagnetic properties

Without electric charges, neutrinos have electromagnetic interactions with the photon via quantum loops.

Given the SM interactions, a massive Dirac neutrino can only have a tiny magnetic dipole moment:



$$\boxed{\mu_\nu \sim \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu = 3\times 10^{-20} \frac{m_\nu}{0.1\,\mathrm{eV}} \mu_B}$$

A massive Majorana neutrino can not have magnetic & electric dipole moments, as its antiparticle is itself.

Proof: Dirac neutrino's electromagnetic vertex can be parametrized as

$$\boxed{\Gamma_\mu(p,p') = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)\left(q^2\gamma_\mu - q_\mu q^\nu\gamma_\nu\right)\gamma_5}$$

Majorana neutrinos

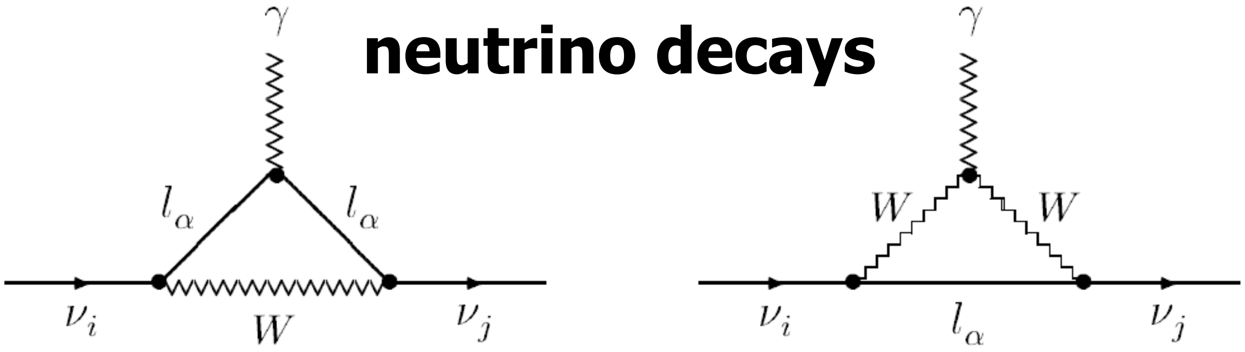
$$\boxed{\overline{\psi}\underline{\Gamma_\mu}\psi = \overline{\psi^c}\Gamma_\mu\psi^c = \psi^T \mathcal{C}\Gamma_\mu\mathcal{C}\overline{\psi}^T = \left(\psi^T\mathcal{C}\Gamma_\mu\mathcal{C}\overline{\psi}^T\right)^T = -\overline{\psi}\mathcal{C}^T\Gamma_\mu^T\mathcal{C}^T\psi = \overline{\psi}\underline{\mathcal{C}\Gamma_\mu^T\mathcal{C}^{-1}}\psi}$$

$\Rightarrow$ $\boxed{f_Q(q^2) = f_M(q^2) = f_E(q^2) = 0}$ **intrinsic property of Majorana ν's.**

Transition dipole moments

Both Dirac & Majorana neutrinos can have *transition* dipole moments (of a size comparable with μ_ν) that may give rise to neutrino decays, scattering with electrons, interactions with external magnetic field & contributions to ν masses. (Data: < a few × 10^-11 Bohr magneton).

neutrino decays

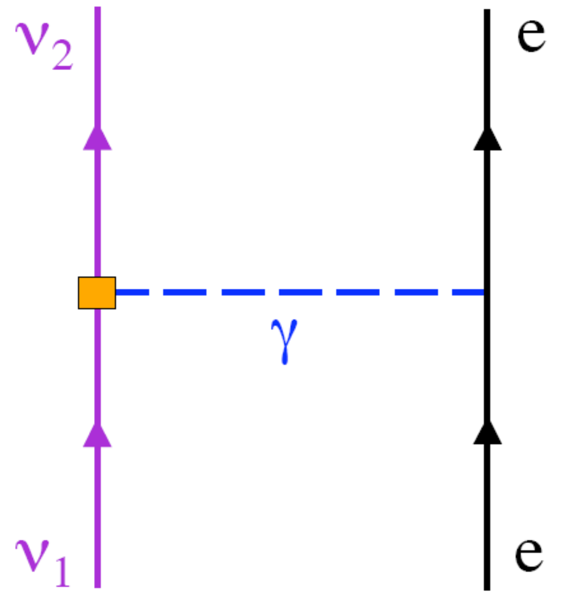


$$\mu_{\text{eff}} \equiv \sqrt{|\mu_{ij}|^2 + |\epsilon_{ij}|^2}$$

$$\Gamma_{\nu_i\to\nu_j+\gamma} = 5.3\times\left(1-\frac{m_j^2}{m_i^2}\right)^3\left(\frac{m_i}{1\text{ eV}}\right)^3\left(\frac{\mu_{\text{eff}}}{\mu_{\text{B}}}\right)^2\text{s}^{-1}$$

$$\frac{\mathrm{d}\sigma'_\mu}{\mathrm{d}T} = \frac{\alpha^2\pi}{m_e^2}\sum_{k=1}^{3}\left|\sum_{j=1}^{3} e^{iq_jL}V_{ej}\left(i\frac{\mu_{jk}}{\mu_{\text{B}}}+\frac{\epsilon_{jk}}{\mu_{\text{B}}}\right)\right|^2\left(\frac{1}{T}-\frac{1}{E_\nu}\right)$$

scattering



Summary

(A) Three reasons for neutrinos to be massless in the SM.

(B) The Dirac mass term and lepton number conservation.

(C) The Majorana mass term and lepton number violation.
---- the Majorana mass matrix must be symmetric;
---- factor 1/2 in front of the mass term makes sense.

(D) The $0\nu\beta\beta$ decay can determine the nature of neutrinos.
---- if a signal is seen, neutrinos must be of Majorana;
---- if a signal is not seen, then there is no conclusion.

(E) Electromagnetic dipole moment of massive neutrinos.
---- Dirac neutrinos have magnetic dipole moments;
---- Majorana neutrinos have no dipole moments;
---- Dirac & Majorana neutrinos: transition moments.

The phenomenology of massive neutrinos will be explored