有效场论

与

大 N 展开

清华大学物理系　　王　青

Back

Close

基础篇：有效场论

之

量子色动力学

与

赝标介子手征有效拉氏量

引　　言

有效场论： 在特定的能量范围针对特定的对象构造的量子场理论

- 存在各种各样的有效场理论 泛泛地介绍
- 以赝标介子的手征有效拉氏量最早最完善的有效场论为例进行介绍
- 对其它低能强子简要介绍，对标准模型请听廖益老师的课

为什么要谈QCD?

- 授课对象:从事核及强子物理研究的青年教师和研究生
- QCD是核物理及强子物理背后的基础理论
- 核及强子物理的模型可看成是QCD低能有效场论的领头阶

赝标介子手征有效拉氏量：

- 跨强子物理与核物理；系统和成功的理论；有QCD基础
- 需群论，粒子物理，量子场论和规范场的知识
- 集成散居各处的知识，抓几个点细讲 很多推导只给结果，，

细节留给同学自己练习； 希望主要纪录在PPT上修改了的或没写的讲解内容！

有效场论基础文献：

- **Effective Field Theories** (Schladming lectures)
  A. Manohar, hep-ph/9606222
- **Goldstone and pseudoGoldstone bosons in nuclear, particle and condensed matter physics**
  C.Burgess, hep-th/9808176
- **Goldstone Boson primer**
  C. Burgess, hep-ph/9812468
- **Lectures on Effective Field Theories** (TASI lectures)
  I. Rothstein, hep-ph/0308266
- **Effective field theories**, Encyclopedia of Mathematical Physics
  G. Ecker, hep-ph/0507056
- **Five lectures on effective field theory**
  D.B. Kaplan, nucl-th/0510023
- **Effective field theory, past and future**
  Steven Weinberg, ArXiv: 0908.1964 [hep-th]

手征有效拉氏量基础文献：

- Departures from chiral symmetry: a review
  Heinz Pagels, Phys. Rept. 16, 219(1975)

- Phenomenological lagrangians
  Steven Weinberg, Physica A96, 327(1979)

- Approaching the chiral limit in qcd
  J. Gasser and A. Zepeda, Nucl. Phys. B174, 445(1980)

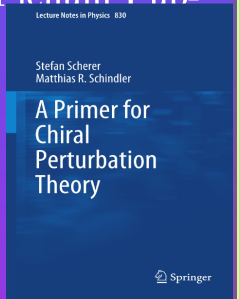
- Chiral perturbation theory to one loop
  J. Gasser and H. Leutwyler, Annals Phys. 158, 142(1984)

- Chiral perturbation theory: expansions in the mass of the strange quark
  J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465(1985)

手征有效拉氏量介绍文献：

- Hadrons 94 lectures, H. Leutwyler, hep-ph/9406283, local ps.gz version (hep-ph version has figures stored inside the tex file)
- Chiral Dynamics 1997 introduction, J. Gasser, hep-ph/9711503
- Les Houches Lectures, A. Pich, hep-ph/9806303
- Benasque lectures, G.Ecker, hep-ph/0011026
- Boris Ioffe Festschrift, H. Leutwyler, hep-ph/0008124
- Frascati Spring school lectures, G. Colangelo and G. Isidori, hep-ph/0101264
- Introduction to chiral perturbation theory,S. Scherer, in Advances in Nuclear Physics, (Editors: J.W. Negele and E. Vogt, Kluwer Academic / Plenum Publishers New York, 2003), Vol. 27 pages 277-538, hep-ph/0210398
- Schladming Lectures, J. Gasser, hep-ph/0312367
- Lectures at ECT*, S. Scherer and M. Schindler, hep-ph/0505265
- Lectures given at Workshop on Perspectives in Lattice QCD
  S. Sharpe, hep-lat/0607016
- Lectures at "Physics and Astrophysics of Hadrons and Hadronic Matter"
  B. Kubis, hep-ph/0703274
- Chiral perturbation theory beyond one loop
  J.Bijnens, Prog. Part. Nucl. Phys. 58, 521(2007)

http://home.thep.lu.se/ bijnens/chpt/



- 有效场论

  历史，背景与评述

- 量子色动力学与手征对称性
  - 量子色动力学的基本内容
  - QCD拉氏量的整体对称性
  - QCD拉氏量的分立对称性
  - 流流格林函数
  - 外场与QCD的格林函数

  在QCD水平上建立产生流流格林函数的生成泛函，讨论其满足的对称性

- 赝标介子手征有效拉氏量
  - 强子谱对称性分类与手征对称性自发破缺
  - 夸克对凝聚与手征对称性自发破缺
  - Goldstone场的变换性质与群定义
  - 赝标介子手征有效拉氏量
  - 圈图计算与重整化
  - 树图

有效场论的历史，背景和评述

上世纪六十年代初的量子场论：

- 以具体拉氏量 或哈密顿量为基础的量子场论不受待见 甚至要被抛弃！
- 因为所有已发现的量子场论都具有“零荷问题”：
  - 有限的裸荷 ⇒ 为零的重整化荷 ⇒ 无相互作用 ！
  - 所有“非渐近自由”的量子场论都具有这个性质
  - 当时的结论：量子场论的高能 强子物理 不可靠

- Dyson对描述强作用的具体理论 哈密顿量 甚至预言：
  the correct theory will not be found in the next hundred years
- 代之以S矩阵理论；色散关系；流代数...... 它们不依赖于具体的拉氏量

有效场理论的诞生：

- 1966年Weinberg应用流代数计算核子碰撞时的多个软π发射
- 发现虽然可算，但极其复杂
- 但2个和3个软π发射的振幅很像量子场论的最低阶费曼图结果
  只要π只从核子外线发射，核子可以与一个或多个π发生相互作用
- Weinberg发现只要 π的相互作用正比于其动量，就可达此目的
- 只要从线性σ模型出发，重新定义场构造：非线性σ模型
- 从它出发得到了用流代数通过非常困难计算得到的 低能$\pi-\pi$和π−核子散射的结果
- J.Schwinger提醒：非线性σ模型可直接从非线性手征变换构造
- Callan, Coleman, Wess, Zumino进一步把结果推广到一般情形
- 1976年受Wilson重整化理论的启发Weinberg解决了高阶计算和重整化问题
- 1979年提出folk theorem作为有效场论的理论基础
- Gasser和 Leutwyler 把π的有效场论建成了路径积分的表达形式
- 以后又陆续发展起其它各种有效场论……

有效场论的基础 Folk Theorem：

If one writes down

the most general possible Lagrangian,

including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties.

有效场论的基本想法：

- 低能或长距离的动力学不依赖于高能或短距离的动力学!

  高能区的效应只影响低能理论中的参数

- 低能物理可以用只包含少数自由度的 有效拉氏量 描写.

- 在高能时才出现的额外的自由度如重粒子可以被略去.

  高能自由度在低能区的效应可以用低能自由度构造的有效算符 来描写

有效场论与规范场论：

- 1973年发现很多非阿贝尔规范场论具有渐近自由性质04年诺贝尔物理奖
- 只有渐近自由的量子场论可以逃脱“零荷问题”
- 粒子物理的“标准模型”就是用非阿贝尔规范场论描写的
- 非阿贝尔规范场论的出现导致量子场论的“复兴”
- 但非阿贝尔规范场论渐近自由的确认晚于π介子有效场论的建立
- 即使不存在渐近自由，有效场论也会使量子场论“重生”
- 描述四种基本相互作用的粒子物理标准模型和广义相对论是有效场论的 领头阶!
- 有效场论是 Theory of Everything 的另一面极端

相对论情形下Goldstone粒子的有效场论的特殊性：

- 连续对称性自发破缺时每个破缺生成元对应一个零质量粒子Goldstone粒子

  08年诺贝尔物理奖！

- 若连续对称性的自发破缺是近似的,则Goldstone粒子具有小质量赝Goldstone粒子

- 对发生近似连续对称性自发破缺的体系,产生的赝Goldstone为其低能区主要自由度

- 以赝Goldstone为自由度的有效场论描写近似连续对称性自发破缺导致的各种性质

- 对称性及破缺在基本相互作用中起核心作用,赝Goldstone有效场论因而有特殊地位

- 强作用低能的两个基本物理性质: 近似的手征对称性自发破缺; 颜色禁闭

- 赝标介子被理解为赝Goldstone粒子, 其有效场论描述了强作用的低能物理

量子色动力学与手征对称性

Table 1: 夸克与胶子

粒子	电荷	自旋	质量
上夸克 (up)	2/3	1/2	2.08MeV
下夸克 (down)	-1/3	1/2	4.73MeV
奇异夸克 (strange)	-1/3	1/2	104MeV
粲夸克 (charm)	2/3	1/2	1.27GeV
底夸克 (bottom)	-1/3	1/2	4.2GeV
顶夸克 (top)	2/3	1/2	171GeV
胶子(gluon)	0	1	0

- 每种夸克分别有“红”、“绿”、“蓝”三种颜色都具有相同的质量
- 胶子有八种分别带不同的颜色

QCD拉氏量:

$$\mathcal{L}_{QCD} = \sum_{f=u,d,s,c,b,t} \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,i}\mathcal{G}_i^{\mu\nu} + \frac{g^2\bar{\theta}}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}\sum_{i=1}^{8}\mathcal{G}_{\mu\nu,i}\mathcal{G}_{\rho\sigma,i}$$

夸克q_f是色三重态

$$\not{D} \equiv \gamma^\mu D_\mu \qquad D_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} = \partial_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} - ig\sum_{i=1}^{8}\frac{\lambda_i^C}{2}\mathcal{A}_{\mu,i}\begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}$$

$q_{f,\alpha}$ 是Dirac旋量 $\quad \bar{q}_f(x)\frac{\lambda_i^C}{2}\not{\mathcal{A}}_i(x)q_f(x) = \bar{q}_{f,\alpha,s}(x)q_{f,\beta,s'}(x)\mathcal{A}_{\mu,i}(x)\frac{\lambda_{i,\alpha\beta}^C}{2}\gamma_{ss'}^\mu$

味道空间：$f = u, d, s, c, b, t$ 颜色空间：$\alpha, \beta = r, g, b$ $\quad i = 1, .., 8$ 旋量空间：$s, s' = 1, 2, 3, 4$

$$\mathcal{G}_{\mu\nu,i} = \partial_\mu\mathcal{A}_{\nu,i} - \partial_\nu\mathcal{A}_{\mu,i} + gf_{ijk}\mathcal{A}_{\mu,j}\mathcal{A}_{\nu,k} \qquad [\lambda_i^C, \lambda_j^C] = if_{ijk}\lambda_k^C$$

QCD拉氏量的整体对称性： 左手与右手夸克场，流代数

$$\mathcal{L}_{QCD} = \underbrace{\mathcal{L}_{QCD,0}}_{\text{手征极限QCD}} \underbrace{- \sum_{l=u,d,s} m_l \bar{q}_l q_l}_{\mathcal{L}_M} \qquad \mathcal{L}_{QCD,0} = \sum_{l=u,d,s} \bar{q}_l i \not{D} q_l + \mathcal{L}'$$

$$\mathcal{L}' = \sum_{h=c,b,t} \bar{q}_h (i\not{D} - m_h) q_h - \frac{1}{4} \mathcal{G}_{\mu\nu,i} \mathcal{G}_i^{\mu\nu} + \frac{g^2 \bar{\theta}}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} \sum_{i=1}^{8} \mathcal{G}_{\mu\nu,i} \mathcal{G}_{\rho\sigma,i}$$

$$P_R = \frac{1}{2}(1+\gamma_5) = P_R^\dagger, \qquad P_L = \frac{1}{2}(1-\gamma_5) = P_L^\dagger, \qquad P_R P_L = P_L P_R = 0$$

$$P_R^2 = P_R \qquad P_L^2 = P_L \qquad P_R + P_L = 1 \qquad q_R = P_R q \qquad q_L = P_L q$$

$$\bar{q}\Gamma_i q = \begin{cases} \bar{q}_R \Gamma_1 q_R + \bar{q}_L \Gamma_1 q_L & \text{for} \quad \Gamma_1 \in \{\gamma^\mu, \gamma^\mu\gamma_5\} \\ \bar{q}_R \Gamma_2 q_L + \bar{q}_L \Gamma_2 q_R & \text{for} \quad \Gamma_2 \in \{1, \gamma_5, \sigma^{\mu\nu}\} \end{cases}$$

夸克质量项联系左手和右手场；夸克协变微商项联系左手和左手场或右手和右手场！

QCD拉氏量的整体对称性： 左手与右手夸克场，流代数

$$\mathcal{L}_{QCD,0} = \sum_{l=u,d,s} (\bar{q}_{L,l} i\not{D}\, q_{L,l} + \bar{q}_{R,l} i\not{D}\, q_{R,l}) + \sum_{h=c,b,t} \bar{q}_h (i\not{D} - m_h) q_h$$

$$-\frac{1}{4}\mathcal{G}_{\mu\nu,i}\mathcal{G}_i^{\mu\nu} + \frac{g^2\bar{\theta}}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}\sum_{i=1}^{8}\mathcal{G}_{\mu\nu,i}\mathcal{G}_{\rho\sigma,i}$$

$$\not{D}\, q_{L,l} = \not{D}\, q_{R,l} = \bar{q}_{L,l}\overleftarrow{\not{D}} = \bar{q}_{R,l}\overleftarrow{\not{D}} = 0 \qquad \overleftarrow{D}_\mu \equiv \overleftarrow{\partial}_\mu + ig\sum_{i=1}^{8}\frac{\lambda_i^C}{2}\mathcal{A}_{\mu,i}$$

$$q_{l,\alpha,L} \to q'_{l,\alpha,L} = (V_L)_{ll'} q_{l',\alpha,L} \qquad q_{l,\alpha,R} \to q'_{l,\alpha,R} = (V_R)_{ll'} q_{l',\alpha,R}$$

$$V_L = e^{-i\Theta_L^a \frac{\lambda_a}{2}} \qquad V_R = e^{-i\Theta_R^a \frac{\lambda_a}{2}}$$

$$L^{\mu,a} = \bar{q}_L \gamma^\mu \frac{\lambda^a}{2} q_L \quad \partial_\mu L^{\mu,a} = 0 \qquad R^{\mu,a} = \bar{q}_R \gamma^\mu \frac{\lambda^a}{2} q_R \quad \partial_\mu R^{\mu,a} = 0$$

$$\partial_\mu L^{\mu,a} = \bar{q}_L \gamma^\mu (\overleftarrow{\partial}_\mu + ig\sum_{i=1}^{8}\frac{\lambda_i^C}{2}\mathcal{A}_{\mu,i} + \partial_\mu - ig\sum_{i=1}^{8}\frac{\lambda_i^C}{2}\mathcal{A}_{\mu,i})\frac{\lambda^a}{2} q_L = \bar{q}_L\gamma^\mu(\overleftarrow{D}_\mu + D_\mu)\frac{\lambda^a}{2} q_L = 0$$

Back
Close

QCD拉氏量的整体对称性： 左手与右手夸克场，流代数

$$V^{\mu,a} = R^{\mu,a} + L^{\mu,a} = \bar{q}\gamma^\mu \frac{\lambda^a}{2} q \qquad Q^a_L = \int d^3x\, q^\dagger_L(\vec{x},t)\frac{\lambda^a}{2} q_L(\vec{x},t)$$

$$A^{\mu,a} = R^{\mu,a} - L^{\mu,a} = \bar{q}\gamma^\mu\gamma_5 \frac{\lambda^a}{2} q \qquad Q^a_R = \int d^3x\, q^\dagger_R(\vec{x},t)\frac{\lambda^a}{2} q_R(\vec{x},t)$$

$$V^\mu = \bar{q}_R\gamma^\mu q_R + \bar{q}_L\gamma^\mu q_L = \bar{q}\gamma^\mu q$$

对应：$V_L = V_R = e^{-i\Theta^0}$

$$Q_V = \int d^3x \left[q^\dagger_L(\vec{x},t) q_L(\vec{x},t) + q^\dagger_R(\vec{x},t) q_R(\vec{x},t)\right]$$

$$A^\mu = \bar{q}_R\gamma^\mu q_R - \bar{q}_L\gamma^\mu q_L = \bar{q}\gamma^\mu\gamma_5 q$$

对应：$V_L = V^\dagger_R = e^{-i\Theta^0}$

- $j^\mu \equiv (\rho, \vec{j})$ $\partial_\mu j^\mu = 0 \Rightarrow \frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j} = 0$ 流守恒保证荷不随时间演化！
- 轴矢流在经典水平上守恒，量子化后拥有一个反常项

Back
Close

QCD拉氏量的整体对称性：左手与右手夸克场，流代数

$$\{q_{f,\alpha,s}(\vec{x},t), q^{\dagger}_{f',\beta,r}(\vec{y},t)\} = \delta(\vec{x}-\vec{y})\delta_{ff'}\delta_{\alpha\beta}\delta_{sr}$$

$$\{q_{f,\alpha,s}(\vec{x},t), q_{f',\beta,r}(\vec{y},t)\} = 0 \qquad \{q^{\dagger}_{f,\alpha,s}(\vec{x},t), q^{\dagger}_{f',\beta,r}(\vec{y},t)\} = 0$$

$$[q^{\dagger}(\vec{x},t)\Gamma_1 q(\vec{x},t), q^{\dagger}(\vec{y},t)\Gamma_2 q(\vec{y},t)]$$

$$= \delta(\vec{x}-\vec{y})[q^{\dagger}(\vec{x},t)\Gamma_1\Gamma_2 q(\vec{y},t) - q^{\dagger}(\vec{y},t)\Gamma_2\Gamma_1 q(\vec{x},t)]$$

$$[Q^a_L, q_{L;f,\alpha,s}(x)] = -\frac{\lambda^a_{ff'}}{2} q_{L;f',\alpha,s} \qquad [Q^a_L, q_{R;f,\alpha,s}(x)] = 0$$

$$[Q^a_R, q_{R;f,\alpha,s}(x)] = -\frac{\lambda^a_{ff'}}{2} q_{R;f',\alpha,s} \qquad [Q^a_R, q_{L;f,\alpha,s}(x)] = 0$$

QCD拉氏量的整体对称性：左手与右手夸克场，流代数

$$\mathcal{L}_{QCD} \quad \Rightarrow \quad H_{QCD,0} \qquad \vec{P}_{QCD} = \vec{P}_{QCD,0}$$

$$[Q^a_L, H_{QCD,0}] = [Q^a_R, H_{QCD,0}] = [Q_L, \vec{P}_{QCD}] = [Q_R, \vec{P}_{QCD}] = 0$$

$$[Q^a_L, Q^b_L] = if_{abc}Q^c_L \qquad [Q^a_R, Q^b_R] = if_{abc}Q^c_R$$

$$[Q^a_L, Q^b_R] = 0 \qquad [Q^a_L, Q_V] = [Q^a_R, Q_V] = 0$$

$$e^A B e^B = B + [A, B] + \frac{1}{2!}[A, [A, B]] + \frac{1}{3!}[A, [A, [A, B]]] + \cdots$$

$$e^{i\Theta^a_L Q^a_L}\ q_L(x)\ e^{-i\Theta^a_L Q^a_L} = V_L\ q_L(x) \qquad e^{i\Theta^a_L Q^a_L}\ q_R(x) e^{-i\Theta^a_L Q^a_L} = q_R(x)$$

$$e^{i\Theta^a_R Q^a_R}\ q_R(x) e^{-i\Theta^a_R Q^a_R} = V_R\ q_R(x) \qquad e^{i\Theta^a_R Q^a_R}\ q_L(x) e^{-i\Theta^a_R Q^a_L} = q_L(x)$$

QCD拉氏量的整体对称性: 轻夸克质量 对称性明显破缺

$$\mathcal{L}_M = -(\bar{q}_R M^\dagger q_L + \bar{q}_L M q_R) = -\bar{q}[\frac{M+M^\dagger}{2} + \frac{M-M^\dagger}{2}\gamma_5]q$$

自己试着推导:

$$\partial_\mu V^{\mu,a} = i\bar{q}_L[M, \frac{\lambda^a}{2}]q_R + i\bar{q}_R[M^\dagger, \frac{\lambda^a}{2}]q_L$$

$$\partial_\mu A^{\mu,a} = i\left(\bar{q}_L\{\frac{\lambda^a}{2}, M\}q_R - \bar{q}_R\{\frac{\lambda^a}{2}, M^\dagger\}q_L\right)$$

$$\partial_\mu V^\mu = 0$$

$$\partial_\mu A^\mu = 2i\left(\bar{q}_L M q_R - \bar{q}_R M^\dagger q_L\right) + \frac{3g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathcal{G}_i^{\mu\nu}\mathcal{G}_i^{\rho\sigma}, \qquad \epsilon_{0123} = 1$$

- 问 ： 近似的对称性是有对称性还是没有对称性？
- 答 ： 没有对称性
- 问 ： 那还搞个啥？ 对称性都没了
- 答 ： 我们希望对没对称性的系统挖掘出信息来不放弃对它的研究
- 续答： 就把它进一步分类为有近似对称性和无近似对称性的体系
- 续答： 研究发现前者可以有很多性质和对称性有密切的关联
- 续答： 和近似的对称性对应自发破缺的对称性又叫隐藏的对称性
- 续答： 对隐藏的对称性物理体系的对称性同样也是没有滴！

$$\vec{x} \to -\vec{x} \qquad t \to t$$

$$q_{f,\alpha}(\vec{x},t) \to \gamma^0 q_{f,\alpha}(-\vec{x},t) \qquad \bar{q}_{f,\alpha}(\vec{x},t) \to \bar{q}_{f,\alpha}(-\vec{x},t)\gamma^0$$

$$\mathcal{A}_{0,i}(\vec{x},t) \to \mathcal{A}_{0,i}(-\vec{x},t) \qquad \mathcal{A}_{j,i}(\vec{x},t) \to -\mathcal{A}_{j,i}(-\vec{x},t)$$

$$\bar{q}_{f,\alpha}(\vec{x},t)\Gamma q_{f',\beta}(\vec{x},t) \to \bar{q}_{f,\alpha}(-\vec{x},t)\gamma^0\Gamma\gamma^0 q_{f',\beta}(-\vec{x},t)$$

$$\bar{\theta} \to -\bar{\theta}$$

$$\mathcal{L}_{QCD}(\vec{x},t) = \sum_{l=u,d,s}(\bar{q}_{L,l} i\not{D}\, q_{L,l} + \bar{q}_{R,l} i\not{D}\, q_{R,l}) + \sum_{h=c,b,t}\bar{q}_h(i\not{D} - m_h)q_h$$
$$-\frac{1}{4}\mathcal{G}_{\mu\nu,i}\mathcal{G}_i^{\mu\nu} + \frac{g^2\bar{\theta}}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}\sum_{i=1}^{8}\mathcal{G}_{\mu\nu,i}\mathcal{G}_{\rho\sigma,i} \xrightarrow{P} \mathcal{L}_{QCD}(-\vec{x},t)$$

QCD拉氏量的分立对称性： 宇称变换

$$\bar{\mathbf{q}}_{\mathbf{f},\alpha}(\vec{\mathbf{x}},\mathbf{t})\gamma^0\mathbf{P}_\mathbf{L}\mathbf{q}_{\mathbf{f}',\beta}(\vec{\mathbf{x}},\mathbf{t}) \rightarrow \bar{\mathbf{q}}_{\mathbf{f},\alpha}(-\vec{\mathbf{x}},\mathbf{t})\gamma^0\mathbf{P}_\mathbf{R}\mathbf{q}_{\mathbf{f}',\beta}(-\vec{\mathbf{x}},\mathbf{t})$$

$$\bar{\mathbf{q}}_{\mathbf{f},\alpha}(\vec{\mathbf{x}},\mathbf{t})\gamma^0\mathbf{P}_\mathbf{R}\mathbf{q}_{\mathbf{f}',\beta}(\vec{\mathbf{x}},\mathbf{t}) \rightarrow \bar{\mathbf{q}}_{\mathbf{f},\alpha}(-\vec{\mathbf{x}},\mathbf{t})\gamma^0\mathbf{P}_\mathbf{L}\mathbf{q}_{\mathbf{f}',\beta}(-\vec{\mathbf{x}},\mathbf{t})$$

$$\mathbf{Q}^\mathbf{a}_\mathbf{L} \xrightarrow{P} \mathbf{Q}^\mathbf{a}_\mathbf{R} \qquad \mathbf{Q}^\mathbf{a}_\mathbf{R} \xrightarrow{P} \mathbf{Q}^\mathbf{a}_\mathbf{L} \qquad \mathbf{Q}_\mathbf{V} \xrightarrow{P} \mathbf{Q}_\mathbf{V}$$

宇称变换将左手群变成右手群！

QCD拉氏量的分立对称性：电荷共轭变换

$$\vec{x} \to \vec{x} \qquad t \to t$$

$$q_{f,\alpha}(\vec{x}, t) \to C\bar{q}^{\dagger}_{f,\alpha}(\vec{x}, t) \qquad \bar{q}_{f,\alpha}(\vec{x}, t) \to -q^{\dagger}_{f,\alpha}(\vec{x}, t)C^{-1}$$

$$\mathcal{A}_{\mu,i}(\vec{x}, t)\lambda_i^C \to -\mathcal{A}_{\mu,i}(\vec{x}, t)\lambda_i^{C,T}$$

$$C = i\gamma^2\gamma^0 = -C^{-1} = -C^{\dagger} = -C^T$$

$$CI^TC = -1 \qquad C\gamma_5^TC = -\gamma_5 \qquad C\sigma^{\mu\nu,T}C = \sigma^{\mu\nu}$$

$$C\gamma^{\mu,T}C = \gamma^{\mu} \qquad C\gamma_5\gamma^{\mu,T}C = -\gamma_5\gamma^{\mu}$$

QCD拉氏量的分立对称性： 电荷共轭变换

$$\bar{q}_{f,\alpha}(\vec{x},t)\Gamma q_{f',\beta}(\vec{x},t) \to \bar{q}_{f',\beta}(\vec{x},t)C^T\Gamma^T C^{-1,T} q_{f,\alpha}(\vec{x},t)$$
$$= -\bar{q}_{f',\beta}(\vec{x},t)C\Gamma^T C q_{f,\alpha}(\vec{x},t)$$

$$\bar{q}_{f,\alpha}(\vec{x},t)\gamma^\mu q_{f',\beta}(\vec{x},t) \to -\bar{q}_{f',\beta}(\vec{x},t)\gamma^\mu q_{f,\alpha}(\vec{x},t)$$

$$\bar{q}_{f,\alpha}(\vec{x},t) q_{f',\beta}(\vec{x},t) \to \bar{q}_{f',\beta}(\vec{x},t) q_{f,\alpha}(\vec{x},t)$$

$$\bar{\theta} \to \bar{\theta}$$

$$\mathcal{L}_{QCD}(\vec{x},t) \stackrel{C}{\longrightarrow} \mathcal{L}(\vec{x},t)$$

QCD拉氏量的分立对称性: 电荷共轭变换

$$\bar{q}_{f,\alpha}(\vec{x},t)\gamma^0 P_L q_{f',\beta}(\vec{x},t) \to -\bar{q}_{f',\beta}(\vec{x},t)\gamma^0 P_R q_{f,\alpha}(\vec{x},t)$$

$$\bar{q}_{f,\alpha}(\vec{x},t)\gamma^0 P_R q_{f',\beta}(\vec{x},t) \to -\bar{q}_{f',\beta}(\vec{x},t)\gamma^0 P_L q_{f,\alpha}(\vec{x},t)$$

$$Q_L^a(t) \xrightarrow{C} Q_R^{\bar{a}} \qquad Q_R^a \xrightarrow{C} Q_R^{\bar{a}} \qquad Q_V \xrightarrow{C} Q_V$$

$$\bar{1}=2 \quad \bar{2}=1 \quad \bar{3}=3 \quad \bar{4}=5 \quad \bar{5}=4 \quad \bar{6}=7 \quad \bar{7}=6 \quad \bar{8}=8$$

电荷共轭联系的是 $\lambda^{\bar{a}} \equiv \lambda^{a,T}$ 的两个态

流流格林函数

$$\mathbf{S^a(x) = \bar{q}(x)\lambda^a q(x)} \qquad \mathbf{P^a(x) = i\bar{q}(x)\gamma_5\lambda^a q(x)}$$

$$\mathbf{V^{\mu,a}(x) = \bar{q}(x)\gamma^\mu \frac{\lambda^a}{2} q(x)} \qquad \mathbf{A^{\mu,a}(x) = \bar{q}(x)\gamma^\mu\gamma_5 \frac{\lambda^a}{2} q(x)}$$

$$\mathbf{V^\mu = \bar{q}(x)\gamma^\mu q(x)} \qquad \mathbf{A^\mu(x) = \bar{q}(x)\gamma^\mu\gamma_5 q(x)}$$

如此定义的量都是厄米的！特别赝标密度中的i就是为确保其厄米性而引入的

$$\langle \mathbf{0}|\mathbf{T}\ \hat{\phi}_1(\mathbf{x_1})\hat{\phi}_2(\mathbf{x_2})\cdots\hat{\phi}_n(\mathbf{x_n})|\mathbf{0}\rangle$$

$$\mathbf{G}^{\mu,\mathbf{ab}}_{\mathrm{AP}}(\mathbf{x},\mathbf{y})$$

$$= \langle \mathbf{0}|\mathbf{T}[\mathbf{A}^{\mu,\mathbf{a}}(\mathbf{x})\mathbf{P}^{\mathbf{b}}(\mathbf{y})]|\mathbf{0}\rangle$$

$$= \theta(\mathbf{x_0}-\mathbf{y_0})\langle \mathbf{0}|\mathbf{A}^{\mu,\mathbf{a}}(\mathbf{x})\mathbf{P}^{\mathbf{b}}(\mathbf{y})|\mathbf{0}\rangle + \theta(\mathbf{y_0}-\mathbf{x_0})\langle \mathbf{0}|\mathbf{P}^{\mathbf{b}}(\mathbf{y})\mathbf{A}^{\mu,\mathbf{a}}(\mathbf{x})|\mathbf{0}\rangle$$

$$\partial_{\mu,\mathbf{x}}\mathbf{G}^{\mu,\mathbf{ab}}_{\mathrm{AP}}(\mathbf{x},\mathbf{y})$$

$$= \delta(\mathbf{x_0}-\mathbf{y_0})\langle \mathbf{0}|\mathbf{A}^{\mathbf{0},\mathbf{a}}(\mathbf{x})\mathbf{P}^{\mathbf{b}}(\mathbf{y})|\mathbf{0}\rangle - \delta(\mathbf{x_0}-\mathbf{y_0})\langle \mathbf{0}|\mathbf{P}^{\mathbf{b}}(\mathbf{y})\mathbf{A}^{\mathbf{0},\mathbf{a}}(\mathbf{x})|\mathbf{0}\rangle$$
$$+\theta(\mathbf{x_0}-\mathbf{y_0})\langle \mathbf{0}|\partial_{\mu,\mathbf{x}}\mathbf{A}^{\mu,\mathbf{a}}(\mathbf{x})\mathbf{P}^{\mathbf{b}}(\mathbf{y})|\mathbf{0}\rangle + \theta(\mathbf{y_0}-\mathbf{x_0})\langle \mathbf{0}|\mathbf{P}^{\mathbf{b}}(\mathbf{y})\partial_{\mu,\mathbf{x}}\mathbf{A}^{\mu,\mathbf{a}}(\mathbf{x})|\mathbf{0}\rangle$$

$$= \delta(\mathbf{x_0}-\mathbf{y_0})\langle \mathbf{0}|[\mathbf{A}^{\mathbf{0},\mathbf{a}}(\mathbf{x}),\mathbf{P}^{\mathbf{b}}(\mathbf{y})]|\mathbf{0}\rangle + \langle \mathbf{0}|\mathbf{T}\partial_{\mu,\mathbf{x}}\mathbf{A}^{\mu,\mathbf{a}}(\mathbf{x})\mathbf{P}^{\mathbf{b}}(\mathbf{y})|\mathbf{0}\rangle$$

$$\partial_{\mu,\mathbf{x}}\langle\mathbf{0}|\mathbf{T}\{\mathbf{J}^{\mu}(\mathbf{x})\hat{\phi}_{1}(\mathbf{x}_{1})\hat{\phi}_{2}(\mathbf{x}_{2})\cdots\hat{\phi}_{\mathbf{n}}(\mathbf{x}_{\mathbf{n}})\}|\mathbf{0}\rangle$$

$$=\langle\mathbf{0}|\mathbf{T}\{[\partial_{\mu,\mathbf{x}}\mathbf{J}^{\mu}(\mathbf{x})]\hat{\phi}_{1}(\mathbf{x}_{1})\hat{\phi}_{2}(\mathbf{x}_{2})\cdots\hat{\phi}_{\mathbf{n}}(\mathbf{x}_{\mathbf{n}})\}|\mathbf{0}\rangle$$

$$+\delta(\mathbf{x}^{0}-\mathbf{x}_{1}^{0})\langle\mathbf{0}|\mathbf{T}\{[\mathbf{J}^{0}(\mathbf{x}),\hat{\phi}_{1}(\mathbf{x}_{1})]\hat{\phi}_{2}(\mathbf{x}_{2})\cdots\hat{\phi}_{\mathbf{n}}(\mathbf{x}_{\mathbf{n}})\}|\mathbf{0}\rangle$$

$$+\delta(\mathbf{x}^{0}-\mathbf{x}_{2}^{0})\langle\mathbf{0}|\mathbf{T}\{\hat{\phi}_{1}(\mathbf{x}_{1})[\mathbf{J}^{0}(\mathbf{x}),\hat{\phi}_{2}(\mathbf{x}_{2})]\cdots\hat{\phi}_{\mathbf{n}}(\mathbf{x}_{\mathbf{n}})\}|\mathbf{0}\rangle$$

$$+\cdots+\delta(\mathbf{x}^{0}-\mathbf{x}_{\mathbf{n}}^{0})\langle\mathbf{0}|\mathbf{T}\{\hat{\phi}_{1}(\mathbf{x}_{1})\hat{\phi}_{2}(\mathbf{x}_{2})\cdots[\mathbf{J}^{0}(\mathbf{x}),\hat{\phi}_{\mathbf{n}}(\mathbf{x}_{\mathbf{n}})]\}|\mathbf{0}\rangle$$

Back
Close

流流格林函数: 手征Ward恒等式与流代数

$$[V_0^a(\vec{x},t), V^{\mu,b}(\vec{y},t)] = \delta(\vec{x}-\vec{y}) i f^{abc} V^{\mu,c}(\vec{x},t) \qquad [V_0^a(\vec{x},t), V^{\mu}(\vec{y},t)] = 0$$

$$[V_0^a(\vec{x},t), A^{\mu,b}(\vec{y},t)] = \delta(\vec{x}-\vec{y}) i f^{abc} A^{\mu,c}(\vec{x},t)$$

$$[V_0^a(\vec{x},t), S^b(\vec{y},t)] = \delta(\vec{x}-\vec{y}) i f^{abc} S^{\mu,c}(\vec{x},t) \qquad [V_0^a(\vec{x},t), S^0(\vec{y},t)] = 0$$

$$[V_0^a(\vec{x},t), P^b(\vec{y},t)] = \delta(\vec{x}-\vec{y}) i f^{abc} P^c(\vec{x},t) \qquad [V_0^a(\vec{x},t), P^0(\vec{y},t)] = 0$$

$$[A_0^a(\vec{x},t), A^{\mu,b}(\vec{y},t)] = \delta(\vec{x}-\vec{y}) i f^{abc} V^{\mu,c}(\vec{x},t)$$

$$[A_0^a(\vec{x},t), V^{\mu,b}(\vec{y},t)] = \delta(\vec{x}-\vec{y}) i f^{abc} A^{\mu,c}(\vec{x},t) \qquad [A_0^a(\vec{x},t), V^{\mu}(\vec{y},t)] = 0$$

$$[A_0^a(\vec{x},t), S^b(\vec{y},t)] = \delta(\vec{x}-\vec{y}) i f^{abc} P^c(\vec{x},t) \qquad [A_0^a(\vec{x},t), S^0(\vec{y},t)] = 0$$

$$[A_0^a(\vec{x},t), P^b(\vec{y},t)] = \delta(\vec{x}-\vec{y}) i f^{abc} S^c(\vec{x},t) \qquad [A_0^a(\vec{x},t), P^0(\vec{y},t)] = 0$$

流流格林函数：手征Ward恒等式与流代数

$$\partial_{\mu,x} G^{\mu,ab}_{AP}(x,y)$$

$$= \delta(x-y) i f^{abc} \langle 0|S^c(x)|0\rangle + i\langle 0|T[\overline{q}(x)\{\frac{\lambda^a}{2}, M\}\gamma_5 q(x) P^b(y)]|0\rangle$$

$$\overline{q}(x)\{\frac{\lambda^a}{2}, M\}\gamma_5 q(x)$$

$$= [\frac{1}{3}(m_u+m_d+m_s) + \frac{1}{\sqrt{3}}(\frac{m_u+m_d}{2}-m_s)d^{aa8}]P^a(x)$$

$$+[\sqrt{\frac{1}{6}}(m_u-m_d)\delta^{a3} + \frac{\sqrt{2}}{3}(\frac{m_u+m_d}{2}-m_s)\delta^{a8}]P^0(x)$$

$$+\frac{m_u-m_d}{2}\sum_{c=1}^{8} d^{a3c}P^c(x)$$

$$\mathcal{L} = \mathcal{L}_{\rm QCD,0} + \bar{q} J q \qquad J(x) \equiv \not{v}(x) + \not{a}(x)\gamma_5 - s(x) + ip(x)\gamma_5$$

$$v^\mu = \sum_{a=0}^{8} \lambda^a v^{\mu,a} \qquad a^\mu = \sum_{a=0}^{8} \lambda^a a^{\mu,a} \qquad s = \sum_{a=0}^{8} \lambda^a s^a \qquad p = \sum_{a=0}^{8} \lambda^a p^a$$

$$\lambda^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\lambda^1 \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 \equiv \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

真空： $v^\mu = a^\mu = p = 0 \qquad s = \mathrm{diag}(m_u, m_d, m_s)$

$$\exp(iZ[J,\bar{\theta}]) = \langle 0 | T \exp\left[i \int d^4x \;\; \bar{q}(x) J(x) q(x) \right] |0\rangle_{\bar{\theta}} = \langle 0_{\rm out} | 0_{\rm in} \rangle_{J,\bar{\theta}}$$

Back
Close

例子:

一点格林函数

$$\langle 0|\bar{u}u|0\rangle_0 = \frac{1}{2}\left[\frac{2}{3}\frac{\delta}{\delta s^0(x)} + \frac{\delta}{\delta s^3(x)} + \frac{1}{\sqrt{3}}\frac{\delta}{\delta s^8(x)}\right] Z[J,\bar{\theta}]\Big|_{J=0}$$

$$= \frac{1}{2}\Big[\frac{2}{3}(\langle 0|\bar{u}u|0\rangle_0 + \langle 0|\bar{d}d|0\rangle_0 + \langle 0|\bar{s}s|0\rangle_0) + (\langle 0|\bar{u}u|0\rangle_0 - \langle 0|\bar{d}d|0\rangle_0) + \frac{1}{3}(\langle 0|\bar{u}u|0\rangle_0 + \langle 0|\bar{d}d|0\rangle_0 - 2\langle 0|\bar{s}s|0\rangle_0)\Big]$$

两点格林函数

$$\langle 0|TA_\mu^a(x)A_\nu^b(0)|0\rangle_0 = -i\frac{\delta^2}{\delta a^{\mu,a}(x)\delta a^{\nu,b}(0)} Z[J,\bar{\theta}]\Big|_{v=a=p=0,s=\mathrm{diag}(m_u,m_d,m_s)}$$

要求 $\int d^4x \quad \bar{q}(x)J(x)q(x)$不变

$$\int_{x=-\infty}^{x=\infty} dx = -\int_{-x=\infty}^{-x=-\infty} d(-x) = \int_{-x=-\infty}^{-x=\infty} d(-x)$$

宇称变换:

$$v^{\mu} \xrightarrow{P} v_{\mu} \qquad a^{\mu} \xrightarrow{P} -a_{\mu} \qquad s \xrightarrow{P} s \qquad p \xrightarrow{P} -p$$

电荷共轭变换:

$$v_{\mu} \xrightarrow{C} -v_{\mu}^{T} \qquad a_{\mu} \xrightarrow{C} a_{\mu}^{T} \qquad s, p \xrightarrow{C} s^{T}, p^{T}$$

要求 $\int d^4x \;\; \bar{q}(x)[i\partial\!\!\!/ + J(x)]q(x)$不变 为什么要把整体对称性推广成局域对称性? 见后...

$$[V_R^\dagger P_L + V_L^\dagger P_R][i\partial\!\!\!/ + J'(x)][V_R P_R + V_L P_L] = i\partial\!\!\!/ + J(x)$$

$$J \to J' = [V_R P_L + V_L P_R][J + i\partial\!\!\!/][V_R^\dagger P_R + V_L^\dagger P_L]$$

$$v\!\!\!/ + a\!\!\!/ \gamma_5$$
$$\to v\!\!\!/\,' + a\!\!\!/\,'\gamma_5 = [V_R P_L + V_L P_R][v\!\!\!/ + a\!\!\!/ \gamma_5 + i\partial\!\!\!/][V_R^\dagger P_R + V_L^\dagger P_L]$$
$$-s + ip\gamma_5$$
$$\to -s' + ip'\gamma_5 = [V_R P_L + V_L P_R][-s + ip\gamma_5][V_R^\dagger P_R + V_L^\dagger P_L]$$

$$v_\mu + a_\mu \to v'_\mu + a'_\mu = V_R[v_\mu + a_\mu + i\partial_\mu]V_R^\dagger \qquad \gamma_5 = P_R - P_L$$
$$v_\mu - a_\mu \to v'_\mu - a'_\mu = V_L[v_\mu + a_\mu + i\partial_\mu]V_L^\dagger \qquad 1 = P_R + P_L$$

$$-s + ip \to -s' + ip' = V_L[-s + ip]V_R^\dagger$$
$$s + ip \to s' + ip' = V_R[s + ip]V_L^\dagger$$

Back
Close

外场与QCD的格林函数：外场的对称性变换行为

$$e^{iZ[J,\bar{\theta}]} = \int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar{q}_h\mathcal{D}q_h\mathcal{D}\bar{q}\mathcal{D}q \; e^{i\int d^4x \; [\mathcal{L}_{QCD}+\bar{q}Jq]}$$

$$Z[J,\bar{\theta}] = Z[J^P,\bar{\theta}^P] \qquad\qquad Z[J,\bar{\theta}] = Z[J^C,\bar{\theta}^C]$$

$$\mathcal{D}\mathcal{A}^P_{\mu,i}\mathcal{D}\bar{q}^P_h\mathcal{D}q^P_h\mathcal{D}\bar{q}^P\mathcal{D}q^P = \mathcal{D}\mathcal{A}^C_{\mu,i}\mathcal{D}\bar{q}^C\mathcal{D}q^C\mathcal{D}\bar{q}^C\mathcal{D}q^C = \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar{q}_h\mathcal{D}q_h\mathcal{D}\bar{q}\mathcal{D}q$$

生成泛函在洛伦兹变换下也是不变的!

$$Z[J,\bar{\theta}] = Z[J',\bar{\theta}]\big|_{\text{略去反常}}$$

$$\int \mathcal{D}\bar{q}\mathcal{D}q\ e^{i\int d^4x\ \bar{q}(\cdots)\mathcal{D}q} = \mathrm{Det}(\cdots)$$

$$e^{iZ[J,\bar{\theta}]} = \int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar{q}_h\mathcal{D}q_h\ \mathrm{Det}[i\not\partial + g\not\mathcal{A}_i\lambda_i^C/2 + J]\ e^{i\int d^4x\mathcal{L}'}$$

$$= \int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar{q}_h\mathcal{D}q_h\ \exp\left\{\mathrm{Trln}[i\not\partial + g\not\mathcal{A}_i\lambda_i^C/2 + J] + i\int d^4x\mathcal{L}'\right\}$$

$$\mathrm{Trln}(\cdots) = \mathrm{lnDet}(\cdots)$$

$$e^{iZ[J',\bar{\theta}]} = \int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar{q}_h\mathcal{D}q_h\ \exp\left\{\mathrm{Trln}[i\not\partial + g\not\mathcal{A}_i\lambda_i^C/2 + J'] + i\int d^4x\mathcal{L}'\right\}$$

$$= \int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar{q}_h\mathcal{D}q_h\ \exp\left\{\mathrm{Trln}[i\not\partial + g\not\mathcal{A}_i\lambda_i^C/2 + J]\right.$$

$$\left. + \delta\mathrm{Trln}[i\not\partial + g\not\mathcal{A}_i\lambda_i^C/2 + J] + i\int d^4x\mathcal{L}'\right\}$$

Back

Close

$$V_R(x) = 1 + i\alpha(x) + i\beta(x) + \cdots \qquad V_L(x) = 1 + i\alpha(x) - i\beta(x) + \cdots$$

$$\delta J = i[\alpha(x) - \beta(x)\gamma_5]J(x) - [i\partial\!\!\!/ + J(x)]i[\alpha(x) + \beta(x)\gamma_5]$$

$$\delta \mathrm{Tr}\ln[i\partial\!\!\!/ + gA\!\!\!/_i\lambda_i^C/2 + J]$$
$$= \mathrm{Tr}\left[[i\partial\!\!\!/ + gA\!\!\!/_i\lambda_i^C/2 + J]^{-1}\delta[i\partial\!\!\!/ + gA\!\!\!/_i\lambda_i^C/2 + J]\right]$$
$$= i\mathrm{Tr}\left[[i\partial\!\!\!/ + gA\!\!\!/_i\lambda_i^C/2 + J]^{-1}\{[\alpha(x) - \beta(x)\gamma_5][i\partial\!\!\!/ + gA\!\!\!/_i\lambda_i^C/2 + J] - [i\partial\!\!\!/ + gA\!\!\!/_i\lambda_i^C/2 + J][\alpha(x) + \beta(x)\gamma_5]\}\right]$$
$$= -2i\mathrm{Tr}[\beta\gamma_5] = -2i\lim_{\Lambda\to\infty}\mathrm{Tr}\left[\beta\gamma_5 e^{\frac{[i\partial\!\!\!/ + gA\!\!\!/_i\lambda_i^C/2 + J]^2}{\Lambda^2}}\right]$$
$$= -i\int d^4x\, \mathrm{tr_f}\left[\beta(x)[\tilde{\Omega}(x) + \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathcal{G}_{\mu\nu,i}(x)\mathcal{G}_{\rho\sigma,i}(x)]\right]$$

Fujikawa, Phys.Rev.D21,2848(1980); Phys.Rev.D22,1499(1980); Phys.Rev.D29,285(1984)

$$\tilde{\Omega}(x) = \frac{N_c}{16\pi^2}\epsilon^{\mu\nu\mu'\nu'}\left\{V_{\mu\nu}(x)V_{\mu'\nu'}(x) + \frac{4}{3}d_\mu a_\nu(x)d_{\mu'}a_{\nu'}(x) + \frac{2i}{3}\{V_{\mu\nu}(x), a_{\mu'}(x)a_{\nu'}(x)\} + \frac{8i}{3}a_\mu(x)V_{\mu'\nu'}(x)a_\nu(x) + \frac{4}{3}a_\mu(x)a_\nu(x)a_{\mu'}(x)a_{\nu'}(x)\right\}$$

$$V_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu], \quad d_\mu a_\nu = \partial_\mu a_\nu - i[v_\mu, a_\nu]$$

Back
Close

外场与QCD的格林函数: 外场的对称性变换行为

$$e^{iZ[J',\bar\theta]}$$
$$= \int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar{q}_h\mathcal{D}q_h \quad \exp\{\mathrm{Trln}[i\partial\!\!\!/ + g\mathcal{A}\!\!\!/_i\lambda_i^C/2 + J(x)]$$
$$+\delta\mathrm{Trln}[i\partial\!\!\!/ + g\mathcal{A}\!\!\!/_i\lambda_i^C/2 + J] + i\int d^4x\mathcal{L}'\}$$

$$+\frac{g^2\bar\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}\sum_{i=1}^{8}\mathcal{G}_{\mu\nu,i}\mathcal{G}_{\rho\sigma,i}$$

$$= \exp\{-i\int d^4x\ \mathrm{tr_f}[\beta(x)\tilde\Omega(x)]\}\int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar{q}_h\mathcal{D}q_h$$
$$\times\exp\{\mathrm{Trln}[i\partial\!\!\!/ + g\mathcal{A}\!\!\!/_i\lambda_i^C/2 + J(x)] + i\int d^4x[\mathcal{L}' \underbrace{-\frac{g^2\mathrm{tr}_f\beta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathcal{G}_{\mu\nu,i}\mathcal{G}_{\rho\sigma,i}}_{U_A(1)\ \text{反常}}]\}$$

$$\bar\theta(x)\to\bar\theta'(x) = \bar\theta(x) + 2\mathrm{tr_f}\beta(x) = \bar\theta(x) - i\ln\det{}_f[V_R(x)V_L^\dagger(x)]$$ 后面证

$$Z[J',\bar\theta'] = Z[J,\bar\theta]\underbrace{-\int d^4x\ \mathrm{tr_f}[\beta(x)\tilde\Omega(x)]}_{\text{反常}}$$

Back
Close

$$\delta \mathbf{Z}[\mathbf{J}, \bar{\theta}] = -\int \mathbf{d}^4\mathbf{x}\ \mathrm{tr}_{\mathbf{f}}[\beta(\mathbf{x})\tilde{\mathbf{\Omega}}(\mathbf{x})]$$

Back
Close

$$\mathbf{Z}[\mathbf{J}',\bar{\theta}'] = \mathbf{Z}[\mathbf{J},\bar{\theta}] - \int \mathbf{d^4x}\ \mathrm{tr_f}[\beta(\mathbf{x})\tilde{\mathbf{\Omega}}(\mathbf{x})]$$

对有限大变换:

$$\mathbf{V_R}(\mathbf{x}) = \mathbf{e}^{\mathbf{i}[\beta(\mathbf{x})+\alpha(\mathbf{x})]} = \lim_{\mathbf{n}\to\infty}\left\{\mathbf{1}+\frac{\mathbf{i}}{\mathbf{n}}[\alpha(\mathbf{x})+\beta(\mathbf{x})]\right\}^{\mathbf{n}}$$

$$\mathbf{V_L}(\mathbf{x}) = \mathbf{e}^{-\mathbf{i}[\beta(\mathbf{x})-\alpha(\mathbf{x})]} = \lim_{\mathbf{n}\to\infty}\left\{\mathbf{1}+\frac{\mathbf{i}}{\mathbf{n}}[\alpha(\mathbf{x})-\beta(\mathbf{x})]\right\}^{\mathbf{n}}$$

$$\mathbf{J}' = [\mathbf{V_R P_L}+\mathbf{V_L P_R}][\mathbf{J}+\mathbf{i}\not{\partial}][\mathbf{V_R^\dagger P_R}+\mathbf{V_L^\dagger P_L}]$$

$$= \lim_{\mathbf{n}\to\infty}\{[\mathbf{1}+\tfrac{\mathbf{i}}{\mathbf{n}}(\alpha+\beta)]\mathbf{P_L}+[\mathbf{1}+\tfrac{\mathbf{i}}{\mathbf{n}}(\alpha-\beta)]\mathbf{P_R}\}^{\mathbf{n}}[\mathbf{J}+\mathbf{i}\not{\partial}]\{[\mathbf{1}+\tfrac{\mathbf{i}}{\mathbf{n}}(\alpha+\beta)]^\dagger\mathbf{P_R}+[\mathbf{1}+\tfrac{\mathbf{i}}{\mathbf{n}}(\alpha-\beta)]^\dagger\mathbf{P_L}\}^{\mathbf{n}}$$

$$= \lim_{\mathbf{n}\to\infty}[\mathbf{1}+\frac{\mathbf{i}}{\mathbf{n}}(\alpha-\beta\gamma_5)]^{\mathbf{n}}[\mathbf{J}+\mathbf{i}\not{\partial}][\mathbf{1}-\frac{\mathbf{i}}{\mathbf{n}}(\alpha+\beta\gamma_5)]^{\mathbf{n}} = \lim_{\mathbf{n}\to\infty}\left(\mathbf{1}+\frac{\delta_\beta}{\mathbf{n}}\right)^{\mathbf{n}}\mathbf{J}$$

$$= \mathbf{e}^{\delta_\beta}\mathbf{J} \qquad \left(\mathbf{1}+\frac{\delta_\beta}{\mathbf{n}}\right)\mathbf{J} \equiv [\mathbf{1}+\frac{\mathbf{i}}{\mathbf{n}}(\alpha-\beta\gamma_5)][\mathbf{J}+\mathbf{i}\not{\partial}][\mathbf{1}-\frac{\mathbf{i}}{\mathbf{n}}(\alpha+\beta\gamma_5)]$$

$$\delta_\beta\mathbf{J} = \mathbf{i}(\alpha-\beta\gamma_5)\mathbf{J} - [\mathbf{i}\not{\partial}+\mathbf{J}]\mathbf{i}(\alpha+\beta\gamma_5) + \frac{\mathbf{1}}{\mathbf{n}}(\alpha-\beta\gamma_5)[\mathbf{i}\not{\partial}+\mathbf{J}](\alpha+\beta\gamma_5)$$

$$\xlongequal{\mathbf{n}\to\infty} \mathbf{i}(\alpha-\beta\gamma_5)\mathbf{J} - [\mathbf{i}\not{\partial}+\mathbf{J}]\mathbf{i}(\alpha+\beta\gamma_5)$$

Back
Close

$$\delta_\beta \mathbf{J} = \mathbf{i}(\alpha-\beta\gamma_5)\mathbf{J} - [\mathbf{i}\not\partial+\mathbf{J}]\mathbf{i}(\alpha+\beta\gamma_5)$$

$$\delta_\beta(\mathbf{v}^\mu + \mathbf{a}^\mu\gamma_5) = \mathbf{i}(\alpha+\beta\gamma_5)(\mathbf{v}^\mu + \mathbf{a}^\mu\gamma_5) - [\mathbf{i}\partial^\mu + \mathbf{v}^\mu + \mathbf{a}^\mu\gamma_5]\mathbf{i}(\alpha+\beta\gamma_5)$$

$$\delta_\beta(\mathbf{v}^\mu + \mathbf{a}^\mu) = \mathbf{i}(\alpha+\beta)(\mathbf{v}^\mu + \mathbf{a}^\mu) - [\mathbf{i}\partial^\mu + \mathbf{v}^\mu + \mathbf{a}^\mu]\mathbf{i}(\alpha+\beta)$$

$$\delta_\beta(\mathbf{v}^\mu - \mathbf{a}^\mu) = \mathbf{i}(\alpha-\beta)(\mathbf{v}^\mu - \mathbf{a}^\mu) - [\mathbf{i}\partial^\mu + \mathbf{v}^\mu - \mathbf{a}^\mu]\mathbf{i}(\alpha-\beta)$$

$$\delta_\beta(-\mathbf{s} + \mathbf{i}\mathbf{p}\gamma_5) = \mathbf{i}(\alpha-\beta\gamma_5)(-\mathbf{s} + \mathbf{i}\mathbf{p}\gamma_5) - (-\mathbf{s} + \mathbf{i}\mathbf{p}\gamma_5)\mathbf{i}(\alpha+\beta\gamma_5)$$

$$\delta_\beta(-\mathbf{s} + \mathbf{i}\mathbf{p}) = \mathbf{i}(\alpha-\beta)(-\mathbf{s} + \mathbf{i}\mathbf{p}) - (-\mathbf{s} + \mathbf{i}\mathbf{p})\mathbf{i}(\alpha+\beta)$$

$$\delta_\beta(\mathbf{s} + \mathbf{i}\mathbf{p}) = \mathbf{i}(\alpha+\beta)(\mathbf{s} + \mathbf{i}\mathbf{p}) - (\mathbf{s} + \mathbf{i}\mathbf{p})\mathbf{i}(\alpha-\beta)$$

$$\bar\theta' - \bar\theta = -\mathbf{i}\ln\det{}_\mathbf{f}[\mathbf{V_R}\mathbf{V_L^\dagger}] = -\mathbf{i}\lim_{\mathbf{n}\to\infty}\ln\det{}_\mathbf{f}\left[[\mathbf{1}+\frac{\mathbf{i}}{\mathbf{n}}(\alpha+\beta)][\mathbf{1}-\frac{\mathbf{i}}{\mathbf{n}}(\alpha-\beta)]\right]^\mathbf{n}$$

$$= -\mathbf{i}\lim_{\mathbf{n}\to\infty}\left[\ln\det{}_\mathbf{f}[\mathbf{1}+\frac{\mathbf{2i}}{\mathbf{n}}\beta+\frac{\mathbf{1}}{\mathbf{n^2}}(\alpha^\mathbf{2}-\beta^\mathbf{2})]^\mathbf{n}\right] = \mathbf{2}\mathrm{tr}_\mathbf{f}(\beta)$$

$$\bar\theta' \equiv \mathbf{e}^{\delta_\beta}\bar\theta = \lim_{\mathbf{n}\to\infty}\left(\mathbf{1}+\frac{\mathbf{1}}{\mathbf{n}}\delta_\beta\right)^\mathbf{n}\bar\theta \qquad \delta_\beta\bar\theta = \mathbf{2}\mathrm{tr}_\mathbf{f}(\beta) \qquad \delta_\beta^\mathbf{n}\bar\theta = \mathbf{0} \quad \mathbf{n} > \mathbf{1}$$

$\alpha,\ \beta$ 有限大:

$$\delta_\beta \mathbf{J} = \mathbf{i}(\alpha - \beta\gamma_5)\mathbf{J} - [\mathbf{i}\partial\!\!\!/ + \mathbf{J}]\mathbf{i}(\alpha + \beta\gamma_5) \qquad \mathbf{J}' = \mathbf{e}^{\delta_\beta}\mathbf{J}$$

$$\delta_\beta \bar{\theta} = \mathbf{2}\mathrm{tr}_\mathbf{f}(\beta) \qquad \delta_\beta^\mathbf{n}\bar{\theta} = \mathbf{0} \quad \mathbf{n} > \mathbf{1} \qquad \bar{\theta}' = \mathbf{e}^{\delta_\beta}\bar{\theta}$$

$$\mathbf{f}(\mathbf{J}') = \mathbf{e}^{\delta_\beta}\mathbf{f}(\mathbf{J}) \qquad \mathbf{g}(\bar{\theta}') = \mathbf{e}^{\delta_\beta}\mathbf{g}(\bar{\theta})$$

$$\mathbf{e}^{\delta_\beta} = \mathbf{1} - \frac{\mathbf{1} - \mathbf{e}^{\delta_\beta}}{\delta_\beta}\delta_\beta = \mathbf{1} - \int_\mathbf{0}^\mathbf{1} \mathbf{dt}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \delta_\beta$$

$$\mathbf{Z}[\mathbf{J}', \bar{\theta}'] = \mathbf{e}^{\delta_\beta}\mathbf{Z}[\mathbf{J}, \bar{\theta}] = \mathbf{Z}[\mathbf{J}, \bar{\theta}] - \int_\mathbf{0}^\mathbf{1} \mathbf{dt}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \delta_\beta \mathbf{Z}[\mathbf{J}, \bar{\theta}]$$

$$= \mathbf{Z}[\mathbf{J}, \bar{\theta}] - \int_\mathbf{0}^\mathbf{1} \mathbf{dt} \int \mathbf{d}^\mathbf{4}\mathbf{x}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \mathrm{tr}_\mathbf{f}[\beta(\mathbf{x})\tilde{\mathbf{\Omega}}(\mathbf{x})]$$

Back
Close

赝标介子手征有效拉氏量

强子谱对称性分类与手征对称性自发破缺:

$$Q_V^a = Q_R^a + Q_L^a \qquad Q_A^a = Q_R^a - Q_L^a$$

$$Q_V^a = \int d^3x\, q^\dagger(\vec{x},t)\frac{\lambda^a}{2}q(\vec{x},t) \qquad Q_A^a = \int d^3x\, q^\dagger(\vec{x},t)\frac{\lambda^a}{2}\gamma_5 q(\vec{x},t)$$

$$[Q_V^a, Q_V^b] = if^{abc}Q_V^c$$

$$[Q_A^a, Q_A^b(t)] = if^{abc}Q_V^c \qquad [Q_V^a, Q_A^b] = if^{abc}Q_A^c$$

$$PQ_L^aP^{-1} = Q_R^a \qquad PQ_R^aP^{-1} = Q_L^a$$

$$PQ_V^aP^{-1} = Q_V^a \qquad PQ_A^aP^{-1} = -Q_A^a$$

$$[Q_V^a, H_{QCD,0}] = [Q_A^a, H_{QCD,0}] = [Q_V^a, \vec{P}_{QCD}] = [Q_A^a, \vec{P}_{QCD}] = 0$$

$$[P, H_{QCD,0}] = [P, \vec{P}_{QCD}] = [Q_V^a, P] = 0$$

$$H_{QCD}|i,-\rangle = E_i|i,-\rangle \quad P|i,-\rangle = -|i,-\rangle \quad Q_V^a|i,-\rangle = t_{ij}^a|j,-\rangle$$

$$H_{QCD,0}|i,-\rangle \approx E_i|i,-\rangle \approx E_-|i,-\rangle \qquad [t^a, t^b] = if^{abc}t^c$$

$$H_{QCD,0}Q_V^a|i,-\rangle = Q_V^a H_{QCD,0}|i,-\rangle = E_i Q_V^a|i,-\rangle$$

$$|i,-\rangle \equiv a_i^\dagger|0\rangle \qquad t_{ij}^a a_j^\dagger|0\rangle = t_{ij}^a|j,-\rangle = Q_V^a a_i^\dagger|0\rangle = [Q_V^a, a_i^\dagger]|0\rangle + a_i^\dagger Q_V^a|0\rangle$$

$$|0\rangle' \equiv e^{iQ_V^a\Theta^a}|0\rangle = |0\rangle \quad \Rightarrow \quad Q_V^a|0\rangle = 0 \quad \Longrightarrow \quad [Q_V^a, a_i^\dagger] = t_{ij}^a a_j^\dagger$$

Q_V^a 对赝标介子态的作用是将其变成它自己和其它的赝标介子态的线性叠加！

Back
Close

强子谱对称性分类与手征对称性自发破缺

1000MeV以下的几个质量最低的强子谱为:

赝标介子族($J^P = 0^-$):

$\pi^{\pm}(140)$ $\pi^0(135)$ $K^{\pm}(494)$ $K^0, \bar{K}^0(498)$ $\eta(547)$ $\eta'(958)$

矢量介子家族($J^P = 1^-$):

$\rho^{\pm}, \rho^0(769)$ $K^{*\pm}(892)$ $K^{*0}, \bar{K}^{*0}(896)$ $\omega(782)$

标量介子家族($J^P = 0^+$): f_0或$\sigma(500)$ K_0^*或$\kappa(800)$ $f_0(980)$ $a_0(980)$

重子家族($J^P = (\frac{1}{2})^+$): $p(938)$ $n(940)$

赝标和矢量介子形成$SU(3)_V$的八重态（伴随）表示。$\Longrightarrow$ $Q_V^a|0\rangle = 0$ 理论证明！

Q_A^a 对赝标介子态的作用：

$$Q_A^a|i,-\rangle \equiv \tilde{t}_{is}^a|s,+\rangle \qquad P|s,+\rangle = |s,+\rangle$$

$$H_{QCD,0}Q_A^a|s,-\rangle = Q_A^a H_{QCD,0}|s,-\rangle = E_- Q_A^a|s,-\rangle$$

$$H_{QCD,0}|s,+\rangle = E_+|s,+\rangle \qquad E_+ \approx E_-$$

$$Q_A^a|i,-\rangle = \tilde{t}_{is}^a b_s^\dagger|0\rangle = \tilde{t}_{is}^a|s,+\rangle = Q_A^a|i,-\rangle = Q_A^a a_i^\dagger|0\rangle$$

$$= [Q_A^a, a_i^\dagger]|0\rangle + a_i^\dagger Q_A^a|0\rangle$$

$$|0\rangle' \equiv e^{iQ_A^a\Theta^a}|0\rangle \neq |0\rangle \quad \Rightarrow \quad Q_A^a|0\rangle \neq 0 \quad \Rightarrow \quad a_i^\dagger Q_A^a|0\rangle$$ 不是单粒子态！

Q_A^a 对赝标介子态的作用除单粒子态外还包含有将其变成多粒子态的部分！

Back
Close

强子谱对称性分类与手征对称性自发破缺

$$Q_A^a|0\rangle \neq 0 \quad Q_V^a|0\rangle = 0 \;\rightarrow\; SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$$

理论证明？ $\vec{P}_{QCD} Q_A^a|0\rangle = Q_A^a \vec{P}_{QCD}|0\rangle = 0 \qquad Q_A^a|0\rangle = \tilde{b}^{a,\dagger}(\vec{0})|0\rangle$

$$H_{QCD,0}\tilde{b}^{a,\dagger}(\vec{0})|0\rangle = H_{QCD,0}Q_A^a|0\rangle = Q_A^a H_{QCD,0}|0\rangle = 0$$ Goldstone粒子

$$P\tilde{b}^{a,\dagger}(\vec{0})|0\rangle = PQ_A^a|0\rangle = PQ_A^a P^{-1}P|0\rangle = -Q_A^a|0\rangle$$

$\tilde{b}^{a,\dagger}(\vec{p})|0\rangle \equiv |\phi^a(\vec{p})\rangle$ 定义了用赝标介子场 $\phi^a(\vec{p})$ 标记的动量为 $\vec{p}$ 赝标介子态

$$[Q_V^a, \tilde{b}^{b,\dagger}(\vec{0})]|0\rangle = [Q_V^a, Q_A^b]|0\rangle = if^{abc}Q_A^c|0\rangle = if^{abc}\tilde{b}^{c,\dagger}(\vec{0})|0\rangle$$

$$[Q_V^a, \tilde{b}^{b,\dagger}(\vec{0})] = if^{abc}\tilde{b}^{c,\dagger}(\vec{0}) \xRightarrow{\text{换参考系：}Q_V^a\text{洛伦兹不变}} [Q_V^a, \tilde{b}^{b,\dagger}(\vec{p})] = if^{abc}\tilde{b}^{c,\dagger}(\vec{p})$$

$$Q_V^a|\phi^b(\vec{p})\rangle = if^{abc}|\phi^c(\vec{p})\rangle \qquad [Q_V^a, \phi^b(\vec{p})] = if^{abc}\phi^c(\vec{p})$$

Back
Close

$$T_3 = Q_V^3 \qquad Y = \frac{2}{\sqrt{3}} Q_V^8 \qquad Q_V^a |\phi^b(\vec{p})\rangle = i f^{abc} |\phi^c(\vec{p})\rangle$$

$$K^0(-\frac{1}{2}, 1) \qquad K^+(\frac{1}{2}, 1)$$

$$\pi^-(-1, 0) \qquad \pi^0(0, 0) \qquad \eta(0, 0) \qquad \pi^+(1, 0)$$

$$K^-(-\frac{1}{2}, -1) \qquad \bar{K}^0(\frac{1}{2}, -1)$$

$$T_3(|\phi^1\rangle - i|\phi^2\rangle) = -(|\phi^1\rangle - i|\phi^2\rangle) \qquad Y(|\phi^1\rangle - i|\phi^2\rangle) = 0$$

$$T_3(|\phi^1\rangle + i|\phi^2\rangle) = |\phi^1\rangle + i|\phi^2\rangle \qquad Y(|\phi^1\rangle + i|\phi^2\rangle) = 0$$

$$T_3(|\phi^4\rangle - i|\phi^5\rangle) = -\frac{1}{2}(|\phi^4\rangle - i|\phi^5\rangle) \qquad Y(|\phi^4\rangle - i|\phi^5\rangle) = -(|\phi^4\rangle - i|\phi^5\rangle)$$

$$T_3(|\phi^4\rangle + i|\phi^5\rangle) = \frac{1}{2}(|\phi^4\rangle + i|\phi^5\rangle) \qquad Y(|\phi^4\rangle + i|\phi^5\rangle) = |\phi^4\rangle + i|\phi^5\rangle$$

$$T_3(|\phi^6\rangle - i|\phi^7\rangle) = \frac{1}{2}(|\phi^6\rangle - i|\phi^7\rangle) \qquad Y(|\phi^6\rangle - i|\phi^7\rangle) = -(|\phi^6\rangle - i|\phi^7\rangle)$$

$$T_3(|\phi^6\rangle + i|\phi^7\rangle) = -\frac{1}{2}(|\phi^6\rangle + i|\phi^7\rangle) \qquad Y(|\phi^6\rangle + i|\phi^7\rangle) = |\phi^6\rangle + i|\phi^7\rangle$$

$$T_3|\phi^3\rangle = Y|\phi^3\rangle = T_3|\phi^8\rangle = Y|\phi^8\rangle = 0$$

$$\sum_{\mathbf{a}=1}^{\mathbf{8}} \lambda^{\mathbf{a}}|\phi^{\mathbf{a}}\rangle \equiv \begin{pmatrix} |\phi^{\mathbf{3}}\rangle + \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}|\phi^{\mathbf{8}}\rangle & |\phi^1\rangle - i|\phi^2\rangle & |\phi^4\rangle - i|\phi^5\rangle \\ |\phi^1\rangle + i|\phi^2\rangle & -|\phi^3\rangle + \frac{1}{\sqrt{3}}|\phi^8\rangle & |\phi^6\rangle - i|\phi^7\rangle \\ |\phi^4\rangle + i|\phi^5\rangle & |\phi^6\rangle + i|\phi^7\rangle & -\frac{2}{\sqrt{3}}|\phi^8\rangle \end{pmatrix}$$

$$= \begin{pmatrix} |\pi^{\mathbf{0}}\rangle + \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}|\eta\rangle & \sqrt{\mathbf{2}}|\pi^-\rangle & \sqrt{\mathbf{2}}|\mathbf{K}^-\rangle \\ \sqrt{\mathbf{2}}|\pi^+\rangle & -|\pi^{\mathbf{0}}\rangle + \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}|\eta\rangle & \sqrt{\mathbf{2}}|\bar{\mathbf{K}}^{\mathbf{0}}\rangle \\ \sqrt{\mathbf{2}}|\mathbf{K}^+\rangle & \sqrt{\mathbf{2}}|\mathbf{K}^{\mathbf{0}}\rangle & -\frac{\mathbf{2}}{\sqrt{\mathbf{3}}}|\eta\rangle \end{pmatrix}$$

Back
Close

夸克对凝聚与手征对称性自发破缺

$$\langle 0|A^a_\mu(0)|\phi^b(p)\rangle = ip_\mu F_0\delta^{ab} \qquad Q^a_A|0\rangle \neq 0 \Longrightarrow F_0 \neq 0$$

$$S^a(y) = \bar{q}(y)\lambda^a q(y)$$

$$[Q^a_V, S^0(y)] = 0 \qquad [Q^a_V, S^b(y)] = i\sum_{c=1}^{8} f^{abc}S^c(y)$$

$$Q^a_V = \int d^3x q^\dagger(\vec{x},t)\frac{\lambda^a}{2}q(\vec{x},t)$$

$$Q^a_V|0\rangle = 0 \quad \rightarrow \quad \langle 0|S^a(y)|0\rangle = \langle 0|S^a(0)|0\rangle \equiv \langle S^a\rangle = 0$$

$$\rightarrow \langle\bar{u}u\rangle - \langle\bar{d}d\rangle = 0;\ \langle\bar{u}u\rangle + \langle\bar{d}d\rangle - 2\langle\bar{s}s\rangle = 0 \rightarrow \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{s}s\rangle$$

$$\langle\bar{q}q\rangle \equiv \langle\bar{u}u + \bar{d}d + \bar{s}s\rangle = 3\langle\bar{u}u\rangle = 3\langle\bar{d}d\rangle = 3\langle\bar{s}s\rangle \stackrel{?}{\neq} 0$$

夸克对凝聚与手征对称性自发破缺

$$\mathrm{P^a(y) = i\bar{q}(y)\gamma_5\lambda^a q(y)} \qquad \mathrm{\langle 0|i[Q_A^a, P^a(y)]|0\rangle = \frac{2}{3}\langle \bar{q}q\rangle}$$

$$\mathrm{i[Q_A^a, P^a(y)]} = \begin{cases} \mathrm{\bar{u}u + \bar{d}d} & a = 1,2,3 \\ \mathrm{\bar{u}u + \bar{s}s} & a = 4,5 \\ \mathrm{\bar{d}d + \bar{s}s} & a = 6,7 \\ \frac{1}{3}(\mathrm{\bar{u}u + \bar{d}d + 4\bar{s}s}) & a = 8 \end{cases}$$

$$-\frac{\mathrm{2i}}{3}\langle \mathrm{\bar{q}q}\rangle$$

$$= \sum_{\mathrm{n}} \mathrm{[\langle 0|Q_A^a|n\rangle\langle n|P^a(y)|0\rangle - \langle 0|P^a(y)|n\rangle\langle n|Q_A^a|0\rangle]}$$

$$= \int \frac{\mathrm{d\vec{p}}}{2\sqrt{\mathrm{\vec{p}^2}}} \mathrm{[\langle 0|Q_A^a|\phi^b(\vec{p})\rangle\langle \phi^b(\vec{p})|P^a(y)|0\rangle}$$

$$\mathrm{-\langle 0|P^a(y)|\phi^b(\vec{p})\rangle\langle \phi^b(\vec{p})|Q_A^a|0\rangle]}$$

Back

Close

Goldstone场的变换性质与群定义

考虑Goldstone场： π^a 假设在群g变换下： $\pi^a \xrightarrow{g} \phi^a[g,\pi]$

$\phi^a[I,\pi] = \pi^a$ $\phi[g_1,\phi[g_2,\pi]] = \phi[g_1 g_2,\pi]$ 注意这里的结合顺序是先右再左！

真空 $\rightarrow \pi^a = 0$ $\phi^a[I,0] = 0$ $\phi^a[I,\phi[g,0]] = \pi^a[g,0]$

$\phi[g_1,\phi[g_2,\phi[g,0]]] = \phi[g_1 g_2 g,0] = \phi[g_1 g_2,\phi[g,0]]$

$\rightarrow$ $\phi^a[g,0]$ 可以用来描述赝标Goldstone粒子场 ！

$\rightarrow$ 独立群元素和独立Goldstone粒子场一一对应 ！

$\rightarrow$ 赝标Goldstone粒子场的局域性要求对应的群元必须是局域的 ！

这就是为什么要把原来拉氏量具有的整体手征对称性局域化的原因

Goldstone场的变换性质与群定义

所有使$\phi(h,0)=0$的$h\subset \mathbf{SU(3)}_L\times\mathbf{SU(3)}_R$形成子群**H**。

单位元：$\pi=\mathbf{0}\ \to\phi(\mathbf{I},\mathbf{0})=\mathbf{0}$

乘法：$\phi(\mathbf{h}_1,\mathbf{0})=\mathbf{0}\qquad\phi(\mathbf{h}_2,\mathbf{0})=\mathbf{0}$
$\to\ \phi(\mathbf{h}_1\mathbf{h}_2,\mathbf{0})=\phi[\mathbf{h}_1,\phi(\mathbf{h}_2,\mathbf{0})]=\phi[\mathbf{h}_1,\mathbf{0}]=\mathbf{0}$

由于此乘法就是原来群的乘法，自然是满足结合律

逆元：$\mathbf{0}=\phi(\mathbf{h}^{-1}\mathbf{h},\mathbf{0})=\phi[\mathbf{h}^{-1},\phi(\mathbf{h},\mathbf{0})]=\phi[\mathbf{h}^{-1},\mathbf{0}]$

H的物理含义在于保持真空不变（即从$\pi=0$变到$\phi=0$）。手征极限下发生对称性自发破缺后剩下 对称性是$\mathbf{SU(3)}_V$ $\to$ $\mathbf{H=SU(3)}_V$!

Back
Close

所有使$\phi(h,0)=0$的$h\subset$**SU(3)**$_L\times$**SU(3)**$_R$形成子群**H!**

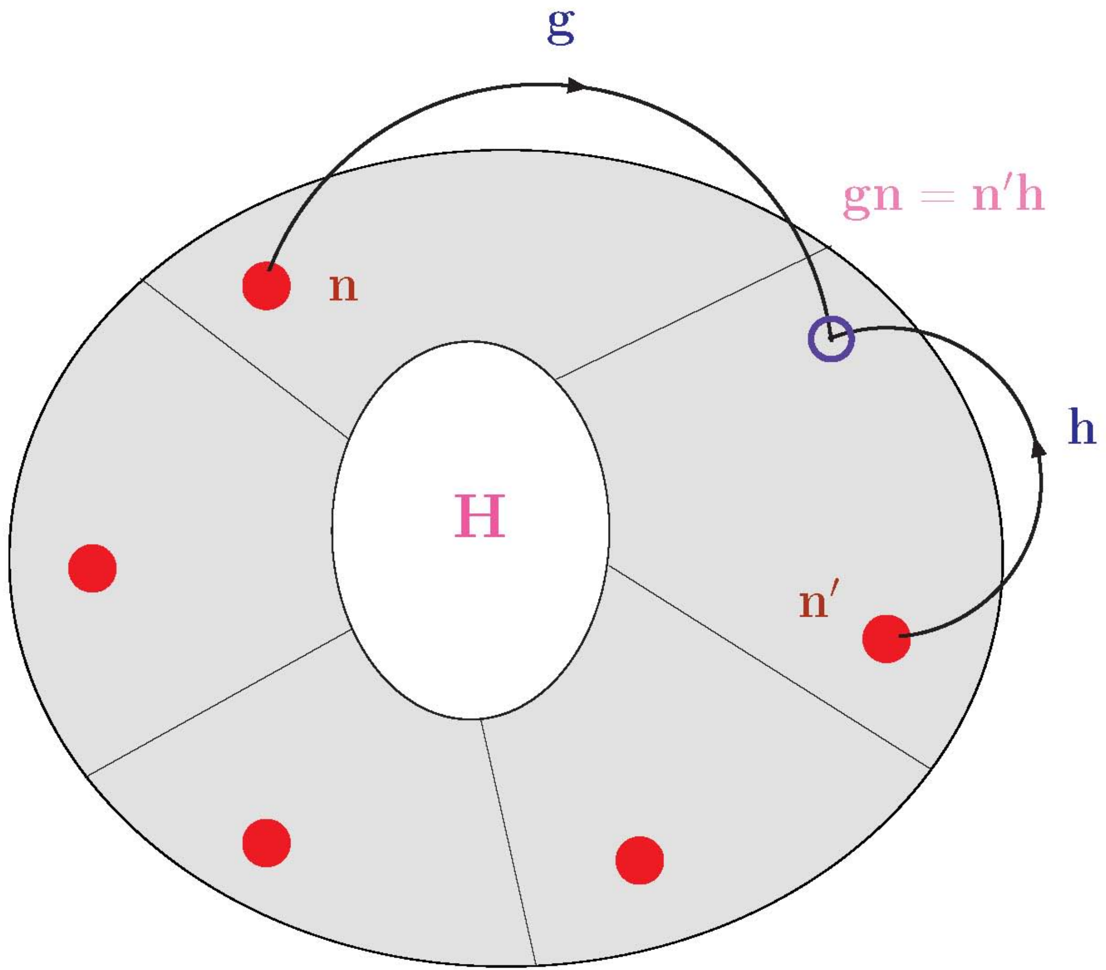
对所有的$g\subset$**SU(3)**$_L\times$**SU(3)**$_R$和$h\subset$**H**有，$\phi(g,0)=\phi(gh,0)$**!**

对所有的$g_1,g_2\subset$**SU(3)**$_L\times$**SU(3)**$_R$,若$\phi(g_1,0)=\phi(g_2,0)$则$g_1^{-1}g_2\subset$**H!**

$$g_2=g_1\bar{g}$$

$$\begin{aligned}\phi(\bar{g},0)&=\phi[g_1^{-1},\phi[g_1,\phi(\bar{g},0)]]=\phi[g_1^{-1},\phi(g_1\bar{g},0)]=\phi[g_1^{-1},\phi(g_2,0)]\\&=\phi[g_1^{-1},\phi(g_1,0)]=\phi(I,0)=0\quad\rightarrow\quad g_1^{-1}g_2=\bar{g}\subset\mathrm{H}\end{aligned}$$

赝标Goldstone粒子场$\phi(g,0)$同构于SU(3)$_L\times$SU(3)$_R$/SU(3)$_V$！ 它可以看在是定义在SU(3)$_L\times$SU(3)$_R$的左陪集上（准到$g'=gh$。选左陪集来自于前面给出的结合律乘法定义）。 可以在每个陪集上选一个代表元素n，用它来代表赝标Goldstone粒子场。 每个陪集上都选定代表元素后，任意的一个群元都可唯一地分解为nh,那么，对任意的群元g及陪集上的代表元n， 我们有$gn=n'h$决定了n在操作g下到的n’变换行为，其中n’也是陪集上的代表元。


g
gn = n′h
n
h
H
n′

Goldstone场的变换性质与群定义

群元用QCD中的V_L, V_R构造：$g=(V_L,V_R)$　$g'=(V'_L,V'_R)$　$gg'\equiv(V_LV'_L, V_RV'_R)$

$\mathbf{n}=(1,U)$　$U(x)=e^{i\phi(x)/F_0}$　Goldstone场：$\phi(x)=\sum_{a=1}^{8}\lambda^a\phi^a(x)$

$U(x)\ \xrightarrow{\phi\to 0}\ 1+\phi(x)/F_0$　真空 $\to\phi=0$　$U=1$

$$gn=(V_L,V_R)(1,U)=(V_L,V_RU)=(1,V_RUV_L^\dagger)(V_L,V_L)$$
$$=(1,U')(V_L,V_L)=n'h$$

$U\to U'=V_RUV_L^\dagger$　$U_{ll'}(x)\sim q_{R,l\alpha s}(x)\bar{q}_{L,l'\alpha s}(x)$　$\det_f U(x)=e^{i\bar{\theta}}$

$$\det_f U'=\det_f(V_RUV_L^\dagger)=(\det U)e^{\ln\det_f(V_RV_L^\dagger)}=e^{i[\bar{\theta}-i\ln\det_f(V_RV_L^\dagger)]}=e^{i\bar{\theta}'}$$

$V1V^\dagger=1$　$A^\dagger 1A^\dagger\neq 1$　$\Rightarrow H=SU(3)_V$

Goldstone场的变换性质与群定义

$$\mathbf{n} = (\Omega^\dagger, \Omega) \quad \mathbf{gn} = (\mathbf{V_L}, \mathbf{V_R})(\Omega^\dagger, \Omega) = (\mathbf{V_L}\Omega^\dagger\tilde{\mathbf{h}}^\dagger, \mathbf{V_R}\Omega\tilde{\mathbf{h}}^\dagger)(\tilde{\mathbf{h}}, \tilde{\mathbf{h}}) = \mathbf{n}'\mathbf{h}$$

$$\mathbf{V_R}\Omega\tilde{\mathbf{h}}^\dagger = [\mathbf{V_L}\Omega^\dagger\tilde{\mathbf{h}}^\dagger]^\dagger = \tilde{\mathbf{h}}\Omega\mathbf{V_L}^\dagger \qquad \Omega \to \Omega' = \mathbf{V_R}\Omega\tilde{\mathbf{h}}^\dagger = \tilde{\mathbf{h}}\Omega\mathbf{V_L}^\dagger$$

$\tilde{h}$ 对应的对称性叫 hidden symmetry。它是一个导引出的矢量型的对称性

$$\Omega^2 \to \Omega^{2\prime} = \mathbf{V_R}\Omega^2\mathbf{V_L}^\dagger \qquad \mathbf{U} = 1 \to \text{真空} \to \Omega = 1 \quad \to \quad \mathbf{U} = \Omega^2$$

和前面定义的陪集代表元相差一个子群变换：$(\Omega^\dagger, \Omega) = (1, \mathbf{U})(\Omega^\dagger, \Omega^\dagger)$

$$\Omega(\mathbf{x}) = \mathbf{e}^{\mathbf{i}\frac{\phi(\mathbf{x})}{2\mathbf{F_0}}} \qquad \phi(\mathbf{x}) = \sum_{\mathbf{a}=1}^{8}\lambda^{\mathbf{a}}\phi^{\mathbf{a}}(\mathbf{x}) \qquad \mathbf{U_{ll'}}(\mathbf{x}) \sim \mathbf{q_{R,l\alpha s}}(\mathbf{x})\bar{\mathbf{q}}_{\mathbf{L,l'\alpha s}}(\mathbf{x})$$

$$\mathbf{V_L} = \mathbf{V_R} = \mathbf{V} \equiv \mathbf{e}^{-\mathbf{i}\frac{\lambda^{\mathbf{a}}}{2}\Theta^{\mathbf{a}}} \;\Rightarrow\; \mathbf{VUV}^\dagger = \mathbf{e}^{\mathbf{i}\Theta^{\mathbf{a}}\mathbf{Q_V^a}}\mathbf{U}\mathbf{e}^{-\mathbf{i}\Theta^{\mathbf{a}}\mathbf{Q_V^a}} \qquad \mathbf{Q_V^a} \equiv \mathbf{Q_L^a} + \mathbf{Q_R^a}$$

$$\mathbf{V}\phi\mathbf{V}^\dagger = \mathbf{e}^{\mathbf{i}\Theta^{\mathbf{a}}\mathbf{Q_V^a}}\phi\mathbf{e}^{-\mathbf{i}\Theta^{\mathbf{a}}\mathbf{Q_V^a}} \;\Rightarrow\; [\mathbf{Q_V^a}, \phi] = [-\frac{\lambda^{\mathbf{a}}}{2}, \phi] \;\Rightarrow\; \underbrace{[\mathbf{Q_V^a}, \phi^{\mathbf{b}}] = \mathbf{i}\mathbf{f}^{\mathbf{abc}}\phi^{\mathbf{c}}}_{[\mathbf{Q_V^a}, \tilde{\mathbf{b}}^{\mathbf{b},\dagger}(\vec{\mathbf{p}})] = \mathbf{i}\mathbf{f}^{\mathbf{abc}}\tilde{\mathbf{b}}^{\mathbf{c},\dagger}(\vec{\mathbf{p}})}$$

Back
Close

赝标介子手征有效拉氏量

$$e^{iZ[J,\bar\theta]} = \int \mathcal{D}U \;\; \exp\left\{i \int d^4x \mathcal{L}_{\rm eff}[U, J, \bar\theta]\right\}$$

为什么选 U 作为有效理论的变量?

$$q_{f,\alpha}(\vec{x}, t) \xrightarrow{P} \gamma^0 q_{f,\alpha}(-\vec{x}, t) \qquad \bar{q}_{f,\alpha}(\vec{x}, t) \xrightarrow{P} \bar{q}_{f,\alpha}(-\vec{x}, t)\gamma^0$$

$$q_{f,\alpha}(\vec{x}, t) \xrightarrow{C} C\bar{q}^{\dagger}_{f,\alpha}(\vec{x}, t) \qquad \bar{q}_{f,\alpha}(\vec{x}, t) \xrightarrow{C} -q^{\dagger}_{f,\alpha}(\vec{x}, t)C^{-1}$$

$$U_{ll'}(x) \sim q_{R,l\alpha s}(x)\bar{q}_{L,l'\alpha s}(x)$$

$$U(\vec{x}, t) \xrightarrow{P} U^P(\vec{x}, t) = U^{\dagger}(-\vec{x}, t) \qquad U(\vec{x}, t) \xrightarrow{C} U^C(\vec{x}, t) = U^T(\vec{x}, t)$$

$$\int \mathcal{D}U \;\; \exp\left\{i \int d^4x \mathcal{L}_{\rm eff}[U, J, \bar\theta]\right\} = e^{iZ[J,\bar\theta]} = e^{iZ[J^P,\bar\theta^P]}$$

$$= \int \mathcal{D}U^P \; e^{\left\{i \int d^4x \;\; \mathcal{L}_{\rm eff}[U^P, J^P, \bar\theta^P]\right\}} = \int \mathcal{D}U \; e^{\left\{i \int d^4x \;\; \mathcal{L}_{\rm eff}[U^P, J^P, \bar\theta^P]\right\}}$$

$$\Rightarrow \;\; \mathcal{L}_{\rm eff}[U, J, \bar\theta] = \mathcal{L}_{\rm eff}[U^P, J^P, \bar\theta^P]$$

$P \Rightarrow C$

Back
Close

赝标介子手征有效拉氏量

$$\int \mathcal{D}\mathbf{U}\ \exp\left\{\mathbf{i}\int \mathbf{d}^4\mathbf{x}\mathcal{L}_{\text{eff}}[\mathbf{U},\mathbf{J},\bar{\theta}]\right\} = \mathbf{e}^{\mathbf{iZ}[\mathbf{J},\bar{\theta}]}$$

$$= \exp\left\{\mathbf{iZ}[\mathbf{J}',\bar{\theta}'] + \mathbf{i}\int_0^1 \mathbf{dt}\int \mathbf{d}^4\mathbf{x}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \text{tr}_{\mathbf{f}}[\beta\tilde{\mathbf{\Omega}}]\right\}$$

$$= \int \mathcal{D}\mathbf{U}\ \exp\left\{\mathbf{i}\int \mathbf{d}^4\mathbf{x}\mathcal{L}_{\text{eff}}[\mathbf{U},\mathbf{J}',\bar{\theta}'] + \mathbf{i}\int_0^1 \mathbf{dt}\int \mathbf{d}^4\mathbf{x}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \text{tr}_{\mathbf{f}}[\beta\tilde{\mathbf{\Omega}}]\right\}$$

$$= \int \mathcal{D}\mathbf{U}'\ \exp\left\{\mathbf{i}\int \mathbf{d}^4\mathbf{x}\mathcal{L}_{\text{eff}}[\mathbf{U}',\mathbf{J}',\bar{\theta}'] + \mathbf{i}\int_0^1 \mathbf{dt}\int \mathbf{d}^4\mathbf{x}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \text{tr}_{\mathbf{f}}[\beta\tilde{\mathbf{\Omega}}]\right\}$$

$$= \int \mathcal{D}\mathbf{U}\ \exp\left\{\mathbf{i}\int \mathbf{d}^4\mathbf{x}\mathcal{L}_{\text{eff}}[\mathbf{U}',\mathbf{J}',\bar{\theta}'] + \mathbf{i}\int_0^1 \mathbf{dt}\int \mathbf{d}^4\mathbf{x}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \text{tr}_{\mathbf{f}}[\beta\tilde{\mathbf{\Omega}}]\right\}$$

$$\int \mathbf{d}^4\mathbf{x}\mathcal{L}_{\text{eff}}[\mathbf{U},\mathbf{J},\bar{\theta}] = \int \mathbf{d}^4\mathbf{x}\mathcal{L}_{\text{eff}}[\mathbf{U}',\mathbf{J}',\bar{\theta}'] + \int_0^1 \mathbf{dt}\int \mathbf{d}^4\mathbf{x}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \text{tr}_{\mathbf{f}}[\beta\tilde{\mathbf{\Omega}}]$$

$$\mathcal{L}_{\rm eff}[\mathbf{U},\mathbf{J},\bar{\theta}]=\mathcal{L}_{\rm eff}[\mathbf{U}^{\mathbf{P}},\mathbf{J}^{\mathbf{P}},\bar{\theta}^{\mathbf{P}}]$$

$$\mathcal{L}_{\rm eff}[\mathbf{U},\mathbf{J},\bar{\theta}]=\mathcal{L}_{\rm eff}[\mathbf{U}^{\mathbf{C}},\mathbf{J}^{\mathbf{C}},\bar{\theta}^{\mathbf{C}}]$$

手征有效拉氏量在洛伦兹变换下也是不变的!

$$\mathcal{L}_{\rm eff}[\mathbf{U},\mathbf{J},\bar{\theta}]=\mathcal{L}_{\rm eff}[\mathbf{U}',\mathbf{J}',\bar{\theta}']+\int_{\mathbf{0}}^{\mathbf{1}}\mathbf{dt}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \mathrm{tr}_{\mathbf{f}}[\beta(\mathbf{x})\tilde{\mathbf{\Omega}}(\mathbf{x})]$$

$$\mathcal{L}_{\rm eff}[\mathbf{U},\mathbf{J},\bar{\theta}]=\mathcal{L}_{\rm eff,N}[\mathbf{U},\mathbf{J},\bar{\theta}]+\mathcal{L}_{\rm eff,A}[\mathbf{U},\mathbf{J},\bar{\theta}]$$

正常项: $\mathcal{L}_{\rm eff,N}[\mathbf{U},\mathbf{J},\bar{\theta}]=\mathcal{L}_{\rm eff,N}[\mathbf{U}',\mathbf{J}',\bar{\theta}']$

反常项: $\mathcal{L}_{\rm eff,A}[\mathbf{U},\mathbf{J},\bar{\theta}]=\mathcal{L}_{\rm eff,A}[\mathbf{U}',\mathbf{J}',\bar{\theta}']+\int_{\mathbf{0}}^{\mathbf{1}}\mathbf{dt}\ \mathbf{e}^{\mathbf{t}\delta_\beta}\ \mathrm{tr}_{\mathbf{f}}[\beta(\mathbf{x})\tilde{\mathbf{\Omega}}(\mathbf{x})]$

Back
Close

赝标介子手征有效拉氏量: 正常项

$$\mathcal{L}_{\mathrm{eff,N}}[\mathbf{U}', \mathbf{J}', \bar{\theta}'] = \mathcal{L}_{\mathrm{eff,N}}[\mathbf{U}, \mathbf{J}, \bar{\theta}]$$

$$\nabla_\mu \mathbf{U} \equiv \partial_\mu \mathbf{U} - \mathbf{i}(\mathbf{v}_\mu + \mathbf{a}_\mu)\mathbf{U} + \mathbf{i}\mathbf{U}(\mathbf{v}_\mu - \mathbf{a}_\mu)$$

$$\begin{aligned}\nabla_\mu \mathbf{U} \to\ & \partial_\mu(\mathbf{V}_\mathrm{R}\mathbf{U}\mathbf{V}_\mathrm{L}^\dagger) - \mathbf{i}[\mathbf{V}_\mathrm{R}(\mathbf{v}_\mu + \mathbf{a}_\mu)\mathbf{V}_\mathrm{R}^\dagger + \mathbf{i}\mathbf{V}_\mathrm{R}\partial_\mu\mathbf{V}_\mathrm{R}^\dagger]\mathbf{V}_\mathrm{R}\mathbf{U}\mathbf{V}_\mathrm{L}^\dagger \\ & +\mathbf{i}\mathbf{V}_\mathrm{R}\mathbf{U}\mathbf{V}_\mathrm{L}^\dagger[\mathbf{V}_\mathrm{L}(\mathbf{v}_\mu - \mathbf{a}_\mu)\mathbf{V}_\mathrm{L}^\dagger + \mathbf{i}\mathbf{V}_\mathrm{L}\partial_\mu\mathbf{V}_\mathrm{L}^\dagger] \\ =\ & \partial_\mu\mathbf{V}_\mathrm{R}\mathbf{U}\mathbf{V}_\mathrm{L}^\dagger + \mathbf{V}_\mathrm{R}\partial_\mu\mathbf{U}\mathbf{V}_\mathrm{L}^\dagger + \mathbf{V}_\mathrm{R}\mathbf{U}\partial_\mu\mathbf{V}_\mathrm{L}^\dagger - \mathbf{i}\mathbf{V}_\mathrm{R}(\mathbf{v}_\mu + \mathbf{a}_\mu)\mathbf{U}\mathbf{V}_\mathrm{L}^\dagger \\ & -\partial_\mu\mathbf{V}_\mathrm{R}\mathbf{U}\mathbf{V}_\mathrm{L}^\dagger + \mathbf{i}\mathbf{V}_\mathrm{R}\mathbf{U}(\mathbf{v}_\mu - \mathbf{a}_\mu)\mathbf{V}_\mathrm{L}^\dagger - \mathbf{V}_\mathrm{R}\mathbf{U}\partial_\mu\mathbf{V}_\mathrm{L}^\dagger \\ =\ & \mathbf{V}_\mathrm{R}[\partial_\mu\mathbf{U} - \mathbf{i}(\mathbf{v}_\mu + \mathbf{a}_\mu)\mathbf{U} + \mathbf{i}\mathbf{U}(\mathbf{v}_\mu - \mathbf{a}_\mu)]\mathbf{V}_\mathrm{L}^\dagger = \mathbf{V}_\mathrm{R}(\nabla_\mu\mathbf{U})\mathbf{V}_\mathrm{L}^\dagger\end{aligned}$$

$$\mathbf{r}_\mu = \mathbf{v}_\mu + \mathbf{a}_\mu \qquad\qquad \mathbf{l}_\mu = \mathbf{v}_\mu - \mathbf{a}_\mu$$

Back
Close

赝标介子手征有效拉氏量：正常项

基元	$SU(3)_L \times SU(3)_R$	C	P
U	$V_R U V_L^\dagger$	U^T	$U^\dagger$
$\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}U$	$V_R\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}UV_L^\dagger$	$(\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}U)^T$	$(\nabla^{\lambda_1}\cdots\nabla^{\lambda_n}U)^\dagger$
$\chi = 2B_0(s+ip)$	$V_R\chi V_L^\dagger$	χ^T	$\chi^\dagger$
$\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}\chi$	$V_R\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}\chi V_L^\dagger$	$(\nabla_{\lambda_1}\cdots\nabla_{\lambda_n}\chi)^T$	$(\nabla^{\lambda_1}\cdots\nabla^{\lambda_n}\chi)^\dagger$
$\bar\theta$	$\bar\theta - i\ln\det_f(V_R V_L^\dagger)$	$\bar\theta$	$-\bar\theta$
$\nabla_\mu\bar\theta \equiv \partial_\mu\bar\theta + 2i\,\mathrm{tr}_f a_\mu$	$\nabla_\mu\bar\theta$	$\nabla_\mu\bar\theta$	$-\nabla_\mu\bar\theta$
$r_\mu = v_\mu + a_\mu$	$V_R r_\mu V_R^\dagger + iV_R\partial_\mu V_R^\dagger$	$-l_\mu^T$	l^μ
$l_\mu = v_\mu - a_\mu$	$V_L l_\mu V_L^\dagger + iV_L\partial_\mu V_L^\dagger$	$-r_\mu^T$	r^μ
$f_{\mu\nu}^R = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]$	$V_R f_{\mu\nu}^R V_R^\dagger$	$-(f_{\mu\nu}^L)^T$	$f_L^{\mu\nu}$
$f_{\mu\nu}^L = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]$	$V_L f_{\mu\nu}^L V_L^\dagger$	$-(f_{\mu\nu}^R)^T$	$f_R^{\mu\nu}$

Table 2: 各元素在手征，电何共轭和宇称变换下的变换性质。

注：$\nabla_\mu\chi \equiv \partial_\mu\chi - i(v_\mu + a_\mu)\chi + i\chi(v_\mu - a_\mu)$

赝标介子手征有效拉氏量: 正常项

数幂: 有效拉氏量由满足对称性要求的所有可能的算符组成。由于算符数目众多, 为了使理论可进行计算并具有可预言性,能进行系统的计算并可估计误差, 必须对算符进行分类,以此 安排计算这些算符的顺序(算符的阶)。分类的标准是寻找一些小参量,并设定各算符 对小参量的依赖。使得理论在展开到这些参量的有限阶幂次,只有有限种算符参与贡献。

低能展开: 按赝标介子之间传递的能动量进行展开。

$$\partial_\mu \sim \mathbf{p} \qquad \mathbf{v}_\nu, \quad \mathbf{a}_\mu \sim \nabla_\mu \sim \partial_\mu \sim \mathbf{p} \qquad \mathbf{p} \sim \mathbf{s} \sim \mathbf{m} \sim \mathbf{M_P^2} \xrightarrow{1/(\mathbf{p}^2-\mathbf{M_P^2})} \mathbf{p}^2$$

$$\mathbf{U}, \quad \bar{\theta} \sim \mathbf{p}^0$$

$$\mathcal{L}_{\mathrm{eff,N}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

洛伦兹对称性要求奇数阶项为零!

$$\mathcal{L}_2 = \frac{\mathbf{F_0^2}}{4}\mathbf{tr_f}[\nabla_\mu \mathbf{U}(\nabla^\mu \mathbf{U})^\dagger] + \frac{\mathbf{F_0^2}}{4}\mathbf{tr_f}(\chi \mathbf{U}^\dagger + \mathbf{U}\chi^\dagger) + \frac{\mathbf{H_0}}{12}\mathbf{tr_f}(\nabla_\mu\bar{\theta}\nabla^\mu\bar{\theta})$$

Back

Close

赝标介子手征有效拉氏量： 正常项

- 如何理解没有 p^0 阶项？
- 赝标介子无势能，赝标介子的自作用当动量趋于零时为零 ！
- 如何实现的？ $U^\dagger U = 1$
- 另一种理解方式： S.Weinberg
  - 若存在一个用赝标介子场构造的有效场论
  - 其拉氏量在整体的手征对称性 $SU(3)_L \times SU(3)_R$ 变换下应是不变的
  - 因可以用群元素（陪集代表元）代表赝标介子场
  - 可将介子场作为群元素，通过居域群变换把介子场转成真空单位元
  - 若略去转动的局域性，拉氏量的手征对称性使拉氏量在群变换后变为零
  - 考虑局域性，拉氏量含微商的项无法变成零，剩下含微商的项 !
  - 这里采用的就是拉氏量所不具有的局域化的手征对称性变换 ！

赝标介子手征有效拉氏量: 正常项

$$
\begin{aligned}
\mathcal{L}_4 \;=\; & \mathbf{L_1}\{\mathbf{tr_f}[\nabla_\mu \mathbf{U}(\nabla^\mu \mathbf{U})^\dagger]\}^2 + \mathbf{L_2}\mathbf{tr_f}[\nabla_\mu \mathbf{U}(\nabla_\nu \mathbf{U})^\dagger \mathbf{tr_f}[\nabla^\mu \mathbf{U}(\nabla^\nu \mathbf{U})^\dagger] \\
& + \mathbf{L_3}\mathbf{tr_f}[\nabla_\mu \mathbf{U}(\nabla^\mu \mathbf{U})^\dagger \nabla_\nu \mathbf{U}(\nabla^\nu \mathbf{U})^\dagger] \\
& + \mathbf{L_4}\mathbf{tr_f}[\nabla_\mu \mathbf{U}(\nabla^\mu \mathbf{U})^\dagger]\mathbf{tr_f}(\chi \mathbf{U}^\dagger + \mathbf{U}\chi^\dagger) \\
& + \mathbf{L_5}\mathbf{tr_f}[\nabla_\mu \mathbf{U}(\nabla^\mu \mathbf{U})^\dagger(\chi \mathbf{U}^\dagger + \mathbf{U}\chi^\dagger)] + \mathbf{L_6}[\mathbf{tr_f}(\chi \mathbf{U}^\dagger + \mathbf{U}\chi^\dagger)]^2 \\
& + \mathbf{L_7}[\mathbf{tr_f}(\chi \mathbf{U}^\dagger - \mathbf{U}\chi^\dagger)]^2 + \mathbf{L_8}\mathbf{tr_f}(\mathbf{U}\chi^\dagger \mathbf{U}\chi^\dagger + \chi \mathbf{U}^\dagger \chi \mathbf{U}^\dagger) \\
& - i\mathbf{L_9}\mathbf{tr_f}[\mathbf{f}^{\mathbf{R}}_{\mu\nu}\nabla^\mu \mathbf{U}(\nabla^\nu \mathbf{U})^\dagger + \mathbf{f}^{\mathbf{L}}_{\mu\nu}(\nabla^\mu \mathbf{U})^\dagger \nabla^\nu \mathbf{U}] \\
& + \mathbf{L_{10}}\mathbf{tr_f}(\mathbf{U}\mathbf{f}^{\mathbf{L}}_{\mu\nu}\mathbf{U}^\dagger \mathbf{f}^{\mu\nu}_{\mathbf{R}}) + \mathbf{H_1}\mathbf{tr_f}(\mathbf{f}^{\mathbf{R}}_{\mu\nu}\mathbf{f}^{\mu\nu}_{\mathbf{R}} + \mathbf{f}^{\mathbf{L}}_{\mu\nu}\mathbf{f}^{\mu\nu}_{\mathbf{L}}) + \mathbf{H_2}\mathbf{tr_f}(\chi\chi^\dagger)
\end{aligned}
$$

$$\bar{\theta} = 1, \qquad \mathbf{tr}_f v_\mu = 0, \qquad \mathbf{tr}_f a_\mu = 0$$

Back

Close

另一种方式： 逆的轴转动$\mathbf{V_L = V_R^\dagger = \Omega}$

$$\mathbf{U_\Omega(x) = V_R(x)U(x)V_L^\dagger(x) = \Omega^\dagger(x)U(x)\Omega^\dagger(x) = 1}$$

$$\begin{aligned}\mathbf{J_\Omega(x)} &\equiv \mathbf{\not{v}_\Omega(x) + \not{a}_\Omega(x)\gamma_5 - s_\Omega(x) + ip_\Omega(x)\gamma_5} \\ &= \mathbf{[V_R P_L + V_L P_R][J + i\not{\partial}][V_R^\dagger P_R + V_L^\dagger P_L]} \\ &= \mathbf{[\Omega(x)P_R + \Omega^\dagger(x)P_L]\ [J(x) + \not{\partial}_x]\ [\Omega(x)P_R + \Omega^\dagger(x)P_L]}\end{aligned}$$

$$\mathbf{s_\Omega = \frac{1}{2}[\Omega(s - ip)\Omega + \Omega^\dagger(s + ip)\Omega^\dagger]}$$

$$\mathbf{p_\Omega = \frac{i}{2}[\Omega(s - ip)\Omega - \Omega^\dagger(s + ip)\Omega^\dagger]}$$

$$\mathbf{v_{\Omega,\mu} = \frac{1}{2}[\Omega^\dagger(v_\mu + a_\mu + i\partial_\mu)\Omega + \Omega(v_\mu - a_\mu + i\partial_\mu)\Omega^\dagger]}$$

$$\mathbf{a_{\Omega,\mu} = \frac{1}{2}[\Omega^\dagger(v_\mu + a_\mu + i\partial_\mu)\Omega - \Omega(v_\mu - a_\mu + i\partial_\mu)\Omega^\dagger]}$$

赝标介子手征有效拉氏量: 正常项

$$1 = U_\Omega \to U'_\Omega \equiv \Omega^{\dagger\prime}U'\Omega^{\dagger\prime} = \tilde{h}\Omega^\dagger V_R^\dagger V_R U V_L^\dagger V_L \Omega^\dagger \tilde{h}^\dagger = 1$$

$$\begin{aligned} J_\Omega \to J'_\Omega &\equiv [\Omega' P_R + \Omega'^\dagger P_L]\ [J' + \not\partial]\ [\Omega' P_R + \Omega'^\dagger P_L] \\ &= [\tilde{h}\Omega V_L^\dagger P_R + \tilde{h}\Omega^\dagger V_R^\dagger P_L] \\ &\quad \times [V_R P_L + V_L P_R][J + \not\partial][V_R^\dagger P_R + V_L^\dagger P_L] \\ &\quad \times [V_R \Omega \tilde{h}^\dagger P_R + V_L \Omega^\dagger \tilde{h}^\dagger P_L] \\ &= \tilde{h}[\Omega P_R + \Omega^\dagger P_L][J + \not\partial][\Omega P_R + \Omega^\dagger P_L]\tilde{h}^\dagger = \tilde{h}[J_\Omega + \not\partial]\tilde{h}^\dagger \end{aligned}$$

$$s_\Omega \to s'_\Omega = \tilde{h} s_\Omega \tilde{h}^\dagger \qquad p_\Omega \to p'_\Omega = \tilde{h} p_\Omega \tilde{h}^\dagger$$

$$v_{\Omega,\mu} \to v'_{\Omega,\mu} = \tilde{h}[v_{\Omega,\mu} + i\partial_\mu]\tilde{h}^\dagger \qquad a_{\Omega,\mu} \to a'_{\Omega,\mu} = \tilde{h} a_{\Omega,\mu} \tilde{h}^\dagger$$

$$\overline{\nabla}_\mu = \partial_\mu - i v_{\Omega,\mu} \qquad \overline{\nabla}_\mu \to \overline{\nabla}'_\mu = \tilde{h}\overline{\nabla}_\mu \tilde{h}^\dagger$$

在手征变换下，转动的外场按hidden symmetry变换，其中 v_Ω 起规范场作用！

赝标介子手征有效拉氏量: 正常项

$$\mathcal{L}_{\mathrm{eff,N}}[\mathbf{U},\mathbf{J}] = \mathcal{L}_{\mathrm{eff,N}}[\mathbf{1},\mathbf{J}_\Omega] = \mathcal{L}_{\mathrm{eff,N}}[\mathbf{1},\mathbf{J}'_\Omega]$$

$$\mathcal{L}_2 = \mathbf{F}_0^2\mathrm{tr}_\mathbf{f}[\mathbf{a}_\Omega^2 + \mathbf{B}_0\mathbf{s}_\Omega]$$

$$\begin{aligned}\mathcal{L}_4 = \mathbf{tr}_\mathbf{f}\Big[& -\mathcal{K}_1[\mathbf{d}_\mu\mathbf{a}_\Omega^\mu]^2 - \mathcal{K}_2(\mathbf{d}^\mu\mathbf{a}_\Omega^\nu - \mathbf{d}^\nu\mathbf{a}_\Omega^\mu)(\mathbf{d}_\mu\mathbf{a}_{\Omega,\nu} - \mathbf{d}_\nu\mathbf{a}_{\Omega,\mu}) + \mathcal{K}_3[\mathbf{a}_\Omega^2]^2 \\ & +\mathcal{K}_4\mathbf{a}_\Omega^\mu\mathbf{a}_\Omega^\nu\mathbf{a}_{\Omega,\mu}\mathbf{a}_{\Omega,\nu} + \mathcal{K}_5\mathbf{a}_\Omega^2\mathrm{tr}_\mathbf{f}[\mathbf{a}_\Omega^2] + \mathcal{K}_6\mathbf{a}_\Omega^\mu\mathbf{a}_\Omega^\nu\mathrm{tr}_\mathbf{f}[\mathbf{a}_{\Omega,\mu}\mathbf{a}_{\Omega,\nu}] + \mathcal{K}_7\mathbf{s}_\Omega^2 \\ & +\mathcal{K}_8\mathbf{s}_\Omega\mathrm{tr}_\mathbf{f}[\mathbf{s}_\Omega] + \mathcal{K}_9\mathbf{p}_\Omega^2 + \mathcal{K}_{10}\mathbf{p}_\Omega\mathrm{tr}_\mathbf{f}[\mathbf{p}_\Omega] + \mathcal{K}_{11}\mathbf{s}_\Omega\mathbf{a}_\Omega^2 + \mathcal{K}_{12}\mathbf{s}_\Omega\mathrm{tr}_\mathbf{f}[\mathbf{a}_\Omega^2] \\ & -\mathcal{K}_{13}\mathbf{V}_\Omega^{\mu\nu}\mathbf{V}_{\Omega,\mu\nu} + \mathbf{i}\mathcal{K}_{14}\mathbf{V}_\Omega^{\mu\nu}\mathbf{a}_{\Omega,\mu}\mathbf{a}_{\Omega,\nu} + \mathcal{K}_{15}\mathbf{p}_\Omega\mathbf{d}^\mu\mathbf{a}_{\Omega,\mu}\Big]\end{aligned}$$

$$\mathbf{d}^\mu\mathbf{a}_\Omega^\nu = [\overline{\nabla}^\mu, \mathbf{a}_\Omega^\nu], \qquad \mathbf{V}_\Omega^{\mu\nu} = \mathbf{i}[\overline{\mathbf{v}}_\Omega^\mu, \overline{\mathbf{v}}_\Omega^\nu]$$

Back
Close

赝标介子手征有效拉氏量: 正常项

$$\mathrm{d}^\mu \mathrm{a}_\Omega^\nu - \mathrm{d}^\nu \mathrm{a}_\Omega^\mu = \frac{1}{2}[\Omega^\dagger \mathrm{f}_R^{\mu\nu}\Omega - \Omega \mathrm{f}_L^{\mu\nu}\Omega^\dagger]$$

$$\mathrm{s}_\Omega = \frac{1}{2}[\Omega(\mathrm{s} - \mathrm{ip})\Omega + \Omega^\dagger(\mathrm{s} + \mathrm{ip})\Omega^\dagger]$$

$$\mathrm{p}_\Omega = \frac{\mathrm{i}}{2}[\Omega(\mathrm{s} - \mathrm{ip})\Omega - \Omega^\dagger(\mathrm{s} + \mathrm{ip})\Omega^\dagger]$$

$$\mathrm{a}_\Omega^\mu = \frac{\mathrm{i}}{2}\Omega^\dagger[\nabla^\mu \mathrm{U}]\Omega^\dagger(\mathrm{x})$$

$$\mathrm{V}_\Omega^{\mu\nu} = \frac{\mathrm{i}}{4}\Omega^\dagger[-(\nabla^\mu \mathrm{U})\mathrm{U}^\dagger(\nabla^\nu \mathrm{U}) + (\nabla^\nu \mathrm{U})\mathrm{U}^\dagger(\nabla^\mu \mathrm{U})]\Omega^\dagger + \frac{1}{2}[\Omega^\dagger \mathrm{f}_R^{\mu\nu}\Omega + \Omega \mathrm{f}_L^{\mu\nu}\Omega^\dagger]$$

$$\mathrm{d}_\mu \mathrm{a}_\Omega^\mu = -\mathrm{B}_0[\mathrm{p}_\Omega - \frac{1}{3}\mathrm{tr}_f(\mathrm{p}_\Omega)]$$

Back
Close

膺标介子手征有效拉氏量: 正常项

$$L_1 = \frac{1}{32}\mathcal{K}_4 + \frac{1}{16}\mathcal{K}_5 + \frac{1}{16}\mathcal{K}_{13} - \frac{1}{32}\mathcal{K}_{14}$$

$$L_2 = \frac{1}{16}(\mathcal{K}_4 + \mathcal{K}_6) + \frac{1}{8}\mathcal{K}_{13} - \frac{1}{16}\mathcal{K}_{14}$$

$$L_3 = \frac{1}{16}(\mathcal{K}_3 - 2\mathcal{K}_4 - 6\mathcal{K}_{13} + 3\mathcal{K}_{14})$$

$$L_4 = \frac{\mathcal{K}_{12}}{16B_0} \qquad L_5 = \frac{\mathcal{K}_{11}}{16B_0} \qquad L_6 = \frac{\mathcal{K}_8}{16B_0^2}$$

$$L_7 = -\frac{\mathcal{K}_1}{16N_f} - \frac{\mathcal{K}_{10}}{16B_0^2} - \frac{\mathcal{K}_{15}}{16B_0N_f}$$

$$L_8 = \frac{1}{16}[\mathcal{K}_1 + \frac{1}{B_0^2}\mathcal{K}_7 - \frac{1}{B_0^2}\mathcal{K}_9 + \frac{1}{B_0}\mathcal{K}_{15}]$$

$$L_9 = \frac{1}{8}(4\mathcal{K}_{13} - \mathcal{K}_{14}) \qquad L_{10} = \frac{1}{2}(\mathcal{K}_2 - \mathcal{K}_{13}) \qquad H_1 = -\frac{1}{4}(\mathcal{K}_2 + \mathcal{K}_{13})$$

$$H_2 = \frac{1}{8}[-\mathcal{K}_1 + \frac{1}{B_0^2}\mathcal{K}_7 + \frac{1}{B_0^2}\mathcal{K}_9 - \frac{1}{B_0}\mathcal{K}_{15}]$$

Back
Close

赝标介子手征有效拉氏量: 反常项

$$\Omega(x) = V_L(x) = V_R^\dagger(x) = e^{-i\beta(x)} \qquad \beta(x) = -\frac{\phi(x)}{2F_0}$$

$$\mathcal{L}_{\rm eff}[U,J] = \mathcal{L}_{\rm eff}[1,J_\Omega] + \int_0^1 dt\ e^{t\delta_\beta}\ {\rm tr}_f[\beta(x)\tilde{\Omega}(x)]$$

$$\mathcal{L}_{\rm eff}[U,J] = \mathcal{L}_{\rm eff,N}[U,J] + \mathcal{L}_{\rm eff,A}[U,J]$$

$$\mathcal{L}_{\rm eff,A}[U,J] = \mathcal{L}_{\rm eff,A}[1,J_\Omega] + \int_0^1 dt\ e^{t\delta_\beta}\ {\rm tr}_f[\beta(x)\tilde{\Omega}(x)]$$

内禀宇称变换:

$$v^\mu \to v_\mu \quad a^\mu \to -a_\mu \quad s \to s \quad p \to -p \quad U \to U^\dagger \quad \Omega \to \Omega^\dagger \quad \delta_\beta \overset{?}{\to} \delta_\beta$$

$$v_\Omega^\mu \to v_{\Omega,\mu} \quad a_\Omega^\mu \to -a_{\Omega,\mu} \quad s_\Omega \to s_\Omega \quad p_\Omega \to -p_\Omega \quad \tilde{\Omega} \to \tilde{\Omega} \quad \beta \to -\beta$$

正常项 = 内禀宇称偶 = 偶数个 ϕ, a^μ, p 　反常项 = 内禀宇称奇 = 奇数个 ϕ, a^μ, p

反常项 $\sim$ 奇数个 $\epsilon_{\mu\nu\epsilon\rho}$

赝标介子手征有效拉氏量: 反常项

$$\delta_\beta \mathbf{v}_\mu = \partial_\mu \alpha + \mathbf{i}[\alpha, \mathbf{v}_\mu] + \mathbf{i}[\beta, \mathbf{a}_\mu] \qquad \delta_\beta \mathbf{s} = \mathbf{i}[\alpha, \mathbf{s}] - \{\beta, \mathbf{p}\}$$

$$\delta_\beta \mathbf{a}_\mu = \partial_\mu \beta + \mathbf{i}[\alpha, \mathbf{a}_\mu] + \mathbf{i}[\beta, \mathbf{v}_\mu] \qquad \delta_\beta \mathbf{p} = \mathbf{i}[\alpha, \mathbf{p}] + \{\beta, \mathbf{s}\}$$

$$\delta'_\beta \mathbf{v}'_\mu = \partial^\mu \alpha + \mathbf{i}[\alpha, \mathbf{v}^\mu] + \mathbf{i}[\beta, \mathbf{a}^\mu] = \delta_\beta \mathbf{v}^\mu = \delta'_\beta \mathbf{v}^\mu$$

$$\delta'_\beta \mathbf{a}'_\mu = -\partial^\mu \beta - \mathbf{i}[\alpha, \mathbf{a}^\mu] - \mathbf{i}[\beta, \mathbf{v}^\mu] = -\delta_\beta \mathbf{a}^\mu = -\delta'_\beta \mathbf{a}^\mu$$

$$\delta'_\beta \mathbf{s}' = \mathbf{i}[\alpha, \mathbf{s}] - \{\beta, \mathbf{p}\} = \delta_\beta \mathbf{s} = \delta'_\beta \mathbf{s}$$

$$\delta'_\beta \mathbf{p}' = -\mathbf{i}[\alpha, \mathbf{p}] - \{\beta, \mathbf{s}\} = -\delta_\beta \mathbf{p} = -\delta'_\beta \mathbf{p}$$

$$\delta'_\beta = \delta_\beta$$

Back
Close

赝标介子手征有效拉氏量: 反常项

$$\mathbf{S_{WZ}[U,J]}$$

$$\equiv \int d^4x \int_0^1 dt\ e^{t\delta_\beta}\ \mathrm{tr_f}[\beta(x)\tilde{\Omega}(x)] = (e^{\delta_\beta}-1)Z[J] = Z[J_\Omega] - Z[J]$$

$$\overset{\Omega(t,x)=e^{-it\beta(x)}}{=\!=\!=\!=\!=\!=\!=\!=} Z[J_{\Omega(1)}] - Z[J_{\Omega(0)}] = \int_0^1 dt\ \frac{dZ[J_{\Omega(t)}]}{dt}$$

$$= -\int_0^1 dt \int d^4x\ \mathrm{tr_f}[\beta(x)\tilde{\Omega}(x)]\big|_{J\to J_{\Omega(t)}}$$

$$= -\frac{i}{2}\int_0^1 dt \int d^4x\ \mathrm{tr_f}[\frac{\partial U(t,x)}{\partial t}U^\dagger(t,x)\tilde{\Omega}(x)]\big|_{J\to J_{\Omega(t)}}$$

$$\overset{J=0}{=\!=\!=\!=} -\frac{N_c i}{48\pi^2}\int_0^1 dt \int d^4x\ \epsilon^{\mu\nu\mu'\nu'}\mathrm{tr_f}[U^\dagger(t,x)\frac{\partial U(t,x)}{\partial t}$$
$$\times L_\mu(t,x)L_\nu(t,x)L_{\mu'}(t,x)L_{\nu'}(t,x)]$$

$$L_\mu \equiv U^\dagger\partial_\mu U \qquad d^\mu a^\nu_\Omega\big|_{J=0} = d^\nu a^\mu_\Omega\big|_{J=0} \qquad V^{\mu\nu}_\Omega\big|_{J=0} = i[a^\mu_\Omega, a^\nu_\Omega]\big|_{J=0}$$

Back
Close

膺标介子手征有效拉氏量: 反常项

$$\frac{\partial}{\partial t}\mathrm{tr_f}\left[L_i(t,y)L_j(t,y)L_k(t,y)L_l(t,y)L_m(t,y)\right]d\Sigma^{ijklm}$$

$$= 5\frac{\partial}{\partial y^m}\mathrm{tr_f}\left[U^\dagger(t,y)\frac{\partial U(t,y)}{\partial t}L_i(t,y)L_j(t,y)L_k(t,y)L_l(t,y)\right]d\Sigma^{ijklm}$$

$$\int_Q d\Sigma^{ijklm}\frac{\partial}{\partial y^m} = \int d^4x\ \epsilon^{\mu\nu\sigma\rho}$$

$$\int_Q d\Sigma^{ijklm}\ \mathrm{tr_f}[L_i(1,y)L_j(1,y)L_k(1,y)L_l(1,y)L_m(1,y)]$$

$$= \int_Q d\Sigma^{ijklm}\int_0^1 dt\ \frac{\partial}{\partial t}\mathrm{tr_f}[L_i(t,y)L_j(t,y)L_k(t,y)L_l(t,y)L_m(t,y)]$$

$$= 5\int d^4x\int_0^1 dt\,\epsilon^{\mu\nu\sigma\rho}\mathrm{tr_f}\left[U^\dagger(t,x)\frac{\partial U(t,x)}{\partial t}L_\mu(t,x)L_\nu(t,x)L_\sigma(t,x)L_\rho(t,x)\right]$$

$$S_{WZ}[U,0] = -\frac{N_c i}{48\pi^2}\int_0^1 dt\int d^4x\ \epsilon^{\mu\nu\mu'\nu'}\mathrm{tr_f}\left[U^\dagger\frac{\partial U}{\partial t}L_\mu L_\nu L_{\mu'}L_{\nu'}\right]$$

$$= -\frac{N_c i}{240\pi^2}\int_Q d\Sigma^{ijklm}\ \mathrm{tr_f}[L_i(1,y)L_j(1,y)L_k(1,y)L_l(1,y)L_m(1,y)]$$

赝标介子手征有效拉氏量: 反常项

$$\mathbf{S}_{\mathrm{WZ}}[\mathbf{U},\mathbf{J}] = \mathbf{S}_{\mathrm{WZ}}[\mathbf{U},\mathbf{J}]|_{\mathbf{J}=0} + \frac{1}{48\pi^2}\int \mathbf{d}^4\mathbf{x}\,\varepsilon_{\mu\nu\alpha\beta}\mathbf{W}_{\mu\nu\alpha\beta}$$

$$\begin{aligned}
\mathbf{W}_{\mu\nu\alpha\beta} = \mathrm{tr}_{\mathbf{f}}\{&[-\mathbf{l}_\mu\mathbf{U}_{\nu\mathbf{L}}\mathbf{U}_{\alpha\mathbf{L}}\mathbf{U}_{\beta\mathbf{L}} + \partial_\mu\mathbf{l}_\nu\mathbf{l}_\alpha\mathbf{U}_{\beta\mathbf{L}} + \mathbf{l}_\mu\partial_\nu\mathbf{l}_\alpha\mathbf{U}_{\beta\mathbf{L}} + (\mathbf{L}\to\mathbf{R})] \\
&+\partial_\mu\mathbf{l}_\nu\mathbf{U}\mathbf{r}_\alpha\mathbf{U}^{-1}\mathbf{U}_{\beta\mathbf{L}} + \mathbf{U}\partial_\mu\mathbf{r}_\nu\mathbf{U}^{-1}\mathbf{l}_\alpha\mathbf{U}_{\beta\mathbf{L}} \\
&-\frac{1}{2}[\mathbf{l}_\mu\mathbf{U}_{\nu\mathbf{L}}\mathbf{l}_\alpha\mathbf{U}_{\beta\mathbf{L}} - \mathbf{L}\to\mathbf{R}] \\
&+\mathbf{l}_\mu\mathbf{U}\mathbf{r}_\nu\mathbf{U}^{-1}\mathbf{U}_{\alpha\mathbf{L}}\mathbf{U}_{\beta\mathbf{L}} - \mathbf{U}\mathbf{r}_\mu\mathbf{U}^{-1}\mathbf{l}_\nu\mathbf{U}_{\alpha\mathbf{L}}\mathbf{U}_{\beta\mathbf{L}} \\
&-\mathbf{l}_\mu\partial_\nu\mathbf{l}_\alpha\mathbf{U}\mathbf{r}_\beta\mathbf{U}^{-1} - \partial_\mu\mathbf{l}_\nu\mathbf{l}_\alpha\mathbf{U}\mathbf{r}_\beta\mathbf{U}^{-1} + \mathbf{r}_\mu\partial_\nu\mathbf{r}_\alpha\mathbf{U}^{-1}\mathbf{l}_\beta\mathbf{U} \\
&+\partial_\mu\mathbf{r}_\nu\mathbf{r}_\alpha\mathbf{U}^{-1}\mathbf{l}_\beta\mathbf{U} + \mathbf{l}_\mu\mathbf{U}\mathbf{r}_\nu\mathbf{U}^{-1}\mathbf{l}_\alpha\mathbf{U}_{\beta\mathbf{L}} + \mathbf{r}_\mu\mathbf{U}^{-1}\mathbf{l}_\nu\mathbf{U}\mathbf{r}_\alpha\mathbf{U}_{\beta\mathbf{R}} \\
&+[\mathbf{l}_\mu\mathbf{l}_\nu\mathbf{l}_\alpha\mathbf{U}_{\beta\mathbf{L}} + (\mathbf{L}\to\mathbf{R})] - \mathbf{l}_\mu\mathbf{l}_\nu\mathbf{l}_\alpha\mathbf{U}\mathbf{r}_\beta\mathbf{U}^{-1} \\
&+\mathbf{r}_\mu\mathbf{r}_\nu\mathbf{r}_\alpha\mathbf{U}^{-1}\mathbf{l}_\beta\mathbf{U}_{\beta\mathbf{L}} - \frac{1}{2}\mathbf{l}_\mu\mathbf{U}\mathbf{r}_\nu\mathbf{U}^{-1}\mathbf{l}_\alpha\mathbf{U}\mathbf{r}_\beta\mathbf{U}^{-1}\}
\end{aligned}$$

$$\mathbf{U}_{\mu\mathbf{L}} = \partial_\mu\mathbf{U}\cdot\mathbf{U}^{-1} \qquad \mathbf{U}_{\mu\mathbf{R}} = \mathbf{U}^{-1}\partial_\mu\mathbf{U}$$

Back
Close

为什么要计算U场的圈图？

- 基本场理论的S矩阵的解析性，幺正性等一系列基本性质要求
- 描述的是物理粒子，应有量子效应
- S矩阵的虚部通常采用树图(经典理论)是算不出来的，要通过虚粒子对的产生才能得到
- S矩阵的非局域，非解析项，如对能量或质量的对数依赖项采用树图是算不出来的，只有通过圈图计算才能得到

圈图的特点：

- 传播子是无质量的标量场的传播子
- 相互作用顶角是带动量幂次，包括含微商或含外场

Back
Close

一个有L个圈的费曼图，有E条赝标介子外线， I条赝标介子内线，n_i个动量幂次为 2n（包括所含的微商和外场产生的动量幂次）， 含b_i个赝标介子线的p^{2n}阶第i 种类型顶角。其对动量（包括内线动量和外线动量及外场）的总依赖幂次为ω

$$\omega = 4L - 2I + \sum_i 2n\ n_i$$

$$L = I + 1 - \sum_i n_i$$

$$\omega = 2L + 2 + 2\sum_i (n-1)n_i$$

$$p^2 \to \omega = 2(L+1) \qquad k\text{个}p^4,\text{其它}p^2 \to \omega = 2(L+1) + 2k$$

圈图的动量幂次高；高动量的相互作用项不会通过圈图对本阶或更低阶的项产生影响

- 在每一个展开动量阶，已经写出了所有可能的相互作用项
- 在每一个展开动量阶，都存在足够的抵消项抵消发散

$$\mathbf{F}_0 = \mathbf{F}_0^r \qquad \mathbf{B}_0 = \mathbf{B}_0^r$$

$$\mathbf{L}_i = \mathbf{L}_i^r + \mathbf{\Gamma}_i\lambda, \qquad \mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{10}$$

$$\mathbf{H}_i = \mathbf{H}_i^r + \mathbf{\Delta}_i\lambda, \qquad \mathbf{i} = \mathbf{1}, \mathbf{2}$$

$$\lambda = (\mathbf{4}\pi)^{-\mathbf{2}}\mu^{\mathbf{d}-\mathbf{4}}\left[\frac{\mathbf{1}}{\mathbf{d}-\mathbf{4}} - \frac{\mathbf{1}}{\mathbf{2}}[\ln(\mathbf{4}\pi) + \gamma + \mathbf{1}]\right]$$

Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Δ_1	Δ_2
$\frac{3}{32}$	$\frac{3}{16}$	0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{11}{144}$	0	$\frac{5}{48}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{8}$	$\frac{5}{24}$

Back
Close

树图

$$\mathbf{Z}[\mathbf{J},\bar\theta] = -\mathbf{i}\ln\int \mathcal{D}\mathcal{A}_{\mu,\mathbf{i}}\mathcal{D}\bar{\mathbf{q}}_\mathbf{h}\mathcal{D}\mathbf{q}_\mathbf{h}\mathcal{D}\bar{\mathbf{q}}_\mathbf{l}\mathcal{D}\mathbf{q}_\mathbf{l}\ \mathbf{e}^{\mathbf{i}\int \mathbf{d}^4\mathbf{x}[\mathcal{L}_{\mathrm{QCD}}+\bar{\mathbf{q}}\mathbf{J}\mathbf{q}]}$$

$$= -\mathbf{i}\ln\int \mathcal{D}\mathbf{U}\ \mathbf{e}^{\mathbf{i}\int \mathbf{d}^4\mathbf{x}\mathcal{L}_{\mathrm{eff}}[\mathbf{U},\mathbf{J},\bar\theta]} = \int \mathbf{d}^4\mathbf{x}\ \mathcal{L}_2[\mathbf{U}_\mathbf{c},\mathbf{J},\bar\theta] + \mathbf{O}(\mathbf{p}^4)$$

R.Jackiw, Phys.Rev. D9, 1686(1974)

$$\frac{\delta}{\delta\mathbf{U}_\mathbf{c}(\mathbf{x})}\int \mathbf{d}^4\mathbf{y}\ \{\mathcal{L}_2[\mathbf{U}_\mathbf{c},\mathbf{J},\bar\theta] + \mathbf{i}\mathbf{B}_0\mathbf{F}_0^2\lambda[\ln\det{}_\mathbf{f}\mathbf{U}(\mathbf{y}) + \mathbf{i}\bar\theta(\mathbf{y})]\} = \mathbf{0}$$

$$\delta\mathcal{L}_2 = \delta\{\frac{\mathbf{F}_0^2}{4}\mathrm{tr}_\mathbf{f}[\nabla_\mu\mathbf{U}(\nabla^\mu\mathbf{U})^\dagger] + \frac{\mathbf{F}_0^2}{4}\mathrm{tr}_\mathbf{f}(\chi\mathbf{U}^\dagger + \mathbf{U}\chi^\dagger) + \frac{\mathbf{H}_0}{12}\mathrm{tr}_\mathbf{f}(\nabla_\mu\bar\theta\nabla^\mu\bar\theta)\}$$

$$= \{\frac{\mathbf{F}_0^2}{4}\mathrm{tr}_\mathbf{f}[\mathbf{U}^\dagger(\nabla^\mu\nabla_\mu\mathbf{U})\mathbf{U}^\dagger - (\nabla^\mu\nabla_\mu\mathbf{U}^\dagger)] + \frac{\mathbf{F}_0^2}{4}\mathrm{tr}_\mathbf{f}(\chi^\dagger - \mathbf{U}^\dagger\chi\mathbf{U}^\dagger)\}\delta\mathbf{U}$$

$$= -\mathbf{i}\mathbf{F}_0^2\Omega^\dagger[\mathbf{d}_\mu\mathbf{a}_\Omega^\mu + \mathbf{B}_0\mathbf{p}_\Omega]\Omega^\dagger\delta\mathbf{U}$$

$\mathbf{p}^2$场方程： $\mathbf{d}_\mu\mathbf{a}_\Omega^\mu + \mathbf{B}_0[\mathbf{p}_\Omega - \frac{1}{3}\mathrm{tr}_\mathbf{f}\mathbf{p}_\Omega] = \mathbf{0}$ $\quad \lambda = \frac{1}{3}\mathrm{tr}_\mathbf{f}\mathbf{p}_\Omega \quad \mathrm{tr}_\mathbf{f}[\mathbf{d}_\mu\mathbf{a}_\Omega^\mu] = \mathbf{0}$

$$\Omega_\mathbf{c}^\dagger(\nabla^\mu\nabla_\mu\mathbf{U}_\mathbf{c})\Omega_\mathbf{c}^\dagger - \Omega_\mathbf{c}(\nabla^\mu\nabla_\mu\mathbf{U}_\mathbf{c}^\dagger)\Omega_\mathbf{c} + [\Omega_\mathbf{c}\chi^\dagger\Omega_\mathbf{c} - \Omega_\mathbf{c}^\dagger\chi\Omega_\mathbf{c}^\dagger]|_{\mathrm{traceless}} = \mathbf{0}$$

Back
Close

树图

$$\Omega^\dagger(\nabla^\mu\nabla_\mu U)\Omega^\dagger - \Omega(\nabla^\mu\nabla_\mu U^\dagger)\Omega + [\Omega\chi^\dagger\Omega - \Omega^\dagger\chi\Omega^\dagger]|_{\mathrm{traceless}} = 0$$

真空：$v_\mu = a_\mu = p = 0 \quad s = M = \mathrm{diag}(m_u, m_d, m_s) \quad U = \Omega = 1$

$$\frac{\delta Z[J,\bar\theta]}{\delta J(x)} = \int d^4y\ \frac{\partial\mathcal{L}_2[U_c, J, \bar\theta]}{\partial J(x)} + O(p^4)$$

$$\langle 0|\bar q(x)\lambda^a q(x)|0\rangle_{J=0} = \frac{\delta Z[J,\bar\theta]}{\delta s_a(x)}\Big|_{v=a=p=0;s=M}$$

$$= \int d^4y\ \frac{\partial\mathcal{L}_2[U_c, J, \bar\theta]}{\partial s_a(x)}\Big|_{J=0;U_c=1} + O(p^4) = F_0^2 B_0 \mathrm{tr}_f(\lambda^a) + O(p^4)$$

Back
Close

$$\langle 0|T\bar{q}(x)\lambda^a\gamma_5 q(x)\bar{q}(x')\lambda^b\gamma_5 q(x')|0\rangle_{v=a=p=0;s=M}$$

$$= -i\frac{\delta^2 Z[J,\bar{\theta}]}{\delta p_a(x)\delta p_b(x')}\Big|_{v=a=p=0;s=M}$$

$$= -i\int d^4y\ \frac{\delta}{\delta p_b(x')}\frac{\partial\mathcal{L}_2[U_c,J,\bar{\theta}]}{\partial p_a(x)}\Big|_{v=a=p=0;s=M,U_c=1} + O(p^4)$$

$$= \frac{1}{2}F_0^2 B_0 \mathrm{tr_f}\{\lambda^a[\frac{\delta U_c^\dagger(x)}{\delta p_b(x')} - \frac{\delta U_c(x)}{\delta p_b(x')}]\}\Big|_{v=a=p=0;s=M,U_c=1} + O(p^4)$$

$$= -F_0^2 B_0 \mathrm{tr_f}\{\lambda^a\frac{\delta U_c(x)}{\delta p_b(x')}\}\Big|_{v=a=p=0;s=M,U_c=1} + O(p^4)$$

$$U_c^\dagger(x)U_c(x) = 1 \qquad [\delta U_c^\dagger(x)]U_c(x) + U_c^\dagger(x)\delta U_c(x) = 0$$

$$\{\Omega_c^\dagger(\partial^2 U_c)\Omega_c^\dagger - \Omega_c(\partial^2 U_c^\dagger)\Omega_c$$
$$+2B_0[\Omega_c(M - ip)\Omega_c - \Omega_c^\dagger(M + ip)\Omega^\dagger]|_{\text{traceless}}\}|_{v=a=0;s=M} = 0$$

$$\partial^2 \delta U_c - \partial^2 \delta U_c^\dagger + 2B_0[\{\delta\Omega_c, M\} - \{\delta\Omega_c^\dagger, M\} - 2i\delta p]|_{\text{traceless}} = 0$$

$$\partial^2 \delta U_c + B_0[\{\delta U_c, M\} - 2i\delta p] - \frac{1}{3}B_0 \mathrm{tr}_f[\{\delta U_c, M\} - 2i\delta p] = 0$$

$$B_0\{M, \lambda_P\} - \frac{2}{3}B_0 \mathrm{tr}_f(M\lambda_P) = \breve{M}_P^2 \lambda_P \qquad \mathrm{tr}_f(\lambda_P \lambda_{P'}^\dagger) = 2\delta_{PP'}$$

$$\delta U_c = \delta U_P \lambda_P \qquad \delta p = \delta p_P \lambda_P$$

$$(\partial^2 + \breve{M}_P^2)\delta U_P - 2iB_0 \delta p_P = 0 \qquad \frac{\delta U_P(x)}{\delta p_{P'}(y)} = \frac{2iB_0 \delta_{PP'}}{\partial_x^2 + \breve{M}_P^2}\delta^4(x - y)$$

$$\langle 0|T\bar{q}(x)\lambda^a\gamma_5 q(x)\bar{q}(y)\lambda_{P'}\gamma_5 q(y)|0\rangle_{v=a=p=0;s=M}$$

$$= -F_0^2 B_0 \mathrm{tr}_f(\lambda^a\lambda_P)\frac{\delta U_P(x)}{\delta p_{P'}(y)}\Big|_{\text{真空}} = -i\frac{2B_0^2F_0^2\mathrm{tr}_f(\lambda^a\lambda_{P'})}{\partial_x^2+\breve{M}_{P'}^2}\delta^4(x-y)$$

$$\int d^4x d^4y\ e^{iq\cdot x+iq'\cdot y}\langle 0|T\bar{q}(x)\lambda^a\gamma_5 q(x)\bar{q}(y)\lambda_{P'}\gamma_5 q(y)|0\rangle_{v=a=p=0;s=M}$$

$$= -i\int d^4x d^4y\ e^{iq\cdot x+iq'\cdot y}\frac{2B_0^2F_0^2\mathrm{tr}_f(\lambda^a\lambda_{P'})}{-q^2+\breve{M}_{P'}^2}\delta^4(x-y)$$

$$= i\frac{2B_0^2F_0^2\mathrm{tr}_f(\lambda^a\lambda_{P'})}{q^2-\breve{M}_{P'}^2}(2\pi)^4\delta^4(q+q')$$

$$= i\frac{\langle 0|\bar{q}(0)\lambda^a\gamma_5 q(0)|\vec{p},\sigma\rangle\langle\vec{p},\sigma|\bar{q}(0)\lambda_{P'}\gamma_5 q(0)|0\rangle}{q^2-\breve{M}_{P'}^2}(2\pi)^4\delta^4(q+q')$$

$$\langle 0|\bar{q}(0)\lambda_P^\dagger\gamma_5 q(0)|\vec{p},\sigma\rangle = \langle\vec{p},\sigma|\bar{q}(0)\lambda_P\gamma_5 q(0)|0\rangle = 2B_0F_0$$

$$B_0\{M, \lambda_P\} - \frac{2}{3}B_0 \mathrm{tr_f}(M\lambda_P) = \breve{M}_P^2 \lambda_P \qquad \mathrm{tr_f}(\lambda_P \lambda_{P'}^\dagger) = 2\delta_{PP'}$$

$$\lambda_{\pi^+} = -\sqrt{\frac{1}{2}}(\lambda^1 + i\lambda^2) \qquad \lambda_{\pi^-} = \sqrt{\frac{1}{2}}(\lambda^1 - i\lambda^2)$$

$$\lambda_{K^+} = -\sqrt{\frac{1}{2}}(\lambda^4 + i\lambda^5) \qquad \lambda_{K^-} = \sqrt{\frac{1}{2}}(\lambda^4 - i\lambda^5)$$

$$\lambda_{K^0} = -\sqrt{\frac{1}{2}}(\lambda^6 + i\lambda^7) \qquad \lambda_{\bar{K}^0} = \sqrt{\frac{1}{2}}(\lambda^6 - i\lambda^7)$$

$$\lambda_{\pi^0} = \lambda^3 \cos\epsilon + \lambda^8 \sin\epsilon \qquad \lambda_{\eta'} = -\lambda^3 \sin\epsilon + \lambda^8 \cos\epsilon$$

树图

$$\tan(2\epsilon) = \frac{\sqrt{3}}{2}\frac{m_d - m_u}{m_s - \hat{m}} \qquad \hat{m} = \frac{1}{2}(m_u + m_d)$$

$$\breve{M}^2_{\pi^\pm} = (m_u + m_d)B_0$$

$$\breve{M}^2_{K^\pm} = (m_u + m_s)B_0$$

$$\breve{M}^2_{K^0} = \breve{M}^2_{\bar{K}^0} = (m_d + m_s)B_0$$

$$\breve{M}^2_{\pi^0} = (m_u + m_d)B_0 - \frac{4}{3}(m_s - \hat{m})B_0\frac{\sin^2\epsilon}{\cos(2\epsilon)}$$

$$\breve{M}^2_{\eta} = \frac{2}{3}(\hat{m} + 2m_s)B_0 + \frac{4}{3}(m_s - \hat{m})B_0\frac{\sin^2\epsilon}{\cos(2\epsilon)}$$

Back
Close

- 讨论了QCD所具有的手征,宇称和电荷共轭对称性
- 建立了流流格林函数的生成泛函,并导出它满足的对称性
- 用赝标介子的有效拉氏量来描述QCD的流流格林函数生成泛函
- 生成泛函的对称性被转嫁到赝标介子的有效拉氏量上 可无基本理论 !
- 根据对称性按照低能展开写下最一般的赝标介子的有效拉氏量
- 在低能展开的意义上可以计算圈图和进行重整化
- 最后计算流流格林函数,它被表达为有效拉氏量中参数的函数
- 由流流格林函数读出各种物理量,它也为有效拉氏量中参数的函数
- $\overrightarrow{\text{从上至下}}$: 基本理论 $\Leftrightarrow$ 有效场论 $\Leftrightarrow$ 现象学 :$\overleftarrow{\text{上至下从}}$

提高篇：有效场论

之

赝标介子手征有效拉氏量与QCD的关系

与

其它低能强子的手征有效拉氏量

Back
Close

- 赝标介子手征有效拉氏量与QCD的关系
  - 从QCD推导手征有效拉氏量的重要性、必要性
  - 详细推导过程
    1. 形式积掉胶子和重夸克场
    2. 积进双局域介子场和其共轭场并积掉轻夸克场
    3. 积进局域赝标介子场
    4. 利用手征转动将赝标介子场吸进外源
    5. 积掉双局域介子场和其共轭场及赝标介子约束的共轭场
  - 另类QCD推导
  - 从QCD定义手征有效拉氏量中的低能常数
- 其它低能强子的手征有效拉氏量
  - 矢量介子
  - 标量介子
  - η'
  - 统一描述轻介子
  - 重子

赝标介子手征有效拉氏量

与QCD的关系

从QCD推导手征有效拉氏量的重要性、必要性

是否必要？即使推出来，也和由对称性限制写下的没差别!

- 能够给出手征有效拉氏量中低能常数的QCD定义，奠定第一原理计算它们的基础 检验QCD，预言实验
- 对无穷阶低能展开进行求和，扩展有效理论的适用范围，超出低能赝标介子范围
- 不依赖QCD的对称性，没有近似，反应QCD的动力学结构

对理论物理暑期学校，应着重于基本理论及其与应用理论的关系!

详细推导过程：1.形式积掉胶子和重夸克场

$$e^{iZ[J,\bar\theta]} = \int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar q_h \mathcal{D}q_h \mathcal{D}\bar q \mathcal{D}q\ e^{i\int d^4x\ [\mathcal{L}_{QCD}+\bar q J q]}$$

$$= \int \mathcal{D}\bar q \mathcal{D}q\ e^{i\int d^4x \bar q(i\partial\!\!\!/+J)q} e^{iZ'[\bar q\gamma^\mu \frac{\lambda_i^C}{2} q,\bar\theta]}$$

$$e^{iZ'[I_i^\mu,\bar\theta]} = \int \mathcal{D}\mathcal{A}_{\mu,i}\mathcal{D}\bar q_h \mathcal{D}q_h\ e^{i\int d^4x\ [-gA_{\mu,i}I_i^\mu+\mathcal{L}']}$$

$$\mathcal{L}' = \sum_{f=c,b,t} \bar q_f(iD\!\!\!/ - m_f)q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,i}\mathcal{G}_i^{\mu\nu} + \frac{g^2\bar\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}\sum_{i=1}^{8}\mathcal{G}_{\mu\nu,i}\mathcal{G}_{\rho\sigma,i}$$

$$Z'[I_i^\mu,\bar\theta] = \sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x_n \frac{(-i)^n g^n}{n!} \underbrace{G^{i_1\cdots i_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n) I_{i_1}^{\mu_1}(x_1)\cdots I_{i_n}^{\mu_n}(x_n)}_{\to GCM\to NJL}$$

$$G^{i_1\cdots i_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n\left[\bar q_{f_1\alpha_1}(x_1)(\frac{\lambda^C_{i_1}}{2})_{\alpha_1\beta_1}\gamma^{\mu_1}q_{f_1\beta_1}(x_1)\right]\cdots\left[\bar q_{f_n\alpha_n}(x_n)(\frac{\lambda^C_{i_n}}{2})_{\alpha_n\beta_n}\gamma^{\mu_n}q_{f_n\beta_n}(x_n)\right]$$

$$= \int d^4x_1'\cdots d^4x_n' g^{n-2}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x_1',\cdots,x_n,x_n')\bar q^{\sigma_1}_{\alpha_1}(x_1)q^{\rho_1}_{\alpha_1}(x_1')\cdots\bar q^{\sigma_n}_{\alpha_n}(x_n)q^{\rho_n}_{\alpha_n}(x_n')$$

$$e^{iZ[J,\bar\theta]} = \int \mathcal{D}\bar q\mathcal{D}q\ \exp\{i\int d^4x\bar q(i\partial\!\!\!/+J)q + \sum_{n=2}^{\infty}\int d^4x_1 d^4x_1'\cdots d^4x_n d^4x_n'$$

$$\times\frac{(-i)^n(g^2)^{n-1}}{n!}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x_1',\cdots,x_n,x_n')\bar q^{\sigma_1}_{\alpha_1}(x_1)q^{\rho_1}_{\alpha_1}(x_1')\cdots\bar q^{\sigma_n}_{\alpha_n}(x_n)q^{\rho_n}_{\alpha_n}(x_n')\}$$

Back
Close

详细推导过程：1.形式积掉胶子和重夸克场

$$G^{i_1 i_2}_{\mu_1\mu_2}(x_1,x_2)\left[\bar{q}_{f_1\alpha_1}(x_1)(\frac{\lambda^C_{i_1}}{2})_{\alpha_1\beta_1}\gamma_{\mu_1}q_{f_1\beta_1}(x_1)\right]\left[\bar{q}_{f_2\alpha_2}(x_2)(\frac{\lambda^C_{i_2}}{2})_{\alpha_2\beta_2}\gamma_{\mu_2}q_{f_2\beta_2}(x_2)\right]$$

$$G^{i_1 i_2}_{\mu_1\mu_2}(x_1,x_2)=\delta_{i_1 i_2}G_{\mu_1\mu_2}(x_1,x_2) \qquad (\lambda^C_{i_1})_{\alpha_1\beta_1}(\lambda^C_{i_1})_{\alpha_2\beta_2}=2\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_1}-\frac{2}{N_c}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2}$$

$$=\frac{1}{2}G_{\mu_1\mu_2}(x_1,x_2)\left[\bar{q}_{f_1\alpha_1}(x_1)\gamma^{\mu_1}q_{f_1\beta_1}(x_1)\right]\left[\bar{q}_{f_2\alpha_2}(x_2)\gamma^{\mu_2}q_{f_2\beta_2}(x_2)\right](\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_1}-\frac{1}{N_c}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2})$$

$$=\int d^4x'd^4x'_2 G_{\mu_1\mu_2}(x_1,x_2)[-\frac{1}{2}\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_1}\delta(x'_1-x_2)\delta(x'_2-x_1)$$
$$-\frac{1}{2N_c}\gamma^{\mu_1}_{\sigma_1\rho_1}\gamma^{\mu_2}_{\sigma_2\rho_2}\delta(x'_1-x_1)\delta(x'_2-x_2)]\bar{q}^{\sigma_1}_{\alpha_1}(x_1)q^{\rho_1}_{\alpha_1}(x'_1)\bar{q}^{\sigma_2}_{\alpha_2}(x_1)q^{\rho_2}_{\alpha_1}(x'_2)$$

$$\overline{G}^{\sigma_1\sigma_2}_{\rho_1\rho_2}(x_1,x'_1,x_2,x'_2)=-\frac{1}{2}G_{\mu_1\mu_2}(x_1,x_2)[\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_1}\delta(x'_1-x_2)\delta(x'_2-x_1)+\frac{1}{N_c}\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_1}\delta(x'_1-x_2)\delta(x'_2-x_1)]$$

核心是可以把色盖尔曼矩阵转化为 $\delta_{\alpha\beta}$!

$$G^{i_1 i_2 i_3}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)=gf^{i_1 i_2 i_3}G^{(0)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)+gd^{i_1 i_2 i_3}G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)$$

$$f^{i_1 i_2 i_3}=-2i\mathrm{tr}\left[[\frac{\lambda^C_{i_1}}{2},\frac{\lambda^C_{i_2}}{2}]\frac{\lambda^C_{i_3}}{2}\right]=\frac{i}{4}\lambda^{i_2}_{\alpha\beta}\lambda^{i_1}_{\beta\gamma}\lambda^{i_3}_{\gamma\alpha}-\frac{i}{4}\lambda^{i_1}_{\alpha\beta}\lambda^{i_2}_{\beta\gamma}\lambda^{i_3}_{\gamma\alpha}$$

$$d^{i_1 i_2 i_3}=-2i\mathrm{tr}\left[\{\frac{\lambda^C_{i_1}}{2},\frac{\lambda^C_{i_2}}{2}\}\frac{\lambda^C_{i_3}}{2}\right]=\frac{1}{4}\lambda^{i_2}_{\alpha\beta}\lambda^{i_1}_{\beta\gamma}\lambda^{i_3}_{\gamma\alpha}+\frac{1}{4}\lambda^{i_1}_{\alpha\beta}\lambda^{i_2}_{\beta\gamma}\lambda^{i_3}_{\gamma\alpha}$$

$$f^{i_1 i_2 i_3}\lambda^{i_1}_{\alpha_1\beta_1}\lambda^{i_2}_{\alpha_2\beta_2}\lambda^{i_3}_{\alpha_3\beta_3}=2i(\delta_{\alpha_3\beta_2}\delta_{\alpha_2\beta_1}\delta_{\alpha_1\beta_3}-\delta_{\alpha_3\beta_1}\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_3})$$

$$d^{i_1 i_2 i_3}\lambda^{i_1}_{\alpha_1\beta_1}\lambda^{i_2}_{\alpha_2\beta_2}\lambda^{i_3}_{\alpha_3\beta_3}=2(\delta_{\alpha_3\beta_2}\delta_{\alpha_2\beta_1}\delta_{\alpha_1\beta_3}+\delta_{\alpha_3\beta_1}\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_3}-\frac{2}{N_c}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_3}\delta_{\alpha_3\beta_2}$$
$$-\frac{2}{N_c}\delta_{\alpha_1\beta_3}\delta_{\alpha_2\beta_2}\delta_{\alpha_3\beta_1}-\frac{2}{N_c}\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_1}\delta_{\alpha_3\beta_3}+\frac{4}{N_c^2}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2}\delta_{\alpha_3\beta_3})$$

详细推导过程：2.积进双局域介子场和其共轭场并积掉轻夸克场

$$\int \mathcal{D}\Phi \ \delta\left(N_c\Phi^{\sigma\rho}(x,x') - \bar{q}^{\sigma}_{\alpha}(x)q^{\rho}_{\alpha}(x')\right)$$

$$e^{iZ[J,\bar{\theta}]} = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}\Phi\,\delta\left(N_c\Phi^{\sigma\rho}(x,x') - \bar{q}^{\sigma}_{\alpha}(x)q^{\rho}_{\alpha}(x')\right) \exp\Big\{i\int d^4x\bar{q}(i\partial\!\!\!/ + J)q$$

$$+N_c\sum_{n=2}^{\infty}\int d^4x_1 d^4x'_1\cdots d^4x_n d^4x'_n \frac{(-i)^n(N_cg^2)^{n-1}}{n!}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x'_1,\cdots,x_n,x'_n)$$

$$\times\Phi^{\sigma_1\rho_1}(x_1,x'_1)\cdots\Phi^{\sigma_n\rho_n}(x_n,x'_n)\Big\}$$

$$\delta\left(N_c\Phi^{\sigma\rho}(x,x') - \bar{q}^{\sigma}_{\alpha}(x)q^{\rho}_{\alpha}(x')\right) = C\int \mathcal{D}\Pi e^{i\int d^4xd^4x'\Pi^{\sigma\rho}(x,x')[N_c\Phi^{\sigma\rho}(x,x') - \bar{q}^{\sigma}_{\alpha}(x)q^{\rho}_{\alpha}(x')]}$$

$$e^{iZ[J,\bar{\theta}]} = \int \mathcal{D}\Phi\mathcal{D}\Pi \ e^{i\Gamma_0[J,\Phi,\Pi]}$$

$$\Gamma_0[J,\Phi,\Pi] = -iN_c\mathrm{Tr}\ln[i\partial\!\!\!/ + J - \Pi] + N_c\int d^4xd^4x'\Phi^{\sigma\rho}(x,x')\Pi^{\sigma\rho}(x,x')$$

$$+N_c\sum_{n=2}^{\infty}\int d^4x_1 d^4x'_1\cdots d^4x_n d^4x'_n \frac{(-i)^n(N_cg^2)^{n-1}}{n!}$$

$$\times\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x'_1,\cdots,x_n,x'_n)\Phi^{\sigma_1\rho_1}(x_1,x'_1)\cdots\Phi^{\sigma_n\rho_n}(x_n,x'_n)$$

Back
Close

详细推导过程：3.积进局域赝标介子U场

$$\mathbf{\Phi^T(x',x)} \;\rightarrow [\mathbf{V_R(x')P_R + V_L(x')P_L]\Phi^T(x',x)[V_R^\dagger(x)P_L + V_L^\dagger(x)P_R]}$$
$$\mathbf{\Pi(x,x')} \rightarrow [\mathbf{V_R(x)P_L + V_L(x)P_R]\Pi(x,x')[V_R^\dagger(x')P_R + V_L^\dagger(x')P_L]}$$

选择特定手征转动将$\mathbf{\Phi^T(x,x)}$的标量和赝标部分转成纯标量:$\mathbf{V_L(x) = V_R^\dagger(x) = \Omega'(x)}$

记转后的场为：$\sigma(x) = \sigma^\dagger(x)$

$$\mathbf{\Phi^T(x,x)}\Big|_{\text{标量和赝标部分}} = [\mathbf{\Omega'(x)P_R + \Omega'^\dagger(x)P_L]\sigma(x)[\Omega'^\dagger(x)P_L + \Omega'(x)P_R]}$$
$$(\mathbf{1})_{\xi\eta}\mathbf{\Phi^{(b\xi)(a\eta)}(x,x)} = [\mathbf{\Omega'(x)\sigma(x)\Omega'(x) + \Omega'^\dagger(x)\sigma(x)\Omega'^\dagger(x)]^{ab}}$$
$$(\gamma_5)_{\xi\eta}\mathbf{\Phi^{(b\xi)(a\eta)}(x,x)} = [\mathbf{\Omega'(x)\sigma(x)\Omega'(x) - \Omega'^\dagger(x)\sigma(x)\Omega'^\dagger(x)]^{ab}}$$

$$\det\mathbf{\Omega'^2(x)} = \mathbf{e^{i\vartheta(x)}} \qquad \mathbf{\Omega(x) \equiv \Omega'(x)e^{-i\frac{\vartheta(x)}{2N_f}}}$$
$$\mathbf{U(x) \equiv \Omega^2(x)} \qquad \det\mathbf{\Omega^2(x)} = \mathbf{1}$$

$$\mathbf{e^{-i\frac{\vartheta(x)}{N_f}}\Omega^\dagger(x)}\mathrm{tr_l}[\mathbf{P_R\Phi^T(x,x)]\Omega^\dagger(x)} = \mathbf{e^{i\frac{\vartheta(x)}{N_f}}\Omega(x)}\mathrm{tr_l}[\mathbf{P_L\Phi^T(x,x)]\Omega(x)}$$
$$\mathbf{e^{2i\vartheta(x)}} = \frac{\det\{\mathrm{tr_l}[\mathbf{P_R\Phi^T(x,x)}]\}}{\det\{\mathrm{tr_l}[\mathbf{P_L\Phi^T(x,x)}]\}} \qquad \vartheta\mathbf{(x)} = \vartheta^\dagger\mathbf{(x)}$$

详细推导过程：3.积进局域赝标介子U场

$$\mathcal{O} = \mathbf{e}^{-\mathbf{i}\frac{\vartheta(\mathbf{x})}{\mathbf{N_f}}}\mathrm{tr_l}[\mathbf{P_R}\mathbf{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})] \qquad \mathcal{O}^\dagger = \mathbf{e}^{\mathbf{i}\frac{\vartheta(\mathbf{x})}{\mathbf{N_f}}}\mathrm{tr_l}[\mathbf{P_L}\mathbf{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]$$

$$\mathcal{F}[\mathcal{O}]\int\mathcal{D}\mathbf{U}\delta(\mathbf{U}^\dagger\mathbf{U}-\mathbf{1})\delta(\det\mathbf{U}-\mathbf{1})\delta(\mathbf{\Omega}\mathcal{O}^\dagger\mathbf{\Omega}-\mathbf{\Omega}^\dagger\mathcal{O}\mathbf{\Omega}^\dagger) = \mathbf{C}$$

$$\mathcal{F}^{-1}[\mathcal{O}] = \det\mathcal{O}\int\mathcal{D}\sigma\delta(\mathcal{O}^\dagger\mathcal{O}-\sigma^\dagger\sigma)\delta(\sigma-\sigma^\dagger)$$

$$\delta(\mathbf{\Omega}\mathcal{O}^\dagger\mathbf{\Omega}-\mathbf{\Omega}^\dagger\mathcal{O}\mathbf{\Omega}^\dagger) = \mathbf{C}\int\mathcal{D}\mathbf{\Xi}\mathbf{e}^{-\mathbf{i}\mathbf{N_c}\int\mathbf{d^4x}\mathbf{\Xi}^{\sigma\rho}(\mathbf{x})[\mathbf{\Omega}(\mathbf{x})\mathcal{O}^\dagger(\mathbf{x})\mathbf{\Omega}(\mathbf{x})-\mathbf{\Omega}^\dagger(\mathbf{x})\mathcal{O}(\mathbf{x})\mathbf{\Omega}^\dagger(\mathbf{x})]}$$

$$\mathbf{e}^{\mathbf{i}\mathbf{Z}[\mathbf{J},\bar{\theta}]} = \int\mathcal{D}\mathbf{\Phi}\mathcal{D}\mathbf{\Pi}\mathcal{D}\mathbf{\Xi}\mathcal{D}\mathbf{U}\delta(\mathbf{U}^\dagger\mathbf{U}-\mathbf{1})\delta(\det\mathbf{U}-\mathbf{1})\exp\Big\{\mathbf{i}\mathbf{\Gamma_0}[\mathbf{J},\mathbf{\Phi},\mathbf{\Pi}]$$
$$+\mathbf{i}\mathbf{\Gamma_I}[\mathbf{\Phi}]+\mathbf{i}\mathbf{N_c}\int\mathbf{d^4x}\mathrm{tr_f}\Big[\mathbf{\Xi}(\mathbf{x})\big(\mathbf{e}^{-\mathbf{i}\frac{\vartheta(\mathbf{x})}{\mathbf{N_f}}}\mathbf{\Omega}^\dagger(\mathbf{x})\mathrm{tr_l}[\mathbf{P_R}\mathbf{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]\mathbf{\Omega}^\dagger(\mathbf{x})$$
$$-\mathbf{e}^{\mathbf{i}\frac{\vartheta(\mathbf{x})}{\mathbf{N_f}}}\mathbf{\Omega}(\mathbf{x})\mathrm{tr_l}[\mathbf{P_L}\mathbf{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]\mathbf{\Omega}(\mathbf{x})\big)\Big]\Big\}$$

$$\mathbf{e}^{-\mathbf{i}\mathbf{\Gamma_I}[\mathbf{\Phi}]} \equiv \prod_{\mathbf{x}}\mathcal{F}^{-1}[\mathcal{O}(\mathbf{x})]$$
$$= \prod_{\mathbf{x}}\Big\{\big[\{\det\{\mathrm{tr_l}[\mathbf{P_R}\mathbf{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]\}\{\det\{\mathrm{tr_l}[\mathbf{P_L}\mathbf{\Phi}^{\mathbf{T}}(\mathbf{x},\mathbf{x})]\}\big]^{\frac{1}{2}}$$
$$\times\int\mathcal{D}\sigma\delta[(\mathrm{tr_l}\mathbf{P_R}\mathbf{\Phi}^{\mathbf{T}})(\mathrm{tr_l}\mathbf{P_L}\mathbf{\Phi}^{\mathbf{T}})-\sigma^\dagger\sigma]\delta(\sigma-\sigma^\dagger)\Big\}$$

详细推导过程：4.利用手征转动将赝标介子场吸进外源

$$V_L(x)=V_R^\dagger(x)=\Omega^\dagger(x)$$

$$\Phi_\Omega^T(x,y)=[\Omega^\dagger(x)P_R+\Omega(x)P_L]\Phi^T(x,y)[\Omega^\dagger(y)P_R+\Omega(y)P_L]$$

$$\Pi_\Omega(x,y)=[\Omega^\dagger(x)P_R+\Omega(x)P_L]\Pi(x,y)[\Omega^\dagger(y)P_R+\Omega(y)P_L]$$

$$\Gamma_0[J,\Phi,\Pi]=\Gamma_0[J_\Omega,\Phi_\Omega,\Pi_\Omega]+\text{反常项}$$

$$-iN_c\mathrm{Trln}[i\partial\!\!\!/+J-\Pi]=-iN_c\mathrm{Trln}[i\partial\!\!\!/+J_\Omega-\Pi_\Omega]+\underbrace{\text{反常项}}_{-S_{WZ}[\Omega^2,J]}$$

$$\Gamma_I[\Phi]=\Gamma_I[\Phi_\Omega]\qquad \vartheta(x)=\vartheta_\Omega(x)\qquad \mathcal{D}\Phi\mathcal{D}\Pi=\mathcal{D}\Phi_\Omega\mathcal{D}\Pi_\Omega$$

$$e^{iZ[J,\bar\theta]}=\int\mathcal{D}\Phi_\Omega\mathcal{D}\Pi_\Omega\mathcal{D}\Xi\mathcal{D}U\delta(U^\dagger U-1)\delta(\det U-1)e^{i\tilde\Gamma[J_\Omega,\Phi_\Omega,\Pi_\Omega,\Xi,\Omega]}$$

$$\tilde\Gamma[J_\Omega,\Phi_\Omega,\Pi_\Omega,\Xi,\Omega]\equiv\Gamma_0[J_\Omega,\Phi_\Omega,\Pi_\Omega]+\Gamma_I[\Phi_\Omega]+iS_{WZ}[\Omega^2,J]$$

$$+N_c\int d^4x\,\underbrace{\mathrm{tr}_f\left[\Xi(x)\left(e^{-i\frac{\vartheta_\Omega(x)}{N_f}}\mathrm{tr}_f[P_R\Phi_\Omega^T(x,x)]-e^{-i\frac{\vartheta_\Omega(x)}{N_f}}\mathrm{tr}_f[P_R\Phi_\Omega^T(x,x)]\right)\right]}_{\mathrm{tr}_{lf}[\Xi(x)(-i\sin\frac{\vartheta_\Omega(x)}{N_f}+\gamma_5\cos\frac{\vartheta_\Omega(x)}{N_f})\Phi_\Omega^T(x,x)]}$$

详细推导过程：5.积掉双局域介子和其共轭场及赝标介子约束共轭场

R.Jackiw,1686(1974) 圈图展开： $\int \mathcal{D}\phi e^{iS[\phi]} = e^{i\Gamma[\phi_c]}$ $\quad \phi_c(x) = \frac{\int \mathcal{D}\phi(x) e^{iS[\phi]}}{\int \mathcal{D}\phi e^{iS[\phi]}}$

$\frac{\partial \Gamma[\phi_c]}{\partial \phi_c} = 0 \qquad \Gamma[\phi_c] = S[\phi_c] +$ 圈图修正

$$e^{iZ[J,\bar{\theta}]} = \int \mathcal{D}U \delta(U^\dagger U - 1)\delta(\det U - 1)\ e^{iS_{\mathrm{eff}}[J_\Omega,\Phi_{\Omega c},\Pi_{\Omega c},\Xi_c,\Omega]}$$

$$e^{iS_{\mathrm{eff}}[J_\Omega,\Phi_{\Omega c},\Pi_{\Omega c},\Xi_c,\Omega]} = \int \mathcal{D}\Phi_\Omega \mathcal{D}\Pi_\Omega \mathcal{D}\Xi\ e^{i\tilde{\Gamma}[J_\Omega,\Phi_\Omega,\Pi_\Omega,\Xi,\Omega]}$$

$$\frac{\partial S_{\mathrm{eff}}[J_\Omega,\Phi_{\Omega c},\Pi_{\Omega c},\Xi_c,\Omega]}{\partial \Phi^{\sigma\rho}_{\Omega c}(x,y)} = \frac{\partial S_{\mathrm{eff}}[J_\Omega,\Phi_{\Omega c},\Pi_{\Omega c},\Xi_c,\Omega]}{\partial \Pi^{\sigma\rho}_{\Omega c}(x,y)} = 0$$

$$\frac{\partial S_{\mathrm{eff}}[J_\Omega,\Phi_{\Omega c},\Pi_{\Omega c},\Xi_c,\Omega]}{\partial \Xi^{\sigma\rho}_{c}(x,y)} = 0$$

$$S_{\mathrm{eff}}[J_\Omega,\Phi_{\Omega c},\Pi_{\Omega c},\Xi_c,\Omega] = \Gamma_0[J_\Omega,\Phi_{\Omega c},\Pi_{\Omega c}] + iS_{WZ}[\Omega^2,J]$$

$$+N_c\int d^4x\,\mathrm{tr}_{lf}[\Xi_c(x)(-i\sin\frac{\vartheta_{\Omega c}(x)}{N_f} + \gamma_5\cos\frac{\vartheta_{\Omega c}(x)}{N_f})\Phi^T_{\Omega c}(x,x)] + O(\frac{1}{N_c})$$

详细推导过程：5.积掉双局域介子和其共轭场及赝标介子约束共轭场

$$\tilde{\Xi}^{\sigma\rho}(x)\delta(x-y)+\Pi^{\sigma\rho}_{\Omega c}(x,y)+\sum_{n=1}^{\infty}\int d^4x_1d^4x'_1\cdots d^4x_nd^4x'_n\frac{(-i)^{n+1}(Ncg^2)^n}{n!}$$
$$\times\overline{G}^{\sigma\sigma_1\cdots\sigma_n}_{\rho\rho_1\cdots\rho_n}(x,y,x_1,x'_1,\cdots,x_n,x'_n)\Phi^{\sigma_1\rho_1}_{\Omega c}(x_1,x'_1)\cdots\Phi^{\sigma_n\rho_n}_{\Omega c}(x_n,x'_n)$$
$$+O(\frac{1}{N_c})=0$$

$$\Phi^{\sigma\rho}_{\Omega c}(x,y)=-i[(i\partial\!\!\!/+J_{\Omega}-\Pi_{\Omega c})^{-1}]^{\rho\sigma}(y,x)+O(\frac{1}{N_c})=0$$

$$\mathrm{tr}_l[(-i\sin\frac{\vartheta_{\Omega c}(x)}{N_f}+\gamma_5\cos\frac{\vartheta_{\Omega c}(x)}{N_f})\Phi^T_{\Omega c}(x,x)]=0$$

$$\tilde{\Xi}^{\sigma\rho}(x)\equiv$$
$$\frac{\partial}{\partial\Phi^{\sigma\rho}_{\Omega c}(x,x)}\int d^4y\,\mathrm{tr}_{lf}[\Xi_c(y)(-i\sin\frac{\vartheta_{\Omega c}(y)}{N_f}+\gamma_5\cos\frac{\vartheta_{\Omega c}(y)}{N_f})\Phi^T_{\Omega c}(y,y)]_{\Xi_c\text{固定}}$$

另类QCD推导

$$\Phi^{T,\rho\sigma}(x',x) \to \frac{1}{N_c} q^\rho_\alpha(x')\bar{q}^\sigma_\alpha(x)$$

$$e^{iZ[J,\bar\theta]} = \int \mathcal{D}A^i_\mu \mathcal{D}\bar{q}_h \mathcal{D}q_h \mathcal{D}\bar{q} \mathcal{D}q \mathcal{D}\Xi \mathcal{D}U \delta(U^\dagger U-1)\delta(\det U-1)\exp\Big\{ i\Gamma_I[\frac{q\bar{q}}{N_c}]+i\int d^4x\Big[\mathcal{L}_{QCD}+\bar{q}Jq+\mathrm{tr}_f\Big(\Xi(x)\{e^{-i\frac{\vartheta(x)}{N_f}}\Omega^\dagger(x)\mathrm{tr}_l[P_R q(x)\bar{q}(x)]\Omega^\dagger(x) - e^{i\frac{\vartheta(x)}{N_f}}\Omega^\dagger(x)\mathrm{tr}_l[P_L q(x)\bar{q}(x)]\Omega^\dagger(x)\}\Big)\Big]\Big\}$$

$$q_\Omega(x) = [\Omega^\dagger(x)P_L+\Omega(x)P_R]q(x) \qquad \bar{q}_\Omega(x) = \bar{q}[\Omega^\dagger(x)P_L+\Omega(x)P_R]$$

$$\int \mathcal{D}\bar{q}\mathcal{D}q = \int \mathcal{D}\bar{q}_\Omega \mathcal{D}q_\Omega \; e^{-S_{WZ}[\Omega^2,J]}$$

$$e^{iZ[J,\bar\theta]} = \int \mathcal{D}A^i_\mu \mathcal{D}\bar{q}_h \mathcal{D}q_h \mathcal{D}\bar{q}_\Omega \mathcal{D}q_\Omega \mathcal{D}\Xi \int \mathcal{D}U \delta(U^\dagger U-1)\delta(\det U-1)\exp\Big\{ i\Gamma_I[\frac{q_\Omega\bar{q}_\Omega}{N_c}] - S_{WZ}[\Omega^2,J] + i\int d^4x\Big[\mathcal{L}_{QCD,\Omega}+\bar{q}_\Omega J_\Omega q_\Omega + \mathrm{tr}_f\Big(\Xi(x)\{e^{-i\frac{\vartheta_\Omega(x)}{N_f}}\mathrm{tr}_l[P_R q_\Omega(x)\bar{q}_\Omega(x)] - e^{i\frac{\vartheta_\Omega(x)}{N_f}}\mathrm{tr}_l[P_L q_\Omega(x)\bar{q}_\Omega(x)]\}\Big)\Big]\Big\}$$

从QCD定义手征有效拉氏量中的低能常数

$$S_{\rm eff}[U,J,\bar\theta]=\int d^4x\,{\rm tr}_f[F^{ab}(x)s^{ab}_\Omega(x)+F'^{ab}(x)p^{ab}_\Omega(x)]\qquad v^\mu_\Omega\tilde h(v^\mu_\Omega+i\partial^\mu)\tilde h^\dagger$$

$$+\int d^4x d^4y G^{abcd}_{\mu\nu}(x,z)a^{\mu,ab}_\Omega(x)a^{\nu,cd}_\Omega(z)\qquad V^{\mu\nu}_\Omega=\partial^\mu v^\nu_\Omega-\partial^\nu v^\mu_\Omega-i[v^\mu_\Omega,v^\nu_\Omega]$$

$$F^{ab}(x)=-\langle\bar q^a_\Omega(x)q^b_\Omega(x)\rangle=F_0^2B_0\delta^{ab}\qquad F_0^2B_0=-\frac{1}{N_f}\langle\bar q q\rangle$$

$$F'^{ab}(x)=-\langle\bar q^a_\Omega(x)q^b_\Omega(x)\tan\frac{\vartheta_\Omega(x)}{N_f}\rangle=0$$

$$G^{abcd}_{\mu\nu}(x,z)=\frac{i}{2}[\langle\bar q^a_\Omega(x)\gamma_\mu\gamma_5q^b_\Omega(x)\bar q^c_\Omega(z)\gamma_\nu\gamma_5q^d_\Omega(x)\rangle$$

$$-\langle\bar q^a_\Omega(x)\gamma_\mu\gamma_5q^b_\Omega(x)\rangle\langle\bar q^c_\Omega(z)\gamma_\nu\gamma_5q^d_\Omega(x)\rangle]=\delta(x-z)g_{\mu\nu}\delta^{ad}\delta^{bc}F_0^2+\text{含微商的}\delta(x-z)$$

$$\langle O\rangle\equiv\frac{\int d\mu\ O}{\int d\mu}\qquad F_0^2=\frac{1}{4(N_f^2-1)}\int d^4x[G^{\mu,abba}_\mu(0,x)-\frac{1}{N_f}G^{\mu,aabb}_\mu(0,x)]$$

$$d\mu\equiv\mathcal{D}A^i_\mu\mathcal{D}\bar q_h\mathcal{D}q_h\mathcal{D}\bar q_\Omega\mathcal{D}q_\Omega\mathcal{D}\Xi\exp\{i\Gamma_I[\frac{q_\Omega\bar q_\Omega}{N_c}]+i\int d^4x[\mathcal{L}_{QCD,\Omega}+\bar q_\Omega J_\Omega q_\Omega$$

$$+{\rm tr}_f(\Xi(x)\{e^{-i\frac{\vartheta_\Omega(x)}{N_f}}{\rm tr}_l[P_Rq_\Omega(x)\bar q_\Omega(x)]-e^{i\frac{\vartheta_\Omega(x)}{N_f}}{\rm tr}_l[P_Lq_\Omega(x)\bar q_\Omega(x)]\})]\}$$

从QCD定义手征有效拉氏量中的低能常数

$$\overline{\Phi^{(b\xi)(a\xi)}_{\Omega c}}(x,x) = -\frac{1}{N_c}\int d^4y\,\frac{\partial S_{eff}}{\partial J^{(d\xi)(c\xi)}(y)}\frac{\partial J^{(d\xi)(c\xi)}(y)}{\partial s^{ba}_{\Omega}(x)} = -\frac{1}{N_c}\frac{\partial S_{eff}}{\partial s^{ba}_{\Omega}(x)}$$

$$= \frac{1}{N_c}\Big[-F_0^2B_0 - 2K_7 s_\Omega(x) - 2K_8\mathrm{tr}_f[s_\Omega(x)] - K_{11}a^2_\Omega(x) - K_{12}\mathrm{tr}_f[a^2_\Omega(x)]\Big]^{ab}$$

$$(\gamma_5)_{\zeta\xi}\overline{\Phi^{(b\zeta)(a\xi)}_{\Omega c}}(x,x) = -\frac{i}{N_c}\int d^4y\,\frac{\partial S_{eff}}{\partial J^{(d\zeta)(c\xi)}(y)}\frac{\partial J^{(d\zeta)(c\xi)}(y)}{\partial p^{ba}_{\Omega}(x)} = -\frac{i}{N_c}\frac{\partial S_{eff}}{\partial p^{ba}_{\Omega}(x)}$$

$$= -\frac{i}{N_c}\Big[2K_9 p_\Omega(x) + 2K_{10}\mathrm{tr}_f[p_\Omega(x)] - K_{15}d_\mu a^\mu_\Omega(x)\Big]^{ab}$$

$$(\gamma^\mu)_{\zeta\xi}\overline{\Phi^{(b\zeta)(a\xi)}_{\Omega c}}(x,x) = \frac{1}{N_c}\int d^4y\,\frac{\partial S_{eff}}{\partial J^{(d\zeta)(c\xi)}(y)}\frac{\partial J^{(d\zeta)(c\xi)}(y)}{\partial v^{ba}_{\Omega,\mu}(x)} = \frac{1}{N_c}\frac{\partial S_{eff}}{\partial v^{ba}_{\Omega,\mu}(x)}$$

$$= \frac{1}{N_c}\Big[-2iK_1\{[d_\nu a^\nu_\Omega(x)]a^\mu_\Omega(x) a^\mu_\Omega(x)[d_\nu a^\nu_\Omega(x)]\} + iK_{14}d_\nu[a^\mu_\Omega(x), a^\nu_\Omega(x)]$$

$$+4iK_2[a_{\Omega,\nu}(x), [d^\mu a^\nu_\Omega(x) - d^\nu a^\mu_\Omega(x)]] + 4K_{13}d_\nu V^{\mu\nu}_\Omega(x) - iK_{15}[a^\mu_\Omega(x), p_\Omega(x)]\Big]^{ab}$$

从QCD定义手征有效拉氏量中的低能常数

$$(\gamma^\mu\gamma_5)_{\zeta\xi}\overline{\Phi^{(b\zeta)(a\xi)}_{\Omega c}(x,x)} = \frac{1}{N_c}\int d^4y \frac{\partial S_{eff}}{\partial J^{(d\zeta)(c\xi)}(y)} \frac{\partial J^{(d\zeta)(c\xi)}(y)}{\partial a^{ba}_{\Omega,\mu}(x)} = \frac{1}{N_c}\frac{\partial S_{eff}}{\partial a^{ba}_{\Omega,\mu}(x)}$$

$$= \frac{1}{N_c}\Big[2F_0^2 a^\mu_\Omega(x) + 2K_1 d^\mu d^\nu a_{\Omega,\nu}(x) + 4K_2 d_\nu\{d_\nu[d^\nu a^\mu_\Omega(x) - d^\mu a^\nu_\Omega(x)]\}$$

$$+2K_3\{a^\mu_\Omega(x), a^2_\Omega(x)\} + 4K_4 a^\nu_\Omega(x) a^\mu_\Omega(x) a_{\Omega,\nu}(x) + 4K_5 a^\mu_\Omega(x)\mathrm{tr}_f[a^2_\Omega(x)]$$

$$+4K_6 a_{\Omega,\nu}(x)\mathrm{tr}_f[a^\mu_\Omega(x) a^\nu_\Omega(x)] + 4K_{11}\{s_\Omega(x), a^\mu_\Omega(x)\}$$

$$+2K_{12} a^\mu_\Omega(x)\mathrm{tr}_f[s_\Omega(x)] + iK_{14}\{a_{\Omega,\nu}(x), V^{\mu\nu}_\Omega(x)\} - K_{15} d^\mu p_\Omega(x)\Big]^{ab}$$

$$F_0^2 B_0 = -\frac{N_c}{N_f}\overline{\Phi^{(a\xi)(a\xi)}_{\Omega c}(x,x)}\Big|_{J_\Omega=0} = iN_c \mathrm{tr}_{lf}[(i\not{\partial} - \Pi_{\Omega c})^{-1}](x,x)$$

Back
Close

从QCD定义手征有效拉氏量中的低能常数

$$F_0^2[a_\Omega^\mu(x)]^{ab} = \frac{N_c}{2}(\gamma^\mu\gamma_5)_{\zeta\xi}\overline{\Phi^{(b\zeta)(a\xi)}(x,x)}\Big|_{a_\Omega^\mu \text{线性项}}$$

$$= \frac{N_c}{2}\int d^4y\,\mathrm{tr}_l[\gamma^\mu\gamma_5(i\partial\!\!\!/ - \Pi_{\Omega c})^{-1}(x,y)a\!\!\!/(y)\gamma_5(i\partial\!\!\!/ - \Pi_{\Omega c})^{-1}(y,x)]^{ab}$$

$$F_0^2 g^{\mu\nu}\delta(x-y) = \frac{N_c}{2}\int d^4y\,\mathrm{tr}_l[\gamma^\mu\gamma_5(i\partial\!\!\!/ - \Pi_{\Omega c})^{-1}(x,y)\gamma^\nu\gamma_5(i\partial\!\!\!/ - \Pi_{\Omega c})^{-1}(y,x)]$$

$$\Pi_{\Omega c}(x,y) = \int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}\Sigma(p^2)$$

$$F_0^2 = \frac{N_c}{8}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}_f[\gamma^\mu\gamma_5\frac{1}{p\!\!\!/ - \Sigma(p^2)}\gamma_\mu\gamma_5\frac{1}{p\!\!\!/ - \Sigma(p^2)}]$$

$$= -4N_c\int\frac{d^4p}{(2\pi)^4}[\frac{1}{8}\frac{\partial}{\partial p^\mu}(\frac{p^\mu}{p^2 - \Sigma^2(p^2)}) + \underbrace{\frac{[\Sigma(p^2) - \frac{1}{2}p^2\Sigma'(p^2)]\Sigma(p^2)}{[p^2 - \Sigma^2(p^2)]^2}}_{\text{Pagel-Stokar公式}}]$$

其它低能强子的手征有效拉氏量

Back

Close

1GeV以下的强子谱

赝标介子族($J^P = 0^-$)：

$\pi^\pm(140)$ $\pi^0(135)$ $K^\pm(494)$ $K^0, \bar{K}^0(498)$ $\eta(547)$ $\eta'(958)$

矢量介子家族($J^P = 1^-$)：

$\rho^\pm, \rho^0(769)$ $K^{*\pm}(892)$ $K^{*0}, \bar{K}^{*0}(896)$ $\omega(782)$

标量介子家族($J^P = 0^+$)： f_0或$\sigma(500)$ K_0^*或$\kappa(800)$ $f_0(980)$ $a_0(980)$

重子家族($J^P = (\frac{1}{2})^+$)： $p(938)$ $n(940)$

赝标和矢量介子形成$SU(3)_V$的八重态（伴随）表示。

问题与难点：

- 赝标介子是近似的Goldstone粒子，手征极限下质量为零
- 矢量粒子看似与手征对称性无关，即使在手征极限，质量也不为零
- 它的质量需要通过复杂的动力学方程BS方程，能否导出它？来决定
- 能否构造一个在最低阶就有质量的手征有效理论？对其它介子也有意义！
- 如果矢量场是规范场，规范对称性自发破缺可以给矢量场以质量！
- 从前面讨论中引出的局域隐藏对称性 $\tilde{\mathrm{h}}$ 出发！

矢量介子：隐藏对称性理论

$$U(x)=\xi_L^\dagger(x)\xi_R(x)$$

$$\underbrace{\xi_R(x)=e^{i\frac{\tilde\phi(x)}{2F_0}}e^{i\frac{\phi(x)}{2F_0}}\qquad \xi_L(x)=e^{i\frac{\tilde\phi(x)}{2F_0}}e^{-i\frac{\phi(x)}{2F_0}}}_{\tilde\phi:\ \text{新的Goldestone}}$$

手征变换：$\xi_R(x)\to\bar h(x)\xi_R(x)V_R^\dagger(x)\qquad \xi_L(x)\to\bar h(x)\xi_L(x)V_L^\dagger(x)$

$$\xi_R(x)\overset{\tilde\phi(x)=0}{=\!=\!=\!=\!=\!=}\Omega(x)\overset{\tilde\phi(x)=0}{=\!=\!=\!=\!=\!=}\xi_L^\dagger(x)\qquad \bar x\overset{\tilde\phi(x)=0}{=\!=\!=\!=\!=\!=}\tilde h(x)$$

引进描述矢量介子的矢量场：$V_\mu(x)\to\bar h(x)V_\mu(x)\bar h^\dagger(x)+i\bar h\partial_\mu\bar h(x)$

$$D^\mu\xi_R\equiv\partial^\mu\xi_R+i\xi_R(v^\mu+a^\mu)-iV^\mu\xi_R\qquad D^\mu\xi_L\equiv\partial^\mu\xi_L+i\xi_L(v^\mu-a^\mu)-iV^\mu\xi_L$$

$$(D^\mu\xi_R)\xi_R^\dagger\mp(D^\mu\xi_L)\xi_L^\dagger\to\bar h[(D^\mu\xi_R)\xi_R^\dagger\mp(D^\mu\xi_L)\xi_L^\dagger]\bar h^\dagger$$

$$\mathcal{L}_2=F_0^2\mathrm{tr_f}\Big[-\frac{1}{4}[(D^\mu\xi_R)\xi_R^\dagger-(D^\mu\xi_L)\xi_L^\dagger]^2+\frac{1}{2}B_0[\xi_L^\dagger\xi_R(s-ip)+\xi_R^\dagger\xi_L(s+ip)]\Big]$$
$$-a\frac{F_0^2}{4}\mathrm{tr_f}[(D^\mu\xi_R)\xi_R^\dagger+(D^\mu\xi_L)\xi_L^\dagger]^2-a'\frac{F_0^2}{4}\Big[\mathrm{tr_f}[(D^\mu\xi_R)\xi_R^\dagger+(D^\mu\xi_L)\xi_L^\dagger]\Big]^2$$

代入矢量介子场的运动方程解，$\mathcal{L}_2$就自然地回到纯赝标介子的手征有效拉氏量!

加入矢量介子动能项：$-\dfrac{1}{4g_\rho^2}V_{\mu\nu}^aV^{a,\mu\nu}$ 数幂？ $V_{\mu\nu}^a\equiv\partial_\mu V_\nu^a-\partial_\nu V_\mu^a+f^{abc}V_\mu^bV_\nu^v$

Back

Close

矢量介子：反对称张量场理论

$$\mathcal{L}_2' = \frac{F_0^2}{4}\mathrm{tr_f}[\nabla_\mu U(\nabla^\mu U)^\dagger] + \frac{F_0^2}{4}\mathrm{tr_f}(\chi U^\dagger + U\chi^\dagger) - \frac{1}{2}\mathrm{tr_f}[\nabla^\lambda \tilde{V}_{\lambda\mu}\nabla_\nu \tilde{V}^{\nu\mu}]$$
$$+\frac{1}{4}M_V^2\mathrm{tr_f}[\tilde{V}_{\mu\nu}\tilde{V}^{\mu\nu}] + \frac{F_V}{\sqrt{2}}\mathrm{tr_f}[\tilde{V}^{\mu\nu}\hat{V}_{\mu\nu}] + \sqrt{2}G_V i\,\mathrm{tr_f}(\tilde{V}_{\mu\nu}[a_\Omega^\mu, a_\Omega^\nu])$$

$$\nabla^\lambda \tilde{V}_{\lambda\mu} \equiv \partial^\lambda \tilde{V}_{\lambda\mu} - i[v_\Omega^\lambda, \tilde{V}_{\lambda\mu}] \qquad \hat{V}_{\mu\nu} \equiv \frac{1}{2}[\Omega^\dagger f_{\mu\nu}^R\Omega + \Omega f_{\mu\nu}^L\Omega^\dagger]$$

手征变换：$\tilde{V}_{\mu\nu}(x) \to \tilde{h}(x)\tilde{V}_{\mu\nu}(x)\tilde{h}^\dagger(x) \qquad \hat{V}_{\mu\nu}(x) \to \tilde{h}(x)\hat{V}_{\mu\nu}(x)\tilde{h}^\dagger(x)$

$$v_{\Omega\mu} \overset{\text{推广}}{=\!=\!=\!=} \frac{1}{2}[\xi_R(v_\mu + a_\mu + i\partial_\mu)\xi_R^\dagger + \xi_L(v_\mu - a_\mu + i\partial_\mu)\xi_L^\dagger]$$

$$\hat{V}_{\mu\nu} \overset{\text{推广}}{=\!=\!=\!=} \frac{1}{2}[\xi_R f_{\mu\nu}^R\xi_R^\dagger + \xi_L f_{\mu\nu}^L\xi_L^\dagger] \qquad a_\Omega^\mu \overset{\text{推广}}{=\!=\!=\!=} \frac{i}{2}\xi_L[\nabla^\mu U]\xi_R^\dagger$$

$$\mathcal{L}_2'' \equiv \mathcal{L}_2' + \frac{1}{2}\kappa^2\mathrm{tr_f}[(V_\mu - v_{\Omega\mu} - \frac{1}{\kappa}\nabla^\nu\tilde{V}_{\nu\mu})^2]$$

$$= \frac{F_0^2}{4}\mathrm{tr_f}[\nabla_\mu U(\nabla^\mu U)^\dagger] + \frac{F_0^2}{4}\mathrm{tr_f}(\chi U^\dagger + U\chi^\dagger) - \frac{1}{8}\kappa^2\mathrm{tr_f}[(D_\mu\xi_R)\xi_R^\dagger + (D_\mu\xi_L)\xi_L^\dagger]^2$$

$$\frac{1}{2}M_V^2\tilde{V}^{\mu\nu} = -\frac{F_V}{\sqrt{2}}\hat{V}^{\mu\nu} - \sqrt{2}G_V i[a_\Omega^\mu, a_\Omega^\nu] - \kappa(V^{\mu\nu} - [\partial^\mu - iV^\mu, i\partial^\nu + v_\Omega^\nu] + i[v_\Omega^\mu - V^\mu, v_\Omega^\nu - V^\nu])$$

Back
Close

- 最困难和复杂的部分：多夸克态、混杂态、胶球易混合进来
- 经常宽度很大，传统的共振态诠释，理论五花八门PDG单独评述！
- 各种理论至今仍无得到业界的共识
- σ可以被看做标度对称性破坏的赝Goldstone？

$$\text{标度对称性流的散度} = \underbrace{\frac{\pi\beta(g)}{g^2}\mathrm{tr}(\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu})}_{\text{红外固定点}} + [\mathbf{1} + \gamma_{\mathbf{m}}(\mathbf{g})]\bar{\mathbf{q}}\mathbf{M}\mathbf{q}$$

- 线性 σ 模型–标量介子九重态：自作用势的选取？
- 幺正化的手征有效理论–逆散射振幅方法

- $\mathbf{U_L(3) \times U_R(3) = SU_L(3) \times SU_R(3) \times U_V(1) \times U_A(1)}$。
- η' 是对应$\mathbf{U_A(1)}$的Goldestone！但$\mathbf{U_A(1)}$有反常！
- 在大Nc极限下,$\mathbf{U_A(1)}$反常不再出现，η' 是赝Goldstone
- 引入手征不变的描写η'的场：$\mathbf{X(x) \equiv i\bar{\theta} - \mathrm{ln}\det_f U(x)}$

$$\mathcal{L}_0 = \mathbf{W_0[X]} \qquad\qquad \nabla_\mu \mathrm{ln}\det{}_f \mathbf{U} \equiv \partial_\mu \mathbf{U} - \mathbf{2i}\mathrm{tr}_f \mathbf{a}_\mu$$

$$\mathcal{L}_2''' = \mathbf{W_1[X]}\mathrm{tr}_f[\nabla_\mu \mathbf{U}(\nabla^\mu \mathbf{U})^\dagger] + \mathbf{W_2[X]}\mathrm{tr}_f(\chi \mathbf{U}^\dagger + \mathbf{U}\chi^\dagger)$$

$$+\mathbf{iW_3[X]}\mathrm{tr}_f(\chi \mathbf{U}^\dagger - \mathbf{U}\chi^\dagger) + \underbrace{\mathbf{W_4[X]}[\nabla_\mu \mathrm{ln}\det{}_f \mathbf{U}][\nabla^\mu \mathrm{ln}\det \mathbf{U}]}_{\text{利用}U\text{场定义的任意性去掉}}$$

$$+\mathbf{W_5[X]}[\nabla_\mu \mathrm{ln}\det{}_f \mathbf{U}]\nabla^\mu \bar{\theta} + \mathbf{W_6[X]}\nabla_\mu \bar{\theta} \nabla^\mu \bar{\theta}$$

- 除最轻的赝标介子八重态外，考虑$\mathrm{SU_V(3)}$八重态R和单态$\mathrm{R_1}$介子：
  矢量1^-–$\tilde{V}_{\mu\nu}$； 轴矢1^+–$\tilde{A}_{\mu\nu}$； 标量0^+–$\tilde{S}$； 赝标0^-–$\tilde{P}$
- 手征变换：$R(x) \to \tilde{h}(x)R(x)\tilde{h}^\dagger(x) \qquad R_1(x) \to R_1(x)$

$$\mathcal{L}_{\rm kin} = \begin{cases} -\frac{1}{2}\mathrm{tr_f}[\nabla^\lambda R_{\lambda\mu}\nabla_\nu R^{\nu\mu} - \frac{1}{2}M_R^2 R_{\mu\nu}R^{\mu\nu}] \\ -\frac{1}{2}\partial^\lambda R_{1,\lambda\mu}R_1^{\nu\mu} + \frac{1}{4}M_{R_1}^2 R_{1,\mu\nu}R_1^{\mu\nu} & R = \tilde{V}, \tilde{A} \\ \frac{1}{2}\mathrm{tr_f}[\nabla^\mu R\nabla_\mu R - M_R^2 R^2] + \frac{1}{2}[\partial^\mu R_1\partial_\mu R_1 - M_{R_1}^2 R_1^2] & R = \tilde{S}, \tilde{P} \end{cases}$$

$$\mathcal{L}_{\rm int} = \begin{cases} \frac{F_V}{2\sqrt{2}}\mathrm{tr_f}[\tilde{V}^{\mu\nu}\hat{V}_{\mu\nu}] + \frac{iG_V}{\sqrt{2}}\mathrm{tr_f}[\tilde{V}_{\mu\nu}a_\Omega^\mu a_\Omega^\nu] & 1^- \\ \frac{F_A}{2\sqrt{2}}\mathrm{tr_f}[\tilde{A}^{\mu\nu}\hat{A}_{\mu\nu}] & 1^+ \\ c_d\mathrm{tr_f}[\tilde{S}a_{\Omega\mu}a_\Omega^\mu] + c_m\mathrm{tr_f}[\tilde{S}(\Omega^\dagger\chi\Omega^\dagger + \Omega\chi\Omega)] \\ \tilde{c}_d\tilde{S}_1\mathrm{tr_f}[a_{\Omega\mu}a_\Omega^\mu] + \tilde{c}_m\tilde{S}_1\mathrm{tr_f}(\Omega^\dagger\chi\Omega^\dagger + \Omega\chi\Omega) & 0^+ \\ id_m\mathrm{tr_f}[\tilde{P}(\Omega^\dagger\chi\Omega^\dagger - \Omega\chi\Omega) + i\tilde{d}_m\tilde{P}_1\mathrm{tr_f}(\Omega^\dagger\chi\Omega^\dagger - \Omega\chi\Omega) & 0^- \end{cases}$$

$\nabla^\lambda R \equiv \partial^\lambda R - i[v_\Omega^\lambda, R]$ 数幂？ $\hat{A}_{\mu\nu} \equiv \frac{1}{2}[\Omega^\dagger f_{\mu\nu}^R\Omega - \Omega f_{\mu\nu}^L\Omega^\dagger]$

重子：$J^P = (\frac{1}{2})^+$八重态

$$\Theta^-(-\frac{1}{2}, -2, 1322) \qquad (I_3, S, \text{mass MeV}) \qquad \Theta^0(\frac{1}{2}, -2, 1315)$$

$$\Sigma^-(-1,-1,1197)\ \Sigma^0(0,-1,1193)\ \Lambda(0,-1,1116)\ \Sigma^+(1,-1,1189)$$

$$n(-\frac{1}{2}, 0, 940) \qquad p(\frac{1}{2}, 0, 938)$$

$$B = \sum_{i=1}^{8} \frac{B_i \lambda^i}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Theta^- & \Theta^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

$$B(x) \to \tilde{h}(x) B(x) \tilde{h}^\dagger(x) \qquad D^\mu B \equiv \partial^\mu B - i[v^\mu_\Omega, B]$$

$$\mathcal{L}_B = \mathrm{Tr}[\bar{B}(i\not{D} - M_0)B] - \frac{D}{2}\mathrm{Tr}(\bar{B}\gamma_\mu\gamma_5\{a^\mu_\Omega, B\}) - \frac{F}{2}\mathrm{Tr}(\bar{B}\gamma_\mu\gamma_5[a^\mu_\Omega, B])$$

Back
Close

- 用介子场U的拓扑非平凡解来描写重子
- U给出$\mathbf{R}^3$到SU(3)流型的映射
- 要求$\mathbf{U}(\mathbf{r}\to\infty)=\mathbf{1}$，映射可被分类组成同伦群
- $\mathbf{\Pi_3(SU(3))=\Pi_3(SU(2))=Z}$ 确定映射可用整数定义为重子数分类！
- 对应的重子流：$\mathbf{B}^\mu \equiv \frac{\mathbf{i}}{\mathbf{24\pi^2}}\epsilon^{\mu\nu\mu'\nu'}\mathrm{Tr}_\mathbf{f}[\mathbf{L}_\nu\mathbf{L}_{\mu'}\mathbf{L}_{\nu'}]$
- p^2阶拉氏量给出拓扑孤子是不稳定，需要p^4项介入才能稳定。
- 引进重子场使介子相互作用系数改变，拓扑孤子也应该不稳定。

谢 谢！