Lectures on Standard Model Effective Field Theory

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Outline

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Difference between dim-6 and dim-5/7 operators

Caution at the very start:
Effective interactions of dim-5 and dim-7 may arise from UV physics different from those of dim-6, because of different symmetry:

- dim-6 operators include B- and L-conserving ones plus B- and L-violating but $B-L$ conserving ones;
- dim-5 operator violates L but not B (without involving quarks);
- all dim-7 operators violate L; some conserve B ($L=\pm 2$, $B=0$), and others violate both B and L ($L=-B=\pm 1$).

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Dim-5 operator

- Without counting flavors there is a unique dim-5 operator:

$$\mathscr{L}_5 = \frac{1}{4}\kappa_{gf}\overline{(L^g_m)^C}\epsilon^{mn}H_n L^f_j \epsilon^{ji} H_i + \text{h.c.} \quad \begin{array}{l} i,j,m,n:\ SU(2)_L \text{ indices} \\ f,g:\ \text{flavor indices}\end{array} \tag{1}$$

$$\psi^C = C\bar{\psi}^T \leftrightarrow \overline{\psi^C} = \psi^T C \text{ charge conjugation,}$$

$$C^T = C^\dagger = -C,\ C^2 = -1;\ \kappa:\ \text{complex and symmetric}$$

- In unitary gauge, $H^T \to (0, v+h)/\sqrt{2}$,

$$\mathscr{L}_5 = \frac{1}{4}\frac{1}{2}(v+h)^2\kappa_{gf}\overline{(\nu^g)^C}\nu^f + \text{h.c.} \Rightarrow m_\nu = -\frac{1}{4}v^2\kappa \quad \begin{array}{l}\text{complex}\\ \text{symmetric}\end{array} \tag{2}$$

  Difficult to detect $h\nu$ interactions. Not much to do with $\mathscr{L}_5$ beyond m_ν.
- E. Ma (1998): Three ways to realize $\mathscr{L}_5$ at tree level, corresponding exactly to three standard seesaw mechanisms.
- $\mathscr{L}_5$ is induced at a high seesaw scale M, where some heavy particles are integrated out. RG running should be included for low energy phys.

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RG running of $\mathscr{L}_5$

Literature:

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RG running of $\mathscr{L}_5$: Sketch of calculation

κ: symmetric matrix, not necessarily multiplicatively renormalizable. We proceed a bit differently from what we did in Lecture 3.

- Consider $\mathscr{L}_5$ as given in bare quantities κ_{bare}, $L_{\text{bare}} = Z_L^{1/2} L$, $H_{\text{bare}} = Z_H^{1/2} H$:

$$\mathscr{L}_5 = \frac{1}{4}\varepsilon^{mn}\varepsilon^{ji} Z_H \overline{(L_m)^C} (Z_L^{1/2})^T \kappa_{\text{bare}} Z_L^{1/2} L_j H_n H_i + \text{h.c.} \quad (3)$$

$$= \frac{1}{4}\mu^{2\varepsilon}\varepsilon^{mn}\varepsilon^{ji}\overline{(L_m)^C}\,(\kappa + \delta\kappa) L_j H_n H_i + \text{h.c.}, \quad (4)$$

where $Z_L^{1/2}$ is a matrix in flavor space and $\delta\kappa$ is the c.t. defined by

$$Z_H (Z_L^{1/2})^T \kappa_{\text{bare}} Z_L^{1/2} = (\kappa + \delta\kappa)\mu^{2\varepsilon} \quad (5)$$

- Compute one-loop diagrams contributing to $LLHH$ with one insertion of $\mathscr{L}_5$ and any number of SM couplings.

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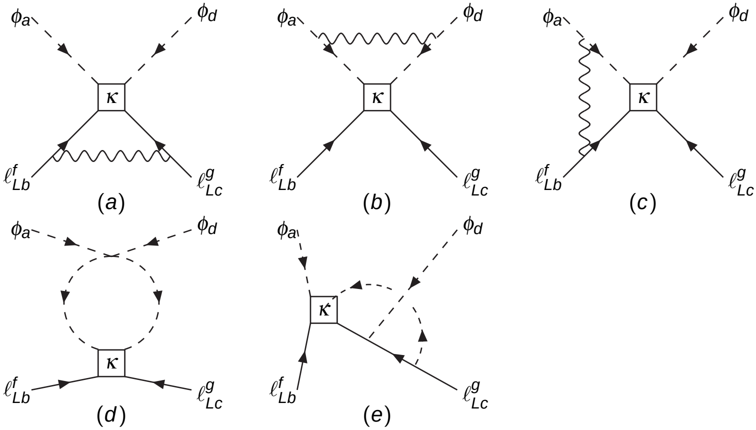

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- Determine c.t. by requiring it to cancel UV divergences in the above diagrams. z_H, z_L are known in SM.
- Denote for simplicity $z_L^{1/2} = A$.

$$\frac{d\kappa_{\rm bare}}{d\mu} = 0 \to \mu\frac{d}{d\mu}\text{eq.(5)}:$$

$$\mu\frac{dZ_H}{d\mu}A^T\kappa_{\rm bare}A + Z_H\mu\frac{dA^T}{d\mu}\kappa_{\rm bare}A + Z_HA^T\kappa_{\rm bare}\mu\frac{dA}{d\mu}$$
$$= \left[\mu\frac{d\kappa}{d\mu} + \mu\frac{d\delta\kappa}{d\mu} + 2\varepsilon(\kappa+\delta\kappa)\right]\mu^{2\varepsilon} \quad (6)$$

$$\Rightarrow \quad \mu\frac{d\ln Z_H}{d\mu}(\kappa+\delta\kappa) + \mu\frac{dA^T}{d\mu}(A^{-1})^T(\kappa+\delta\kappa)A + (\kappa+\delta\kappa)A^{-1}\mu\frac{dA}{d\mu}$$
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RG running of $\mathscr{L}_5$: Sketch of calculation

- Denote renormalization of SM couplings collectively by

$$g_0 = Z_g g \mu^{n_g \varepsilon},\ n_g = \begin{cases} 2 \text{ for } g = \lambda \\ 1 \text{ for gauge/Yukawa couplings} \end{cases} \quad (8)$$

$$\mu \frac{dg_0}{d\mu} = 0 \Rightarrow \beta_g = -n_g \varepsilon g - \mu \frac{d \ln Z_g}{d\mu} g, \quad \begin{array}{l} \text{highest power } \varepsilon^1, \text{ lowest order in } g \\ \text{highest power } \varepsilon^0, \text{ higher order in } g \end{array} \quad (9)$$

- Consider eq.(7). The highest power in ε on lhs is 0, thus

$$\beta_\kappa = \mu \frac{d\kappa}{d\mu} = -2\varepsilon\kappa + O(\varepsilon^0) \quad (10)$$

This is as expected. It also has the following implications.

- Denote $\delta\kappa = \kappa_{pr}(cg)_{pr}$ with $g = \lambda, (g_{1,2,3}, Y_f)^2$.

$$\mu \frac{d\delta\kappa}{d\mu} = \mu \frac{d\kappa_{pr}}{d\mu}(cg)_{pr} + \kappa_{pr}\mu \frac{d(cg)_{pr}}{d\mu} = \left[-2\varepsilon\kappa_{pr} + O(\varepsilon^0)\right](cg)_{pr} + \kappa_{pr}\mu \frac{d(cg)_{pr}}{d\mu} \quad (11)$$

$$\Rightarrow \left[\mu \frac{d\delta\kappa}{d\mu} + 2\varepsilon\delta\kappa\right]_{\varepsilon^0} = \sum_g \left[\mu \frac{dg}{d\mu}\frac{d\delta\kappa}{dg}\right]_{\varepsilon^0} = \sum_g \left[-n_g \varepsilon g \frac{d\delta\kappa}{dg}\right]_{\varepsilon^0} \quad (12)$$

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- β functions have a well-defined limit as $\varepsilon \to 0$.
  I'll not show cancellation of the pole terms in ε.
- Writing $A \approx 1 + \frac{1}{2}\delta Z_L$ and $Z_H = 1 + \delta Z_H$, the result at one loop is

$$\left[\mu \frac{d \ln Z_H}{d\mu}\kappa + \mu \frac{dA^T}{d\mu}\kappa + \kappa\mu\frac{dA}{d\mu}\right]_{\varepsilon^0} = \lim_{\varepsilon\to 0}\beta_\kappa + \sum_g \left[-n_g \varepsilon g \frac{d\delta\kappa}{dg}\right]_{\varepsilon^0}$$

$$\Rightarrow \quad \lim_{\varepsilon\to 0}\beta_\kappa = \sum_g \left\{(-n_g\varepsilon g)\left[\frac{d\delta Z_H}{dg}\kappa - \frac{d\delta\kappa}{dg} + \frac{1}{2}\left(\frac{d\delta Z_L^T}{dg}\kappa + \kappa\frac{d\delta Z_L}{dg}\right)\right]\right\}_{\varepsilon^0} \tag{13}$$

- The final result is

$$\begin{aligned}(4\pi)^2\beta_\kappa &= -\frac{3}{2}\left[\kappa(Y_e Y_e^\dagger) + (Y_e Y_e^\dagger)^T\kappa\right] + 2\lambda\kappa - 3g_2^2\kappa \\ &\quad + 2\mathrm{tr}\left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e\right)\kappa \end{aligned} \tag{14}$$

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Basis of dim-7 operators

- Literature:
  L. Lehman, Extending SMEFT with complete set of dim-7 operators, PRD 90 (2014) 125023
  Y. Liao and X.-D. Ma, RGEn of dim-7 baryon- and lepton-number-violating operators, JHEP 11 (2016) 043
- It is important to use a basis of operators that is complete and independent –
  - complete: consistency of perturbation theory, requirement of renormalizability in EFT
  - independent: correct connection with S matrix
  - Both are required by making correct phenomenological conclusion
- Choices of bases are not unique but there are not many either. It depends usually on convenience.

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Basis of dim-7 operators

- It is a hard job to find a correct basis:
  Completeness relatively easier, but redundancy difficult to remove.
  By redundant operators in $\mathscr{L}_{\text{EFT}}$ we mean those that can be removed by any of the following methods:

  algebraic relations: for reps of Lorentz and gauge groups, including Fierz identities

  integration by parts: total derivative yields zero in perturbation theory

  equations of motion: equivalent to field redefinition without changing S matrix

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  J. F. Nieves and P. B. Pal, Generalized Fierz identities, Am. J. Phys. **72**, 1100 (2004) [hep-ph/0306087]. *involving charge-conjugation*

Basis of dim-7 operators

- It is a hard job to find a correct basis:
  Completeness relatively easier, but redundancy difficult to remove.
  By redundant operators in $\mathscr{L}_{\text{EFT}}$ we mean those that can be removed by any of the following methods:

  algebraic relations: for reps of Lorentz and gauge groups, including Fierz identities

  integration by parts: total derivative yields zero in perturbation theory

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Algebraic relations

- Relations for fundamental reps of gauge group:

$$SU(2) \qquad (T^I)_{jk}(T^I)_{mn} = \frac{1}{2}\delta_{jn}\delta_{mk} - \frac{1}{4}\delta_{jk}\delta_{mn}\,, \; T^I = \frac{1}{2}\sigma^I \text{ (Pauli)} \tag{15}$$

$$\text{Schouten identities} \begin{cases} \varepsilon_{ij}\varepsilon_{mn} = \varepsilon_{im}\varepsilon_{jn} - \varepsilon_{jm}\varepsilon_{in}\,, \\ \varepsilon_{ij}\delta_{mn} = \varepsilon_{im}\delta_{jn} - \varepsilon_{jm}\delta_{in}\,, \end{cases} \tag{16}$$

$$SU(3) \qquad (T^A)_{ab}(T^A)_{cd} = \frac{1}{2}\delta_{ad}\delta_{cb} - \frac{1}{6}\delta_{ab}\delta_{cd}\,, \; T^A = \frac{1}{2}\lambda^A \text{ (Gell-Mann)} \tag{17}$$

- Identities for fermion fields
  Notation for charge conjugation of chiral fields:

$$\Psi^C_{L,R} \equiv (\Psi_{L,R})^C \Rightarrow \overline{\Psi^C_{L,R}} = (\Psi_{L,R})^T C,\; \Psi_{L,R} = (\Psi^C_{L,R})^C \tag{18}$$

  All following identities are equally valid with $L \leftrightarrow R$.
  Identities for fermion field bilinears involving charge conjugation:

$$\overline{\Psi_{1L}}\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_{n-1}}\gamma^{\mu_n}\Psi_{2R} = \pm\overline{\Psi^C_{2R}}\gamma^{\mu_n}\gamma^{\mu_{n-1}}\cdots\gamma^{\mu_2}\gamma^{\mu_1}\Psi^C_{1L} \text{ for } n \begin{cases} \text{even} \\ \text{odd} \end{cases} \tag{19}$$

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Algebraic relations

$$(\overline{\Psi_{1L}}\gamma^\mu\gamma^\nu\Psi_{2R})(\overline{\Psi_{3L}}\gamma_\mu\gamma_\nu\Psi_{4R}) = 8[(\overline{\Psi_{1L}}\Psi_{2R})(\overline{\Psi_{3L}}\Psi_{4R})+(\overline{\Psi_{1L}}\Psi_{4R})(\overline{\Psi_{3L}}\Psi_{2R})], \quad (20)$$

$$(\overline{\Psi_{1L}}\gamma^\mu\gamma^\nu\Psi_{2R})(\overline{\Psi_{3L}}\gamma_\nu\gamma_\mu\Psi_{4R}) = -8(\overline{\Psi_{1L}}\Psi_{4R})(\overline{\Psi_{3L}}\Psi_{2R}) \quad (21)$$

$$(\overline{\Psi_{1L}}\gamma^\mu\gamma^\nu\Psi_{2R})(\overline{\Psi_{3R}}\gamma_\mu\gamma_\nu\Psi_{4L}) = 4(\overline{\Psi_{1L}}\Psi_{2R})(\overline{\Psi_{3R}}\Psi_{4L}) \quad (22)$$

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$$(\overline{\Psi_{1L}}\gamma^\mu\Psi_{2L})(\overline{\Psi_{3L}}\gamma_\mu\Psi_{4L}) = (\overline{\Psi_{1L}}\gamma^\mu\Psi_{4L})(\overline{\Psi_{3L}}\gamma_\mu\Psi_{2L}), \quad (24)$$

$$(\overline{\Psi_{1L}}\gamma^\mu\Psi_{2L})(\overline{\Psi_{3L}}\gamma_\mu\Psi_{4L}) = 2(\overline{\Psi_{1L}}\Psi_{3L}^C)(\overline{\Psi_{4L}^C}\Psi_{2L}), \quad (25)$$

$$(\overline{\Psi_{1L}}\gamma^\mu\Psi_{2L})(\overline{\Psi_{3R}}\gamma_\mu\Psi_{4R}) = -2(\overline{\Psi_{1L}}\Psi_{4R})(\overline{\Psi_{3R}}\Psi_{2L}) \quad (26)$$

$$(\overline{\Psi_{1L}}\gamma^\mu\Psi_{2L})(\overline{\Psi_{3R}^C}\Psi_{4R}) = (\overline{\Psi_{1L}}\Psi_{3R})(\overline{\Psi_{2L}^C}\gamma_\mu\Psi_{4R})+(\overline{\Psi_{1L}}\Psi_{4R})(\overline{\Psi_{2L}^C}\gamma_\mu\Psi_{3R}) \quad (27)$$

$$(\overline{\Psi_{1L}}\Psi_{2R})(\overline{\Psi_{3L}^C}\gamma^\mu\Psi_{4R}) = (\overline{\Psi_{1L}}\gamma_\mu\Psi_{3L})(\overline{\Psi_{4R}^C}\Psi_{2R})-(\overline{\Psi_{1L}}\Psi_{4R})(\overline{\Psi_{3L}^C}\gamma_\mu\Psi_{2R}) \quad (28)$$

$$(\overline{\Psi_{1R}}\gamma^\mu\Psi_{2R})(\overline{\Psi_{3R}^C}\Psi_{4R}) = -(\overline{\Psi_{1R}}\gamma_\mu\Psi_{3R})(\overline{\Psi_{2R}^C}\Psi_{4R})-(\overline{\Psi_{1R}}\gamma_\mu\Psi_{4R})(\overline{\Psi_{2R}^C}\Psi_{3R}) \quad (29)$$

$$(\overline{\Psi_{1L}}\Psi_{2R})(\overline{\Psi_{3R}^C}\Psi_{4R}) = -(\overline{\Psi_{1L}}\Psi_{3R})(\overline{\Psi_{4R}^C}\Psi_{2R})-(\overline{\Psi_{1L}}\Psi_{4R})(\overline{\Psi_{3R}^C}\Psi_{2R}) \quad (30)$$

EoMs

Gauge fields:

$$
\begin{aligned}
D^{\nu}G^{A}_{\mu\nu} &= g_3 \sum_{\Psi=Q,u,d} \bar{\Psi} T^{A} \gamma_{\mu} \Psi, \\
D^{\nu}W^{I}_{\mu\nu} &= g_2 \sum_{\Psi=Q,L} \bar{\Psi} T^{I} \gamma_{\mu} \Psi + i g_2 H^{\dagger} T^{I} \overleftrightarrow{D}_{\mu} H, \\
D^{\nu}B_{\mu\nu} &= g_1 \sum_{\Psi=q,u,d,L,e} \bar{\Psi} Y_{\Psi} \gamma_{\mu} \Psi + i g_1 Y_H H^{\dagger} \overleftrightarrow{D}_{\mu} H. \qquad (31)
\end{aligned}
$$

Matter and Higgs fields:

$$
\begin{aligned}
D^2 H &= \mu^2 H - \lambda (H^{\dagger} H) H - \epsilon^{T} \bar{Q} Y_u u - \bar{d} Y_d^{\dagger} Q - \bar{e} Y_e^{\dagger} L, \\
i \not{D} Q &= Y_u u \tilde{H} + Y_d d H, \\
i \not{D} L &= Y_e e H, \\
i \not{D} u &= Y_u^{\dagger} \tilde{H}^{\dagger} Q, \\
i \not{D} d &= Y_d^{\dagger} H^{\dagger} Q, \\
i \not{D} e &= Y_e^{\dagger} H^{\dagger} L. \qquad (32)
\end{aligned}
$$

Most useful for reducing dim-7 operators are, in explicit indices,

$$
i \gamma^{\mu} D_{\mu} L^{i}_{t} = (Y_e)_{tu} e_u H^{i}, \qquad (33)
$$

Summary of dim-7 operators

	ψ^2H^4		ψ^2H^3D
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^iCL^m)H^jH^n(H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^iC\gamma_\mu e)H^jH^miD^\mu H^n$
	$\psi^2H^2D^2$		ψ^2H^2X
$\mathcal{O}_{LHD1}$	$\epsilon_{ij}\epsilon_{mn}(L^iCD^\mu L^j)H^m(D_\mu H^n)$	$\mathcal{O}_{LHB}$	$\epsilon_{ij}\epsilon_{mn}(L^iCi\sigma_{\mu\nu}L^m)H^jH^nB^{\mu\nu}$
$\mathcal{O}_{LHD2}$	$\epsilon_{im}\epsilon_{jn}(L^iCD^\mu L^j)H^m(D_\mu H^n)$	$\mathcal{O}_{LHW}$	$\epsilon_{ij}(\tau^I\epsilon)_{mn}(L^iCi\sigma_{\mu\nu}L^m)H^jH^nW^{I\mu\nu}$
	ψ^4D		ψ^4H
$\mathcal{O}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iCiD^\mu L^j)$	$\mathcal{O}_{\bar{e}LLLH}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^jCL^m)H^n$
$\mathcal{O}_{\bar{L}QddD}$	$(\bar{L}\gamma_\mu Q)(dCiD^\mu d)$	$\mathcal{O}_{\bar{d}LQLH1}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^jCL^m)H^n$
$\mathcal{O}_{\bar{e}dddD}$	$(\bar{e}\gamma_\mu d)(dCiD^\mu d)$	$\mathcal{O}_{\bar{d}LQLH2}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^jCL^m)H^n$
		$\mathcal{O}_{\bar{d}LueH}$	$\epsilon_{ij}(\bar{d}L^i)(uCe)H^j$
		$\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}(\bar{Q}u)(LCL^i)H^j$
		$\mathcal{O}_{\bar{L}dud\tilde{H}}$	$(\bar{L}d)(uCd)\tilde{H}$
		$\mathcal{O}_{\bar{L}dddH}$	$(\bar{L}d)(dCd)H$
		$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$
		$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\epsilon_{ij}(\bar{L}d)(QCQ^i)\tilde{H}^j$
	redundant operators		
$\mathcal{O}^{(2)}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^iC\sigma^{\mu\nu}D_\nu L^j)$	$\mathcal{O}_{\bar{L}dQdD}$	$(\bar{L}iD^\mu d)(QC\gamma_\mu d)$

Summary of dim-7 operators

Comments:

- Not count flavors and Hermitian conjugates:
  12 ($\leftarrow$ 13) B-conserving $+$ 6 ($\leftarrow$ 7) B-violating operators;
  but $\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$ and $(\bar{L}d)(dCd)H$ vanish *for one generation*.
- Last two operators are redundant.
- $\mathscr{L}_7$: Operators to be multiplied by Wilson coeff. containing flavor indices.

Features:

- All dim-7 operators involve leptons and violate lepton number.
- Most involve H except for three. Rare processes challenging to detect.
- Unique dim-7 operator $\mathscr{O}_{LH}$ for neutrino mass. Actually unique to all dimensions: $\mathscr{O}_5(H^\dagger H)^n$ Liao (2010).

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Demonstration of operator reduction by easy examples

Example 1

A seemingly independent operator is

$$\begin{aligned} &\epsilon_{ij}(L^i C\gamma_\mu e)(\bar{d}\gamma^\mu u)H^j = \epsilon_{ij}(\overline{L_i^C}\gamma_\mu e)(\bar{d}\gamma^\mu u)H^j \\ \overset{(25)}{=} \quad & 2\epsilon_{ij}(\bar{d}L_i)(\overline{u^C}e) = 2\epsilon_{ij}(\bar{d}L^i)(uCe)H^j = 2\mathcal{O}_{\bar{d}LueH}. \end{aligned} \tag{35}$$

Example 2

Redundancy of $\mathcal{O}^{(2)prst}_{\bar{d}uLLD}$:

$$\begin{aligned} \mathcal{O}^{(2)prst}_{\bar{d}uLLD} &= \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L^i_s C\sigma^{\mu\nu}D_\nu L^j_t) \qquad \sigma^{\mu\nu} = i\gamma^\mu\gamma^\nu - ig^{\mu\nu} \\ &= \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L^i_s C\gamma^\mu\gamma^\nu iD_\nu L^j_t) - \epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L^i_s CiD^\mu L^j_t) \\ &\overset{(33)}{=} (Y_e)_{tu}\epsilon_{ij}(\bar{d}_p\gamma_\mu u_r)(L^i_s C\gamma^\mu e_u)H^j - \mathcal{O}^{prst}_{\bar{d}uLLD} \\ &\overset{(35)}{=} 2(Y_e)_{tu}\mathcal{O}^{psru}_{\bar{d}LueH} - \mathcal{O}^{prst}_{\bar{d}uLLD} \end{aligned} \tag{36}$$

RGEs for 6 operators with $B = -L = 1$

They are closed under 1-loop renormalization since other 12 operators have $B = 0, L = -1$.

$$\mathscr{L}_7 = \sum_i C_i \mathscr{O}_i + \text{h.c.}, \quad (37)$$

where we use shortcuts for 6 operators and Wilson coefficients:

$$\begin{aligned}
C_1 &\equiv C^{prst}_{\bar{L}dud\tilde{H}} & \mathscr{O}_1 &\equiv \mathscr{O}^{prst}_{\bar{L}dud\tilde{H}} = \varepsilon_{\alpha\beta\sigma}\varepsilon_{ij}(\bar{L}_{ip}d_{\alpha r})(u_{\beta s}Cd_{\sigma t})H^*_j,\\
C_2 &\equiv C^{prst}_{\bar{L}dddH} & \mathscr{O}_2 &\equiv \mathscr{O}^{prst}_{\bar{L}dddH} = \varepsilon_{\alpha\beta\sigma}\delta_{ij}(\bar{L}_{ip}d_{\alpha r})(d_{\beta s}Cd_{\sigma t})H_j,\\
C_3 &\equiv C^{prst}_{\bar{e}Qdd\tilde{H}} & \mathscr{O}_3 &\equiv \mathscr{O}^{prst}_{\bar{e}Qdd\tilde{H}} = -\varepsilon_{\alpha\beta\sigma}\delta_{ij}(\bar{e}_pQ_{i\alpha r})(d_{\beta s}Cd_{\sigma t})H^*_j,\\
C_4 &\equiv C^{prst}_{\bar{L}dQQ\tilde{H}} & \mathscr{O}_4 &\equiv \mathscr{O}^{prst}_{\bar{L}dQQ\tilde{H}} = -\varepsilon_{\alpha\beta\sigma}\delta_{kl}\delta_{ij}(\bar{L}_{kp}d_{\alpha r})(Q_{l\beta s}CQ_{i\sigma t})H^*_j,\\
C_5 &\equiv C^{prst}_{\bar{L}QddD} & \mathscr{O}_5 &\equiv \mathscr{O}^{prst}_{\bar{L}QddD} = \varepsilon_{\alpha\beta\sigma}\delta_{ij}(\bar{L}_{ip}\gamma_\mu Q_{j\alpha r})(d_{\beta s}CiD^\mu_{\sigma\rho}d_{\rho t}),\\
C_6 &\equiv C^{prst}_{\bar{e}dddD} & \mathscr{O}_6 &\equiv \mathscr{O}^{prst}_{\bar{e}dddD} = \varepsilon_{\alpha\beta\sigma}(\bar{e}_p\gamma_\mu d_{\alpha r})(d_{\beta s}CiD^\mu_{\sigma\rho}d_{\rho t}).
\end{aligned} \quad (38)$$

We study one-loop RGE of 6 B-violating operators:

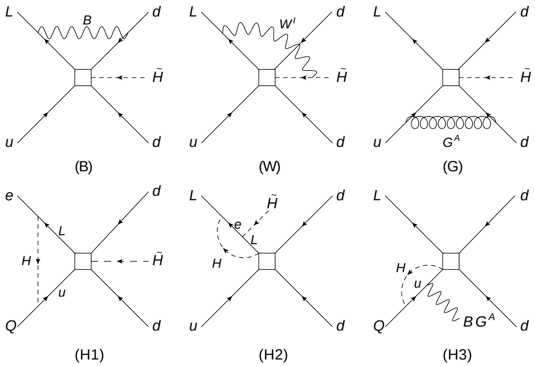
$$\dot{C}_i \equiv 16\pi^2\mu\frac{dC_i}{d\mu} = \sum_{j=1}^{6}\gamma_{ij}C_j \quad (39)$$

RGEs for 6 operators with $B=-L=1$

dimensional regularization, MS

general $R_{\xi_{1,2,3}}$ gauge

representative graphs:



RGEs for 6 operators with $B=-L=1$

We get

$$
\begin{aligned}
\dot{C}_1^{prst} &= +\left(-4g_3^2-\frac{9}{4}g_2^2-\frac{17}{12}g_1^2+W_H\right)C_1^{prst}-\frac{10}{3}g_1^2C_1^{ptsr}-\frac{3}{2}(Y_eY_e^\dagger)_{pv}C_1^{vrst}\\
&+3(Y_d^\dagger Y_d)_{vr}C_1^{pvst}+3(Y_d^\dagger Y_d)_{vt}C_1^{prsv}+2(Y_u^\dagger Y_u)_{vs}C_1^{prvt}-2(Y_d^\dagger Y_u)_{vs}\left(C_2^{pvrt}+v\leftrightarrow r\right)\\
&+4(Y_e)_{pv}(Y_u)_{ws}C_3^{vwrt}-2\Big((Y_u)_{vs}(Y_d)_{wt}+s\leftrightarrow t\Big)C_4^{prvw}-\frac{1}{6}\left(11g_1^2+24g_3^2\right)(Y_u)_{vs}C_5^{pvrt}\\
&+\frac{1}{6}\left(13g_1^2+48g_3^2\right)(Y_u)_{vs}C_5^{pvtr}-\frac{3}{2}(Y_d)_{vt}(Y_d^\dagger Y_u)_{ws}C_5^{pvrw}\\
&-3(Y_u)_{vs}\left((Y_d^\dagger Y_d)_{wt}C_5^{pvrw}-r\leftrightarrow t\right)+\frac{3}{2}(Y_e)_{pv}(Y_d^\dagger Y_u)_{ws}C_6^{vrtw},\\
\dot{C}_2^{prst} &= +\left(-4g_3^2-\frac{9}{4}g_2^2-\frac{13}{12}g_1^2+W_H\right)C_2^{prst}+\frac{5}{2}(Y_eY_e^\dagger)_{pv}C_2^{vrst}\\
&+2\left((Y_d^\dagger Y_d)_{vr}C_2^{pvst}+(Y_d^\dagger Y_d)_{vs}C_2^{prvt}+(Y_d^\dagger Y_d)_{vt}C_2^{prsv}\right)\\
&-\frac{1}{4}\left[\left((Y_u^\dagger Y_d)_{vs}C_1^{prvt}+(Y_u^\dagger Y_d)_{vr}C_1^{psvt}+(Y_u^\dagger Y_d)_{vs}C_1^{ptvr}\right)-s\leftrightarrow t\right]\\
&+\left\{\left[\left(\frac{1}{3}(g_1^2-6g_3^2)(Y_d)_{vr}C_5^{pvst}-\frac{1}{4}g_1^2(Y_d)_{vs}C_5^{pvrt}-\frac{3}{4}(Y_d)_{vr}(Y_u^\dagger Y_d)_{wt}C_5^{pvsw}\right)+r\leftrightarrow t\right]-s\leftrightarrow t\right\}\\
&+\frac{1}{2}(Y_e)_{pv}\left\{\left[g_1^2\left(C_6^{vrst}+r\leftrightarrow s\right)+\frac{3}{4}\left((Y_d^\dagger Y_d)_{wt}\left(C_6^{vrsw}+r\leftrightarrow s\right)+(Y_d^\dagger Y_d)_{wr}C_6^{vtsw}\right)\right]-s\leftrightarrow t\right\},
\end{aligned}
$$

RGEs for 6 operators with $B = -L = 1$

$$
\begin{aligned}
\dot{C}_3^{prst} = {}& +\left(-4g_3^2 - \frac{9}{4}g_2^2 + \frac{11}{12}g_1^2 + W_H\right)C_3^{prst} \\
& +\left[\left((Y_e^\dagger Y_e)_{pv} C_3^{vrst} + \frac{5}{4}(Y_u Y_u^\dagger + Y_d Y_d^\dagger)_{vr} C_3^{pvst} + 3(Y_d^\dagger Y_d)_{vs} C_3^{prvt} - (Y_d^\dagger)_{wr}(Y_d)_{vs} C_3^{pvwt}\right) - s \leftrightarrow t\right] \\
& -\frac{1}{2}(Y_e^\dagger)_{pv}\left[\left((Y_u^\dagger)_{wr} C_1^{vtws} + 2(Y_d)_{ws} C_4^{vtwr} + (Y_d)_{wt} C_4^{vsrw} + 3g_1^2 C_5^{vrst} + 3(Y_d^\dagger Y_d)_{wt} C_5^{vrsw}\right) - s \leftrightarrow t\right] \\
& +\frac{1}{4}(g_1^2 + 12g_3^2)(Y_d^\dagger)_{vr}\left[\left(C_6^{pvst} + C_6^{psvt} + C_6^{pstv}\right) - s \leftrightarrow t\right] \\
& -\frac{3}{4}\left\{\left[(Y_d^\dagger Y_d)_{vs}(Y_d^\dagger)_{wr}\left(C_6^{ptvw} - r \leftrightarrow v\right) + (Y_d^\dagger Y_d)_{ws}(Y_d^\dagger)_{vr}\left(C_6^{ptvw} + 2C_6^{pvtw}\right)\right] - s \leftrightarrow t\right\}, \\
\dot{C}_4^{prst} = {}& +\left(-4g_3^2 - \frac{15}{4}g_2^2 - \frac{19}{12}g_1^2 + W_H\right)C_4^{prst} - 3g_2^2 C_4^{prts} + 3(Y_d^\dagger Y_d)_{vr} C_4^{pvst} \\
& -\frac{1}{2}(Y_e Y_e^\dagger)_{pv}\left(4C_4^{vrts} - C_4^{vrst}\right) + \left(2(Y_u Y_u^\dagger)_{vt} - (Y_d Y_d^\dagger)_{vt}\right)C_4^{prvs} \\
& +\frac{1}{2}\left(5(Y_u Y_u^\dagger)_{vs} + (Y_d Y_d^\dagger)_{vs}\right)C_4^{prvt} + \frac{1}{2}\left(5(Y_d Y_d^\dagger)_{vt} - 3(Y_u Y_u^\dagger)_{vt}\right)C_4^{prsv} \\
& -(Y_d)_{wr}\left((Y_d^\dagger)_{vs} C_4^{pvwt} + (Y_d^\dagger)_{vt} C_4^{pvsw}\right) - \left((Y_u^\dagger)_{vs}(Y_d^\dagger)_{wt}(2C_1^{prvw} + C_1^{pwvr}) + s \leftrightarrow t\right) \\
& -2(Y_e)_{pv}(Y_d^\dagger)_{ws} C_3^{vtwr} - \frac{1}{6}(g_1^2 - 24g_3^2)(Y_d^\dagger)_{vt}(C_5^{psvr} + r \leftrightarrow v) - \frac{3}{2}(Y_e Y_e^\dagger)_{pv}(Y_d^\dagger)_{ws} C_5^{vtrw} \\
& +\frac{3}{2}(Y_d^\dagger Y_d)_{vr}(Y_d^\dagger)_{wt}(C_5^{psvw} + v \leftrightarrow w) + \frac{3}{2}\left((Y_u Y_u^\dagger)_{vs}(Y_d^\dagger)_{wt} + s \leftrightarrow t\right)C_5^{pvrw} \\
& +\frac{3}{2}(Y_e)_{pv}(Y_d^\dagger)_{ws}(Y_d^\dagger)_{xt}\left(C_6^{vxrw} + C_6^{vrwx} + C_6^{vrxw}\right),
\end{aligned}
$$

RGEs for 6 operators with $B = -L = 1$

$$
\begin{aligned}
\dot{C}_5^{prst} &= +\left(\frac{5}{9}g_1^2 - \frac{4}{3}g_3^2\right)C_5^{prst} - \left(\frac{1}{9}g_1^2 - \frac{8}{3}g_3^2\right)C_5^{prts} + \frac{1}{2}(Y_e Y_e^\dagger)_{pv} C_5^{vrst} + \frac{1}{2}(Y_u Y_u^\dagger + Y_d Y_d^\dagger)_{vr} C_5^{pvst} \\
&\quad + (Y_d^\dagger Y_d)_{vs} C_5^{prvt} + (Y_d^\dagger Y_d)_{vt} C_5^{prsv} - (Y_d^\dagger)_{wr}\left((Y_d)_{vs} C_5^{pvwt} + (Y_d)_{vt} C_5^{pvsw}\right) \\
&\quad - (Y_e)_{pv}(Y_d^\dagger)_{wr}\left(C_6^{vwst} + C_6^{vswt} + C_6^{vstw}\right), \\
\dot{C}_6^{prst} &= -\left(\frac{4}{27}g_1^2 + \frac{8}{3}g_3^2\right)C_6^{prst} - \left(\frac{2}{9}g_1^2 - \frac{4}{3}g_3^2\right)\left(C_6^{prts} + C_6^{psrt} + C_6^{pstr} + C_6^{ptrs} + C_6^{ptsr}\right) \\
&\quad + (Y_e^\dagger Y_e)_{pv} C_6^{vrst} + (Y_d^\dagger Y_d)_{vr} C_6^{pvst} + (Y_d^\dagger Y_d)_{vs} C_6^{prvt} + (Y_d^\dagger Y_d)_{vt} C_6^{prsv} - 2(Y_e^\dagger)_{pv}(Y_d)_{wr} C_5^{vwst}
\end{aligned}
$$

where

$$
W_H = \mathrm{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e)
$$

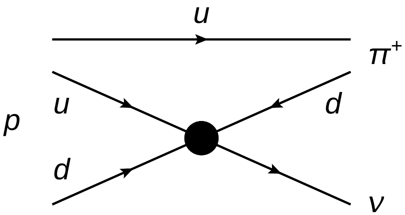
Phenomenological implications: an illustration

$B=-L=1$: rare nucleon decays, e.g., $p\to\nu\pi^+$ $(\nu K^+, e^-\pi^+\pi^+)$, $n\to e^-\pi^+,\ldots$

Not attempt a complete analysis, which requires a sequence of EFT from electroweak scale to nucleon mass scale.

But illustrate potential impact of RGE.

Low energy: $H\to v/\sqrt{2}$, $D\to 0$, only $\mathcal{O}^{p111}_{\bar{L}dud\tilde{H}}$, $\mathcal{O}^{p111}_{\bar{L}dQQ\tilde{H}}$ relevant.



Phenomenological implications: an illustration

Ignore quark mixing, drop all Yukawa couplings except for the top.
RGEs are decoupled ($\alpha_i = g_i^2/(4\pi)$ $(i=1,2,3)$, $\alpha_t = Y_t^2/(4\pi)$):

$$\mu\frac{d}{d\mu}C^{p111}_{\bar{L}dud\tilde{H}} = \frac{1}{4\pi}\left(-4\alpha_3 - \frac{9}{4}\alpha_2 - \frac{57}{12}\alpha_1 + 3\alpha_t\right)C^{p111}_{\bar{L}dud\tilde{H}} \quad (40)$$

$$\mu\frac{d}{d\mu}C^{p111}_{\bar{L}dQQ\tilde{H}} = \frac{1}{4\pi}\left(-4\alpha_3 - \frac{27}{4}\alpha_2 - \frac{19}{12}\alpha_1 + 3\alpha_t\right)C^{p111}_{\bar{L}dQQ\tilde{H}} \quad (41)$$

From $M \sim 10^{15}\ \mathrm{GeV}$(GUT) to $\mu \sim m_p \sim 1\ \mathrm{GeV}$:

$$C^{p111}_{\bar{L}dud\tilde{H}}(m_p) = \left[\frac{\alpha_3(m_p)}{\alpha_3(M)}\right]^{2/\beta_3}\left[\frac{\alpha_2(M_Z)}{\alpha_2(M)}\right]^{9/(8\beta_2)}\left[\frac{\alpha_1(M_Z)}{\alpha_1(M)}\right]^{57/(24\beta_1)}(0.787)C^{p111}_{\bar{L}dud\tilde{H}}(M) \quad (42)$$

$$C^{p111}_{\bar{L}dQQ\tilde{H}}(m_p) = \left[\frac{\alpha_3(m_p)}{\alpha_3(M)}\right]^{2/\beta_3}\left[\frac{\alpha_2(M_Z)}{\alpha_2(M)}\right]^{27/(8\beta_2)}\left[\frac{\alpha_1(M_Z)}{\alpha_1(M)}\right]^{19/(24\beta_1)}(0.787)C^{p111}_{\bar{L}dQQ\tilde{H}}(M) \quad (43)$$

Phenomenological implications: an illustration

$$\beta_3 = 7, \quad \beta_2 = \frac{19}{6}, \quad \beta_1 = -\frac{41}{10}$$
$$\alpha_1(M_Z) = 0.0169225 \pm 0.0000039, \quad \alpha_2(M_Z) = 0.033735 \pm 0.000020,$$
$$\alpha_3(M_Z) = 0.1173 \pm 0.00069, \quad \alpha_t(M_Z) = 0.07514$$

$\Rightarrow$

$$C^{p111}_{\bar{L}dud\tilde{H}}(m_p) = (2.034)(1.158)(1.262)(0.787)C^{p111}_{\bar{L}dud\tilde{H}}(M) = 2.34C^{p111}_{\bar{L}dud\tilde{H}}(M) \quad (44)$$
$$C^{p111}_{\bar{L}dQQ\tilde{H}}(m_p) = (2.034)(1.551)(1.081)(0.787)C^{p111}_{\bar{L}dQQ\tilde{H}}(M) = 2.68C^{p111}_{\bar{L}dQQ\tilde{H}}(M) \quad (45)$$

Outline

Hilbert series

skipped for limited time.

All references can be traced back from latest work by Murayama's group:

B. Henning, et al, Operator bases, S-matrices, and their partition functions, arXiv:1706.08520

B. Henning, et al, $2, 84, 30, 993, \ldots$: Higher dimensional operators in SMEFT, arXiv:1512.03433