Lectures on Standard Model Effective Field Theory

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Outline

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General discussion

- Bottom-up approach:
  SM considered as a low energy EFT below EW scale.
- Why SM renormalizable?
  It includes *all* leading terms (operators with dim ≤ 4) that are consistent with symmetries!
  This is completely consistent with the spirit of EFT.
- If there is any new physics above EW scale (UV theory) and if there are *no light degrees of freedom other than SM fields*, its low energy effects below EW scale (IR theory) can be parameterized by
  - modifications (renormalization) to SM Lagrangian and
  - effective interactions involving high-dim operators.

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SMEFT

- It is therefore an important task to study the list of high-dim operators that are made up exclusively of

  SM fields: G^A_μ, W^I_μ, B_μ; Q, u, d, L, e; H

  and that are consistent with expected symmetries:

  Lorentz invariance and gauge invariance under $SU(3)_C \times SU(2)_L \times U(1)_Y$

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  complete – consistency requirement
  and *independent* (without redundancy) – correct connection with s matrix

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- Since it is a low energy, weakly coupled theory, its power counting rule is simple:
  by the number of suppressed powers of high scale Λ:

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \mathscr{L}_5 + \mathscr{L}_6 + \mathscr{L}_7 + \mathscr{L}_8 + \cdots,$$
$$\mathscr{L}_{n\geq 5} \propto \frac{1}{\Lambda^{n-4}} \tag{1}$$

- Essential steps taken in the continuing efforts:
  $\mathscr{L}_5$: unique (neutrino mass) operator, by Weinberg (1979)
  $\mathscr{L}_6$: Buchmüller-Wyler (1986) ... Grzadkowski et al, 'Warsaw basis' (2010)
  $\mathscr{L}_7$: Lehman (2014), Liao-Ma (2016)

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- Important to recent checks on numbers of complete and independent operators is the mathematical approach of Hilbert series, popularized to the phenomenology community by
  Jenkins-Manohar group in 2009 - 2011
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  B. Henning, et al, *2, 84, 30, 993, ...: Higher dimension operators in SMEFT*, arXiv:1512.03433
- *This Lecture*: dim-6 operators
  Next Lecture: dim-5 and dim-7 operators

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SMEFT

- A major advantage using SMEFT is its generality.
  - All precision data at low energies can be translated into constraints on Wilson coefficients. *Once done and for all, until data updated!*
  - Wilson coefficients worked out for a given new physics model.
  - Comparison of the two provides info on viability of the model from the side of low energy phenomenology.
- Precision measurements include a wide class of processes, such as
  - SLAC, LEP and LEP2: quantities from Z-pole (~ 90 GeV) to $\sqrt{s} = 209$ GeV: m_Z, Γ_Z, $\sigma_{\rm had}$, R, asymmetries, etc;
  - m_W from Tevatron and LEP2;
  - Low energy observables: α, G_F, various ν scattering data, atomic parity violation to measure $\sin^2\theta_W$, etc;
  - Flavor physics such as $b \to s\gamma$, $s\ell\ell$, etc.
- We discuss by examples.

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SMEFT: Dim-6 operators

- Without counting flavors and Hermitian conjugate, the numbers of dim-6 operators are

class	$(\psi)^0$	$(\psi)^2$	$(\psi)^4$	$\not{B}(\psi)^4$
Buchmüller-Wyler	16	35	29	
Grzadkowski et al, 'Warsaw basis'	15	19	25	$5(\to 4)$

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  I will not show the Warsaw basis of dim-6 operators, but discuss some phenomenological consequences.

- RGEs for dim-6 operators are very involved, and were accomplished in a series of papers:

C. Grojean et al, JHEP **1304**, 016 (2013) [arXiv:1301.2588 [hep-ph]].
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S, T parameters

We discuss pure bosonic operators, which

- usually arise from a UV theory whose heavy fields couple only to SM gauge bosons and Higgs field through loop effects,
- affect SM fermion couplings to gauge bosons in a universal manner,
- result in *oblique* effects to fermion processes.

$$\mathscr{L}_6 \supset c_S Q_{HWB} + c_T Q_{HD}, \tag{3}$$

$$\begin{aligned} Q_{HWB} &= H^\dagger \sigma^I H W^I_{\mu\nu} B^{\mu\nu}, \\ Q_{HD} &= (H^\dagger D^\mu H)^* (H^\dagger D_\mu H). \end{aligned} \tag{4}$$

- Q_{HWB} modifies $W^3 - B$ kinetic mixing, contributing to S parameter;
- Q_{HD} violates custodial symmetry by modifying Z mass but not W mass, contributing to T or ρ parameter:

$$S = \frac{4 s_W c_W v^2}{\alpha} c_S, \quad T = -\frac{v^2}{2\alpha} c_T \tag{5}$$

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Global analysis or fitting:

Build a likelihood function to include all data with possible correlations relevant to the S and T parameters;

Experimental results with errors are transformed to combined bounds on S, T with certain statistical significance.

This is very useful to test specific models in which S,T are related.

S, T parameters

Example – contribution from fourth generation heavy quarks T, B

Same $\mathscr{L}$ as for quarks in SM; Yukawa couplings Y_T, Y_B.

Contribution to Q_{HWB} and Q_{HD} takes place at one loop.
Rough estimate from one-loop diagrams involving T, B:

$$H^\dagger H W^I_\mu B_\nu \qquad c_S \sim \frac{N_C}{(4\pi)^2}\frac{g_2 g_1 Y_T^2}{M_T^2} \sim \frac{N_C}{(4\pi)^2}\frac{g_2 g_1}{v^2} \text{ for } M_T \sim M_B \text{ with } Y_T \sim Y_B, \quad (6)$$

$$(H^\dagger)^2 H^2 \qquad c_T \sim \frac{N_C}{(4\pi)^2}\frac{Y_T^4}{M_T^2} \sim \frac{N_C}{(4\pi)^2}\frac{M_T^2}{v^4} \text{ for } M_T \gg M_B \text{ with } Y_T \gg Y_B \quad (7)$$

Better results from calculating one-loop gauge boson self-energies:

$$W^3_\mu W^3_\nu - W^1_\mu W^1_\nu \quad \to \quad T = \frac{N_C}{v^2 \alpha (4\pi)^2}\left[(M_T^2 + M_B^2) - \frac{2M_T^2 M_B^2}{M_T^2 - M_B^2}\ln\frac{M_T^2}{M_B^2}\right] \quad \begin{matrix}\to 0 \text{ for}\\ M_T = M_B\end{matrix} \quad (8)$$

$$W^3_\mu B_\nu \quad \to \quad S = \frac{N_C}{6\pi}\left[1 + \frac{1}{3}\ln\frac{M_B^2}{M_T^2}\right] \quad (9)$$

Precision data disfavor a 4th generation with large mass splitting.

R_K and R_{K^*} anomaly

skipped for limited time.