Lecture 3a on Standard Model Effective Field Theory

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Outline

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General discussions on calculations in EFT

- Suppose we are interested in a physical process occurring at a typical energy scale $E \sim m$.
- Suppose we know physics (full theory or EFT) described by $\mathscr{L}_1$ for EFT_1, whose heaviest particle Φ has mass $M \gg m$ and which has no particles with a mass between m and M.
- We proceed as follows:
- Build EFT_2 so that
(1) Φ has been integrated out, i.e., $\mathscr{L}_2$ for EFT_2 contains no Φ.
(2) Just below scale $\mu = M$ EFT_2 yields same results as EFT_1 for processes involving only particles lighter than Φ. This is called matching calculation.

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$\Longrightarrow$ additional effective interactions $C(\mu)\mathscr{O}(\mu)$ and renormalized effects in existing terms in $\mathscr{L}$.
Decoupling means that additional interactions are from high-dim operators $\mathscr{O}$ suppressed by M and that renormalization effects have no large log at $\mu = M$.
(3) Do RG running from scale $\mu = M$ to $\mu = m$ so that matrix elements of $\mathscr{O}(\mu)$ can be directly evaluated at $\mu = m$.
RG effects are incorporated in Wilson coefficients $C(\mu)$ of $\mathscr{O}(\mu)$.

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- In principle, the end result is independent of regularization and renormalization scheme.
- In practice, dimensional regularization plus mass-independent scheme is easier than Wilson's cutoff plus mass-dependent subtraction.
- A bit formalism follows.
- Suppose there are a field Φ of heavy mass M and fields ϕ of lighter mass in UV theory or EFT_1.
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General discussions on calculations in EFT

All physical quantities can be obtained from connected Green's functions whose generating functional for ϕ is, in EFT$_1$,

$$Z[j] = \int \mathscr{D}\Phi \mathscr{D}\phi \ \exp\left[iS_1[\Phi,\phi] + i \int j\phi \right], \tag{1}$$

where S_1 is the action for EFT$_1$ and j is the source for ϕ.

- EFT$_1$ can be a fundamental theory (UV completion) or an EFT.
- From the point of view of EFT, the only difference is:
  For the former, S_1 contains a finite number of terms which renormalize among themselves.
  For the latter, S_1 has an infinite tower of terms, but is also renormalizable for operators up to any given dimension. This is all right in the sense that experimental accuracy is finite.

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Now integrate out Φ:

$$Z[j] = \int \mathscr{D}\phi \ \exp\left[iS_2[\phi] + i\int j\phi\right] \tag{2}$$

where the 'action' for EFT_2 S_2 is generally nonlocal.
It involves an infinite number of terms when expanded in field derivatives:

$$S_2[\phi] = \int d^4x \ \mathscr{L}_2(\phi, \partial\phi, \partial^2\phi, \dots), \tag{3}$$

$$\mathscr{L}_2(\phi, \partial\phi, \partial^2\phi, \dots) = \mathscr{L}_{n_i \le 4} + \sum_{n_i \ge 5} c_i \mathscr{O}_i. \tag{4}$$

- n_i counts the dimension of fields and derivatives, $[c_i] = 4 - n_i$ and $c_i \sim 1/M^{n_i - 4}$ for $n_i \ge 5$.
- Given n_i, there are a finite number of independent $\mathscr{O}_i$s although the number increases fast with n_i.

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General discussions on calculations in EFT

- Usually all possible $\mathscr{O}_i$s do appear at this order or another in perturbation theory so long as they are consistent with low energy symmetries.
- For a good EFT it should suffice to work with high-dim operators of the first few dimensions.
- Caution: Operators of diff. dim may come from diff. heavy physics.
  - For instance, in SMEFT, dim-5 and dim-7 operators violate lepton number while most dim-6 operators do not.
- By definition S_2 should reproduce low energy ϕ physics of S_1. S_2 renormalizes (part of) existing terms in S_1 and generally introduces new high-dim operators.
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Matching calculation at tree level

First thing first: power counting.

- In strongly coupled theory it can be complicated –
  see Manohar-Georgi's paper in 1980s on naive dimensional analysis.
- In weakly coupled theory this is easy –
  free theory dominates and defines the dimension of fields, and
  $[\mathscr{O}]$ simply counts those of fields and derivatives involved.
- Power counting together with desired accuracy for physical quantities determines to which dimension we should expand and to which order we do perturbation.
- We work with natural units, $\hbar = c = 1$, so that

$$[\text{mass}] = [\text{energy}] = [\text{length}]^{-1}. \tag{5}$$

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Matching calc at tree level – Example 1: heavy scalar

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Consider a toy model, fundamental or EFT_1:

$$\begin{aligned}\mathscr{L}_1(\Phi,\phi,\psi) \quad = \quad & i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 - y_\phi\bar{\psi}\psi\phi + \cdots \\ & + \frac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi) - \frac{1}{2}M^2\Phi^2 + \cdots \\ & - y_\Phi\bar{\psi}\psi\Phi + \cdots \end{aligned} \tag{6}$$

Light fields: massless ψ, ϕ of mass m.
Heavy field: Φ of mass $M \gg m$

Assume Yukawa couplings small enough to allow for pertur. analysis.

We are interested in EFT_2 for light fields ϕ and ψ alone, i.e., we want to integrate out Φ.

Matching calc at tree level – Example 1: heavy scalar

Power counting starts with dimensional analysis:

$$[\bar{\psi}\partial\!\!\!/\psi] = [(\partial^\mu \phi)^2] = [(\partial^\mu \Phi)^2] = 4 \Rightarrow [\psi] = \frac{3}{2},\ [\phi] = [\Phi] = 1 \tag{7}$$

$$[y_\phi \bar{\psi}\psi\phi] = [y_\Phi \bar{\psi}\psi\Phi] = 4 \Rightarrow [y_\phi] = [y_\Phi] = 0 \tag{8}$$

Integrating out Φ will renormalize existing terms for $\phi,\ \psi$ and generate new high-dim operators involving $\phi,\ \psi$.

Here we consider new high-dim operators due to Yukawa coupling of Φ to ψ.

We seek for $\mathscr{L}_2(\phi,\psi)$ in EFT_2 which can reproduce physics of $\mathscr{L}_1(\Phi,\phi,\psi)$ for $\psi,\ \phi$ below scale M.

Matching calc at tree level – Example 1: heavy scalar

First few possible high-dim operators include

$$\text{dim-6} \qquad \mathscr{O}_6 = \bar\psi\psi\bar\psi\psi \tag{9}$$

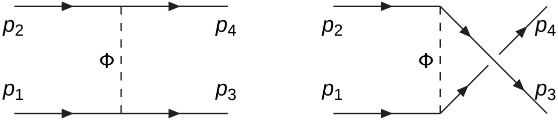
$$\text{dim-8} \qquad \mathscr{O}_8 = (\partial_\mu\bar\psi\partial^\mu\psi)\bar\psi\psi \tag{10}$$

We work to tree level. Consider the process

$$\psi(p_1)+\psi(p_2)\to\psi(p_3)+\psi(p_4)$$

Since we are interested in new effective interactions that are induced by the heavy field Φ, contributions from pure light fields are irrelevant.

There are two Feynman diagrams:



Matching calc at tree level – Example 1: heavy scalar

The amplitude is, from Φ exchange in UV theory (EFT_1),

$$i\mathscr{A}_{\mathrm{UV}} = \bar{u}(p_3)(-iy_\Phi)u(p_1)\frac{i}{(p_1-p_3)^2-M^2}\bar{u}(p_4)(-iy_\Phi)u(p_2)-(3\leftrightarrow 4) \quad (11)$$

Since ps are much smaller than M, we expand the propagator to, e.g., $O(p^2)$,

$$\frac{1}{(p_1-p_3)^2-M^2} = -\frac{1}{M^2}-\frac{(p_1-p_3)^2}{M^4}+O(M^{-6}) \quad (12)$$

Thus,

$$\mathscr{A}_{\mathrm{UV}} = \frac{y_\Phi^2}{M^2}\left[\bar{u}_3u_1\bar{u}_4u_2-(3\leftrightarrow 4)\right]+\frac{y_\Phi^2}{M^4}\left[(p_1-p_3)^2\bar{u}_3u_1\bar{u}_4u_2-(3\leftrightarrow 4)\right] +O(M^{-6}) \quad (13)$$

Matching calc at tree level – Example 1: heavy scalar

The first term can be reproduced by an effective interaction $\propto \mathscr{O}_6$ in IR theory (EFT$_2$):

$$\begin{aligned}\mathscr{L}_2(\phi,\psi) &= i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 - y_\phi\bar{\psi}\psi\phi + \cdots \\ &\quad + \frac{1}{2}G_S\mathscr{O}_6 + \cdots,\ G_S = \frac{y_\Phi^2}{M^2}\end{aligned} \tag{14}$$

The second term is a bit complicated.

We want to get an effective interaction that is valid for both on-shell and off-shell particles.

But to fix its structure we can use any convenient kinematics.

Here we apply on-shell conditions, $p_i^2 = 0$ for ψ particle, so that

$$(p_1 - p_3)^2 = (p_2 - p_4)^2 = -2p_1\cdot p_3 = -2p_2\cdot p_4 = -(p_1\cdot p_3 + p_2\cdot p_4)$$

where symmetrization has been made.

Matching calc at tree level – Example 1: heavy scalar

Only particles are involved here:

$p_{1,2}$ incoming: $p_{1,2} \leftrightarrow i\partial\psi$; $p_{3,4}$ outgoing: $p_{3,4} \leftrightarrow -i\partial\bar{\psi}$.

Thus, the second term in $\mathscr{A}_{\rm UV}$ can be reproduced by an effective interaction in EFT_2:

$$\begin{aligned}\mathscr{L}_2(\phi,\psi) \quad &= \quad i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 - y_\phi\bar{\psi}\psi\phi + \cdots \\ &\quad + \frac{1}{2}G_S\mathscr{O}_6 + c_8\mathscr{O}_8\cdots,\ G_S = \frac{y_\Phi^2}{M^2},\ c_8 = -\frac{y_\Phi^2}{M^4}\end{aligned} \tag{15}$$

But dim-8 operators are not unique; there seem to be 6 possible arrangements of derivatives:

$$((\partial^2\bar{\psi})\psi)\bar{\psi}\psi,\ (\bar{\psi}(\partial^2\psi))\bar{\psi}\psi,$$
$$((\partial_\mu\bar{\psi})(\partial^\mu\psi))\bar{\psi}\psi,$$
$$((\partial_\mu\bar{\psi})\psi)((\partial^\mu\bar{\psi})\psi),\ (\bar{\psi}(\partial_\mu\psi))(\bar{\psi}(\partial^\mu\psi)),\ ((\partial_\mu\bar{\psi})\psi)(\bar{\psi}(\partial^\mu\psi)).$$

Matching calc at tree level – Example 1: heavy scalar

- It is important that all operators consistent with symmetry and of same dimension are included.
- But redundant operators can be removed by
  - integration by parts (IBP) $\leftrightarrow$ action defined up to surface terms $\leftrightarrow$ momentum conservation,
  - eqns of motion (EoM) $\leftrightarrow$ S matrix not changed by field redefinitions,
  - and algebraic relations like Fierz identities associated with representation of Lorenz group.
- It is a hard job to exhaust all possible and independent operators of a given dim in a complicated EFT like SMEFT!

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Back to our example:

$((\partial^2\bar{\psi})\psi)\bar{\psi}\psi$, $(\bar{\psi}(\partial^2\psi))\bar{\psi}\psi$ can be dropped on-shell or by EoM.

$(\bar{\psi}(\partial_\mu\psi))(\bar{\psi}(\partial^\mu\psi))$, $((\partial_\mu\bar{\psi})\psi)(\bar{\psi}(\partial^\mu\psi))$ can be expressed in terms of those kept, dropped, and IBP:

$$
\begin{aligned}
& ((\partial_\mu\bar{\psi})\psi)(\bar{\psi}(\partial^\mu\psi)) \\
= \; & \partial^\mu\left[((\partial_\mu\bar{\psi})\psi)(\bar{\psi}\psi)\right] - ((\partial^2\bar{\psi})\psi)(\bar{\psi}\psi) \\
& -((\partial_\mu\bar{\psi})(\partial^\mu\psi))(\bar{\psi}\psi) - ((\partial_\mu\bar{\psi})\psi)((\partial^\mu\bar{\psi})\psi), \\
& (\bar{\psi}(\partial_\mu\psi))(\bar{\psi}(\partial^\mu\psi)) \\
= \; & \partial_\mu\left[(\bar{\psi}\psi)(\bar{\psi}(\partial^\mu\psi))\right] - ((\partial_\mu\bar{\psi})\psi)(\bar{\psi}(\partial^\mu\psi)) \\
& -((\partial_\mu\bar{\psi})(\partial^\mu\psi))(\bar{\psi}\psi) - (\bar{\psi}\psi)(\bar{\psi}(\partial^2\psi))
\end{aligned}
$$

Matching calc at tree level – Example 1: heavy scalar

Operator $\mathcal{O}_8^1 = ((\partial_\mu \bar{\psi})\psi)((\partial^\mu \bar{\psi})\psi)$ doesn't appear in tree-level matching, but could appear in higher-order matching or get induced by renormalization from $\mathcal{O}_8^2 = ((\partial_\mu \bar{\psi})(\partial^\mu \psi))(\bar{\psi}\psi)$.

Thus we should write for IR theory below scale M:

$$\begin{aligned} \mathscr{L}_2(\phi,\psi) \quad &= \quad i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m^2\phi^2 - y_\phi\bar{\psi}\psi\phi + \cdots \\ &\quad + \frac{1}{2}G_S\mathcal{O}_6 + c_8^1\mathcal{O}_8^1 + c_8^2\mathcal{O}_8^2 \cdots . \end{aligned} \tag{16}$$

Matching calc at tree level – Example 2: 4-Fermi int.

Example 2: 4-Fermi weak interactions

Historically Fermi proposed his 4-Fermi effective interaction to account for nuclear β decays.
In the framework of SM, his interaction is a low energy EFT well below the weak gauge boson masses $m_{W,Z}$.

SM $=$ EFT$_1$ or UV theory bordered at $m_{W,Z}$ 4-Fermi $=$ EFT$_2$ or IR theory

The relevant terms in SM are

$$\mathscr{L}_{\rm SM} = \cdots + \frac{g_2}{\sqrt{2}}\left(W^+_\mu J^{+\mu}_W + W^-_\mu J^{-\mu}_W\right) + \frac{g_2}{\cos\theta_W} Z_\mu J^\mu_Z + \cdots \quad (17)$$

$$J^{+\mu}_W = \bar{\nu}\gamma^\mu P_L e + \bar{u}\gamma^\mu P_L d \quad (18)$$

$$J^{-\mu}_W = (J^{+\mu}_W)^\dagger, \; P_L = \frac{1}{2}(1-\gamma_5) \quad (19)$$

$$J^\mu_Z = \sum_f \bar{f}\gamma^\mu\left(T_3 P_L - Q\sin^2\theta_W\right) f \quad (20)$$

Matching calc at tree level – Example 2: 4-Fermi int.

They yield the amplitudes for a 4-fermion process:

$$i\mathscr{A}_{\rm UV}^{W} = \frac{ig_2}{\sqrt{2}}J_W^{+\mu}\frac{-i}{q^2-m_W^2}\left[g^{\mu\nu}+\cdots\right]\frac{ig_2}{\sqrt{2}}J_W^{-\nu}, \quad (21)$$

$$i\mathscr{A}_{\rm UV}^{Z} = \frac{ig_2}{\cos\theta_W}J_Z^{\mu}\frac{-i}{q^2-m_Z^2}\left[g^{\mu\nu}+\cdots\right]\frac{ig_2}{\cos\theta_W}J_Z^{\nu}, \quad (22)$$

Here spinor wavefunctions are used in currents.
$\cdots$ stand for quadratic q terms, suppressed at low energies.
Crossing terms are possible for Z-exchange.

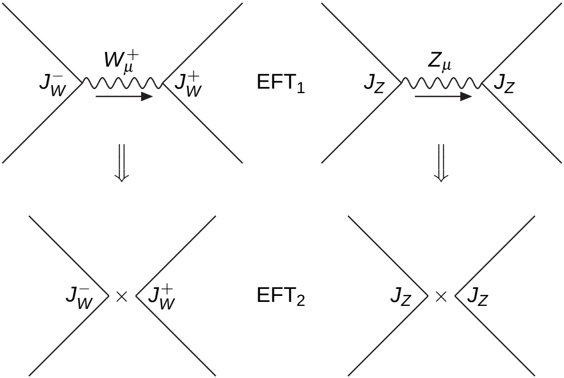
For $|q^2| \ll m_{W,Z}^2$, their leading terms are

$$i\mathscr{A}_{\rm UV}^{W} = -i\left[\frac{g_2}{\sqrt{2}m_W}\right]^2 J_W^{+\mu}J_W^{-\mu}, \quad (23)$$

$$i\mathscr{A}_{\rm UV}^{Z} = -i\left[\frac{g_2}{\sqrt{2}\cos\theta_W m_Z}\right]^2 J_Z^{\mu}J_Z^{\mu} \quad (24)$$

Matching calc at tree level – Example 2: 4-Fermi int.

In Feynman diagrams,



Matching calc at tree level – Example 2: 4-Fermi int.

One process may have both W and Z contributions.
At $E \ll m_{W,Z}$, leading terms are reproduced by effective interactions in EFT_2 where W, Z are integrated out:

$$\mathscr{L}_2 = -\frac{g_2^2}{2m_W^2} J_W^{+\mu} J_{W\mu}^{+} - \frac{1}{2} \frac{g_2^2}{m_Z^2 \cos^2\theta_W} J_Z^{\mu} J_{\mu}^{Z}, \text{ symmetry factor} \tag{25}$$

where the first term corresponds to 4-Fermi charged-current (CC) interaction with the identification

$$\frac{g_2^2}{2m_W^2} = \frac{4G_F}{\sqrt{2}} \tag{26}$$

and the second term is the SM prediction for neutral-current (NC) weak interactions.
The equal couplings for CC and NC interactions are a result of custodial symmetry, $m_W^2 = m_Z^2 \cos^2\theta_W$.

Matching calc at tree level – Example 3: neutrino mass operator from type I seesaw

Example 3: Weinberg's neutrino mass operator from type I seesaw

- Introduce RH fermions N_R, completely neutral under SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, hence the name 'sterile neutrino'. N_R can have gauge-invariant Majorana mass M_N.
- Such a model is a minimal extension of SM and also renormalizable, called EFT_1. Terms containing N_R are

$$\mathscr{L}_{\mathrm{EFT}_1} \supset +i\bar{N}_R \partial\!\!\!/ N_R - \left(\bar{L} Y_N N_R \varepsilon H^* + \frac{1}{2} N_R C M_N N_R + \text{h.c.} \right) \quad (\tilde{H} = \varepsilon H^*) \qquad (27)$$

There are several ways to look at N_R .

- If $M_N = 0$, N_R and ν_L form Dirac neutrinos and gain mass as the electron. But this requires an unnaturally tiny Yukawa coupling Y_N.

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Matching calc at tree level – Example 3: neutrino mass operator from type I seesaw

- Since gauge sym. allows a Majorana mass for N_R , it is more natural $M_N \neq 0$. This has two consequences.

  It breaks lepton number as an accidental sym. of SM.

  It offers a way to induce a tiny neutrino mass via the seesaw mechanism.
- Assume $M_N \gg$ electroweak scale. Integrate out N to get EFT_2.
- Since N couples only to L, H , integrating it out *at tree level* introduces a single effective LH interaction in EFT_2 , which can be found by studying

$$HHLL \to \text{nothing}$$

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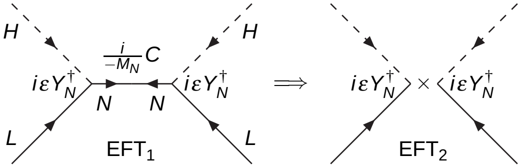
Matching calc at tree level – Example 3: neutrino mass operator from type I seesaw

In Feynman diagrams,



which yields a term in EFT$_2$

$$\mathscr{L}_{\text{EFT}_2} \supset \frac{1}{4}\kappa_{gf}\overline{(L_m^g)^C}\varepsilon^{mn}H_nL_j^f\varepsilon^{ji}H_i + \text{h.c.} \qquad \begin{array}{l} i,j,m,n: \ SU(2)_L \text{ indices} \\ f,g: \ \text{flavor indices}\end{array} \tag{28}$$

$$\frac{1}{2}\kappa_{gf} = -\left(Y_N M_N^{-1} Y_N^T\right)^\dagger_{gf} \tag{29}$$