Lecture 2 on Standard Model Effective Field Theory

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Outline

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Literature

- H. Georgi, *Effective Field Theories*, Annu. Rev. Nucl. Part. Sci. 43 (1993) 209
- W. Skiba, *TASI Lectures on Effective Field Theory and Precision Electroweak Measurements*, arXiv:1006.2142 [hep-ph]
- C.P. Burgess, *Introduction to Effective Field Theory*, arXiv:hep-th/0701053
- I.Z. Rothstein, *TASI Lectures on Effective Field Theories*, arXiv:hep-ph/0308266
- S. Weinberg, *What is Quantum Field Theory and What Did We Think It is?* arXiv:hep-th/9702027

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Basic ideas about effective theory

- Our experience teaches us that phenomena occurring at different scales can be well described by different theories.
  Earth revolution around the Sun, oscillation of strings, etc, are well described by Newton mechanics.
  Electromagnetic properties of macroscopic bodies are well described by classical electrodynamics.
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- Within each mechanics or dynamics, we often make simplifying and also effective approximations:
  Earth's orbit can be explained by assuming a gravitation force $1/r^2$; and if necessary, we know how to include small corrections such as tidal effects.

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Basic ideas about effective theory

- Small-amplitude oscillation of strings can be well described by simple harmonic oscillators; and if necessary, we can include modifications due to dissipative and anharmonic effects.
  Far away from distribution of charges, we can expand electromagnetic fields with respect to the size of distribution – multipole expansion.
- These mechanics and theories are effective theories that are very successful in their ranges of applicability.
  Each theory has its own dynamical variables and laws of dynamics – Each scale has its own physics.
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The outcome of human's endeavors is a sequence of effective theories:

- ..., classical mechanics, classical electrodynamics, kinematics of special relativity, quantum mechanics, quantum electrodynamics, quantum field theory of strong, weak, and electromagnetic interactions, ...
  although some of us may believe there is an ultimate theory of everything.
- Essential to this success is a kind of decoupling theorem –
  Physics at certain distance scale is not affected by degrees of freedom at smaller scales, and the only role played by the latter is defining the 'intrinsic properties' of the former like masses, charges, coupling constants, etc.

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Basic ideas about effective theory

Interactions of particles at microscopic distances are described by quantum field theories –
A sequence of effective field theories

- quantum electrodynamics, ..., 4-Fermi interactions, ..., standard model of electroweak interactions
- phenomenological models of low-mass hadrons, ..., chiral perturbation theory and its extensions, ..., quantum chromodynamics of strong interactions
- ..., supersymmetric theory, ..., grand unification theory, ...?

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Basic ideas about effective theory

Though we are not sure about theories at higher energy scales (smaller distance scales) we can explain and even predict experimental results quite successfully –

because dynamical degrees of freedom of larger masses decouple and leave their footprints only in defining parameters and less important interactions.

Qualitative discussions about effective field theory

There are two approaches to effective field theories.

- Top-down –
  Fundamental theory at high energy scale (ultraviolet theory) is known. Obtain EFT at low energy scale (infrared theory) by integrating out heavy degrees of freedom.
  $\Longrightarrow$ A sequence of EFTs from high to low scales bordered at decreasing masses of particles.
- Bottom-up –
  Unknown or unsolvable UV theory whose low energy (IR) effects we are interested in.
- Important features common to both approaches:
  How symmetry is realized in EFT.
  There exists a power counting law for keeping and discarding effective interactions in $\mathscr{L}$.

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Bottom-up approach –

Example 1: SMEFT (standard model effective field theory)
EFT below electroweak scale without assuming a fundamental theory at high scale:

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \sum_i c_i \mathscr{O}_i \quad (1)$$

where $c_i \mathscr{O}_i$ are effective interactions among SM fields induced by some high scale physics.
(1) Higher dimensional operators $\mathscr{O}_i$ respect SM symmetries and can be exhausted at each dimension of $\mathscr{O}_i$: dim-5, dim-6, dim-7, etc.
(2) c_i are unknown, but can be estimated by dimensional analysis:

$$[\mathscr{O}_i] = n_i \geq 5 \Rightarrow [c_i] = 4 - n_i \leq -1 \Rightarrow c_i \sim \frac{\mathscr{O}(1)}{\Lambda^{n_i - 4}}, \quad (2)$$

with $\Lambda =$ high scale. The higher n_i is, the less important $c_i \mathscr{O}_i$ is.

Qualitative discussions about effective field theory

(3) New terms modify SM predictions in a systematic manner.

Example 2: Chiral perturbation theory

See the lectures by Dr. Qing Wang for detailed discussions.
Effective field theory for pions at low energies

Though we know the fundamental theory is QCD, we are unable to solve pions from it.

Pioneered by Weinberg, Callan-Coleman-Wess-Zumino, et al, and systemized by Gasser and Leutwyler.

Starting point:
dynamical chiral symmetry breaking $SU(2)_L \times SU(2)_R \to SU(2)_I$ by QCD, and pions are the associated Nambu-Goldstone bosons.

Qualitative discussions about effective field theory

Chiral symmetry is nonlinearly realized, and interactions of pions are included in chiral Lagrangian:

$$\mathscr{L} = \mathscr{L}_2 + \mathscr{L}_4 + \mathscr{L}_6 + \cdots \quad (3)$$

where $\mathscr{L}_n$ counts as p^n in power counting law. The leading terms are

$$\mathscr{L}_2 = \frac{1}{4} f_\pi^2 \mathrm{tr}(\partial^\mu U^\dagger \partial_\mu U) + \chi \mathrm{tr}(U + U^\dagger), \; U = \exp(i\sigma^a \pi^a / f_\pi) \quad (4)$$

The unknown $\chi \propto m_{u,d}$ breaks chiral symmetry explicitly, gives squared masses to π^a, and thus counts as p^2.
More terms enter as p power increases, but they become less and less important at energies smaller than $\Lambda \sim 4\pi f_\pi$.

Qualitative discussions about effective field theory

Top-down approach –

- Start from a fundamental theory (UV theory or UV completion) with fields $\phi_1,\ \phi_2,\cdots$.
- Integrate out field ϕ_1 with the largest mass M_1 to obtain EFT_1 at energies $\lesssim M_1$.
- Integrate out field ϕ_2 with the next largest mass M_2 to obtain EFT_2 at energies $\lesssim M_2$.
- Continue until we arrive at energy scales whose physics we are interested in.

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Qualitative discussions about effective field theory

- At the border of two EFTs, we do a matching calculation, requiring that physics just below, e.g., M_1, is identically described by EFT_1 and EFT_2.
- Within one EFT we do a renormalization group (RG) calculation to sum leading logarithms of ratios of scales, e.g., $\ln(\mu/M_1)$, where μ is the scale of our interested physics process.
  Those logs can be large enough to spoil conventional perturbation theory.
- In the bottom-up approach of a specific theory, e.g., SMEFT, we also do matching calculation and RG running. The only difference is that in top-down approach we (supposedly) know everything about EFT_1 from a fundamental theory.

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Summary in one sentence answer to question:
Since decoupling of high scale physics is a general feature of QFT,
why EFT at all in particular when full theory is known and calculable?
EFT simplifies calculation!