Lectures on Standard Model Effective Field Theory

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Outline

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- Lecture 1: Review of Standard Model
  Framework/Symmetries
- Lecture 2: Introduction to Effective Field Theory
  Basic ideas about ET/Qualitative discussions about EFT
- Lecture 3: Techniques in EFT
  General discussions on calculations in EFT/Matching calculation at tree level
  RG running at one loop/Matching calculation at one loop/Summary on EFT calculations
- Lecture 4: Standard Model EFT: Dimension-six Operators
  S, T parameters/R_K, R_{K^*} anomaly
- Lecture 5: Standard Model EFT: Dimension-five and -seven Operators
  Dim-5 operators/Dim-7 operators/Hilbert series: a powerful tool for counting operators

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Introduction

- Please see the lectures by Dr. Jinmin Yang for more detailed discussions. Here I mainly introduce notations, basic ideas, and discuss symmetries.
- The standard model (SM) is based on gauge symmetry and its spontaneous breaking.
- It combines the ideas of gauge theory, with quantum electrodynamics (QED) as a prototype, 4-Fermi interactions, and spontaneous symmetry breaking, and incorporates successful hadron phenomenology.

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Introduction

Some theoretical key steps for the establishment of SM

QED as a consistent quantum field theory (QED);
4-Fermi interaction for nuclear β decay;
Non-Abelian gauge symmetry; renormalizability of non-Abelian gauge theory;
Spontaneous symmetry breaking; renormalizability of spontaneously broken gauge theory;
$SU(2) \times U(1)$ gauge group for unified electromagnetic and weak interactions;
Color number for hadron spectra promoted to dynamical degrees of freedom – $SU(3)$ for QCD;
Asymptotic freedom in non-Abelian gauge theory;
...

Framework

Gauge group: $G = SU(2)_L \times U(1)_Y \times SU(3)_C$, for electroweak and strong interactions

Gauge fields: W_μ^I $(I = 1, 2, 3)$, B_μ, G_μ^A $(A = 1, 2, \cdots, 8)$

Matter (fermion) fields form simplest possible irreps of G:

chiral fields	$SU(3)_C$	$SU(2)_L$	Y
Q	3	2	$\frac{1}{6}$
u	3	1	$\frac{2}{3}$
d	3	1	$-\frac{1}{3}$
L	1	2	$-\frac{1}{2}$
e	1	1	-1

Normalization: $Q_e = T^3 + Y$;

L, Q are left-handed (LH), while u, d, e are right-handed (RH); complete generation/family of chiral fermions guarantees anomaly cancellation.

Framework

Till this point, all interactions are gauge interactions:

$$\mathscr{L}_{\rm SM} \supset \mathscr{L}_{\rm gauge} + \mathscr{L}_{\rm matter}, \qquad (1)$$

$$\mathscr{L}_{\rm gauge} = -\frac{1}{4}G^A_{\mu\nu}G^{A,\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \qquad (2)$$

$$\mathscr{L}_{\rm matter} = \sum_{\psi=Q,u,d;L,e} \bar{\psi} i \not{D} \psi, \qquad (3)$$

where

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_3 f^{ABC} G^B_\mu G^C_\nu, \qquad (4)$$

$$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g_2 \varepsilon^{IJK} W^J_\mu W^K_\nu, \qquad (5)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad (6)$$

$$D_\mu = \partial_\mu - i g_3 T^A G^A_\mu - i g_2 T^I W^I_\mu - i g_1 Y B_\mu, \qquad (7)$$

$$\begin{cases} T^A = \frac{1}{2}\lambda^A, \text{ for } Q,\ u,\ d;\ 0, \text{ otherwise} \\ T^I = \frac{1}{2}\sigma^I, \text{ for } Q,\ L;\ 0, \text{ otherwise} \end{cases}$$

Framework

- But all fermions are massless, and short-distance weak interactions are not yet in sight.
- Introduce a scalar field of appropriate quantum numbers H $(1,2,1/2)$, to realize the desired spontaneous symmetry breaking (SSB) $SU(2)_L \times U(1)_Y \to U(1)_e$:

$$\mathscr{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H) + \mu^2 H^\dagger H - \frac{1}{2}\lambda (H^\dagger H)^2,\ \mu^2 > 0,\ \lambda > 0, \quad (8)$$

$$D_\mu H = \partial_\mu H - i g_2 \frac{1}{2} \sigma^I W^I_\mu H - i g_1 \frac{1}{2} B_\mu H$$

- According to teachings in QFT, we must include all possible operators of mass dimension not exceeding 4 that are consistent with gauge symmetry.

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Framework

The only gap to be filled is

$$\mathscr{L}_{\text{Yukawa}} = -\bar{Q}Y_u u\tilde{H} - \bar{Q}Y_d dH - \bar{L}Y_e eH + \text{h.c.} \quad (9)$$

$$\tilde{H} = i\sigma_2 H^*, \; Y_{u,d,e}: \; 3\times 3 \text{ matrices in generation space}$$

The complete Lagrangian for SM is thus

$$\mathscr{L}_{\text{SM}} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{matter}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}} + \text{gauge-fixing related} \quad (10)$$

SSB from scalar potential $SU(2)_L \times U(1)_Y \to U(1)_e$

In the basis where T^3 is diagonal, the two components of H are eigenstates ofth eelctric charge Q_e,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \; Q_e = \begin{pmatrix} +1 & \\ & 0 \end{pmatrix} \tag{11}$$

that Q_e is not broken, $Q_e \langle H \rangle = 0$, requires that only H^0 can develop a vacuum expectation value (vev):

$$\langle H^0 \rangle = \frac{v}{\sqrt{2}} \tag{12}$$

where v can be made real positive by a global transformation of $U(1)_Y$.

Consequences of SSB

$W^{\pm}$, Z gain mass (Higgs mechanism), identified with weak gauge bosons,
and A keeps massless being the photon:

$$W^{\pm}_{\mu} \frac{1}{\sqrt{2}}(W^1_{\mu} \mp i W^2_{\mu}), \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W^3_{\mu} \\ B_{\mu} \end{pmatrix}; \quad (13)$$

$$m_W = \frac{1}{2} g_2 v, \; m_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2} v, \; \frac{1}{e^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}; \quad (14)$$

$$\cos\theta_W = c_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \; \sin\theta_W = s_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \quad (15)$$

Consequences of SSB

Fermions also gain mass through Yukawa couplings:

$$\mathscr{L}_{\text{Yukawa}} \supset -\bar{u}M_u P_R u - \bar{d}M_d P_R d - \bar{e}M_e P_R e, \; M_{u,d,e} = \frac{v}{\sqrt{2}} Y_{u,d,e}, \qquad (16)$$

where u, d, e now refer to complete Dirac fields, and $M_{u,d,e}$ are general complex 3×3 matrices for 3 generations.

Diagonalization of $M_{u,d,e}$ yields the following physical parameters for 3 generations:

$$3+3 \text{ quark masses}, \; \text{CKM matrix with 3 angles and 1 CP-violating phase};$$
$$\text{3 masses of charged leptons}$$

No neutrino mass, no PMNS matrix for lepton mixing!

Status of SM

- All SM particles have been found.
  No firm experimental anomalies have been discovered, except that neutrino oscillation experiments call for neutrino masses and lepton flavor mixing, and
  existence of dark matter can be best accommodated by dark matter particles.
  Please refer to other lectures.
- General comments:
  (1) The mechanism for SSB has not yet been examined;
  (2) There is a hierarchy problem with the Higgs mass: too sensitive to high scale physics;
  (3) Many free parameters especially in the matter sector;
  etc.

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Symmetries in SM

- Poincare invariance and gauge symmetry completely fix SM interactions when
  (1) a gauge group is assumed and the representations of fields are assigned and
  (2) renormalizability is demanded.
  $\Rightarrow$ “symmetry determines dynamics!"
- It turns out that SM has more (approximate) symmetries, by accident, hence the name “accidental symmetries":
  baryon/lepton number conservation;
  approximate flavor symmetries;
  approximate custodial symmetry.

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Flavor symmetries

- Fermion fields enter only $\mathscr{L}_{\text{matter}} + \mathscr{L}_{\text{Yukawa}}$.
  In $\mathscr{L}_{\text{matter}}$, Q couples to itself but not to others, so do u, d, L, e.
  For n_f generations of fermions, this amounts to a large symmetry:

$$[U(n_f)]^5$$

- But in $\mathscr{L}_{\text{Yukawa}}$, Q couples to u and d and L to e.
  For general couplings this leaves only a small piece of symmetry:

$$U(1) \text{ for a common baryon number } B = 1/3 \text{ of } Q,\ u,\ d;$$
$$U(1) \text{ for electron-flavor number } L_e \text{ of } e,\ \nu_e;$$
$$U(1) \text{ for muon-flavor number } L_\mu \text{ of } \mu,\ \nu_\mu;$$
$$U(1) \text{ for tau-flavor number } L_\tau \text{ of } \tau,\ \nu_\tau$$

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$U(1)$ for tau-flavor number L_τ of τ, ν_τ

Flavor symmetries

The lepton number $L = L_e + L_\mu + L_\tau$.
(In SM B and L are actually violated due to anomaly, but $B - L$ is good.)
If neutrinos have Dirac mass, L is good but $L_{e,\mu,\tau}$ are violated by lepton mixing.
If neutrinos have Majorana mass, nothing is left.

Custodial symmetry

- It refers to symmetry that preserves equal strengths of (4-Fermi) charged-current and neutral-current weak interactions at low energies, or the relation

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1, \tag{17}$$

which has been confirmed to good precision.

- More generally,

$$\rho = \frac{\sum_i \left[T_i(T_i+1) - Y_i^2\right] v_i^2}{\sum_j 2Y_j^2 v_j^2}, \tag{18}$$

where v_i is the vev of the i-th scalar multiplet with $SU(2)_L \times U(1)_Y$ quantum numbers (T_i, Y_i).

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- It is possible to preserve $\rho = 1$ by arranging for relations among vevs, which in turn are based on relations assumed for scalar couplings, as in Georgi-Machacek model.
  It is much nicer that this comes out naturally. This indeed happens for doublet scalars with $T = |Y| = 1/2$.
- The tree-level result $\rho = 1$ in SM can be understood as follows. Define a Higgs matrix, a bi-doublet to be clear later,

$$\Phi = \frac{1}{\sqrt{2}}\left(\tilde{H}, H\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} H^{0*} & H^{+} \\ -H^{-} & H^{0} \end{pmatrix} \quad (19)$$

Note that $\tilde{H}$ transforms under $SU(2)_L$ as H but has an opposite Y, so that under $SU(2)_L \times U(1)_Y$,

$$\Phi \to U_L \Phi \exp\left(-\frac{i}{2}\sigma^3\theta\right) \quad (20)$$

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- $\mathscr{L}_{\text{Higgs}}$ can be written in terms of Φ as

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$$D_\mu\Phi = \partial_\mu\Phi - ig_2\frac{1}{2}\sigma^I W_\mu^I\Phi + ig_1\frac{1}{2}B_\mu\Phi\sigma^3,$$

where σ^3 appears in the B_μ term because $\tilde{H}$ has opposite Y to H.

- The scalar potential has actually a larger symmetry, i.e., under $SU(2)_L \times SU(2)_R$,

$$\Phi \to U_L\Phi U_R \quad \text{bi-doublet} \qquad (22)$$

This is not surprising in itself: H has 4 real components and the scalar potential depends only the 'length' of H, i.e., is invariant under $SO(4) \sim SU(2) \times SU(2)$.

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- $SU(2)_L \times SU(2)_R$ is spontaneously broken by $\langle\Phi\rangle = (v/\sqrt{2})\mathbf{1}_2$ to the diagonal $SU(2)_{L+R}$, which is called custodial symmetry. (Sometimes also the name for $SU(2)_R$.)
- If there is no $U(1)_Y$, i.e., $g_1 = 0$, the kinetic term in $\mathscr{L}_{\text{Higgs}}$ is also invariant under $SU(2)_L \times SU(2)_R$.
  The survived $SU(2)_{L+R}$ guarantees equal mass of all three gauge bosons.
- With $U(1)_Y$, $SU(2)_L \times SU(2)_R$ is explicitly broken by the kinetic term resulting in the mixing between W^3_μ and B_μ.
  But there is still a footprint of custodial symmetry: $\rho = 1$.
- Its radiative corrections due to Higgs-gauge couplings are also $\propto g_1^2$; actually, $[\rho - 1]_{\text{Higgs}} \propto g_1^2 \ln(m_h^2/m_Z^2)$.

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- But if $Y_u = Y_d \equiv Y_q$, we can write the quark Yukawa couplings as

$$\mathscr{L}_{\text{Yukawa}} = -\bar{Q} Y_q \Phi q + \text{lepton term} + \text{h.c.}, \; q = \begin{pmatrix} u \\ d \end{pmatrix} \tag{23}$$

  It is invariant under $SU(2)_L \times SU(2)_R$ by assigning

$$q \to U_R^\dagger q \tag{24}$$

- Thus, radiative corrections due to Yukawa couplings vanish for $m_t = m_b$. Actually $m_t \gg m_b$, $\left[\rho - 1\right]_{\text{top}} \propto G_F^2 m_t^2$.

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