Dark Matter, Dark Energy & Neutrino Mass
暗物质，暗能量和中微子质量

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理论物理前沿暑期讲习班——暗物质，中微子与粒子物理前沿
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Lecture 1: Introduction to Particle Physics and Cosmology

Lecture 2: Some Basic Backgrounds of the Standard Model of Particle Physics and Cosmology

Lecture 3: Neutrino Mass Generation

Lecture 4: Theoretical Understanding of Dark Matter Detections

Lecture 5: Dark Energy and Gravitational Waves

Lecture 3: Neutrino Mass Generation

Outline

- Introduction
- A brief overview of neutrino mass generation
- **A special class of models to generate M_ν**
    - Neutrino mass generation
    - **$0\nu\beta\beta$ decays**
    - **Other physics**

Introduction

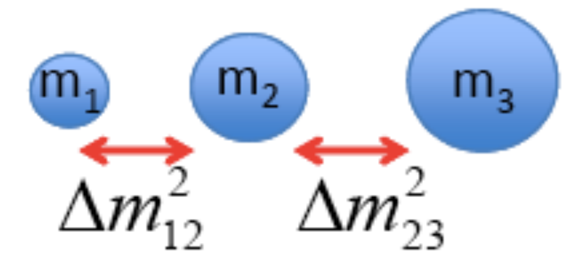
Weak eigenstate (α = e, μ, τ)

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle$$

Mass eigenstate (i = 1, 2, 3)

- **PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix**

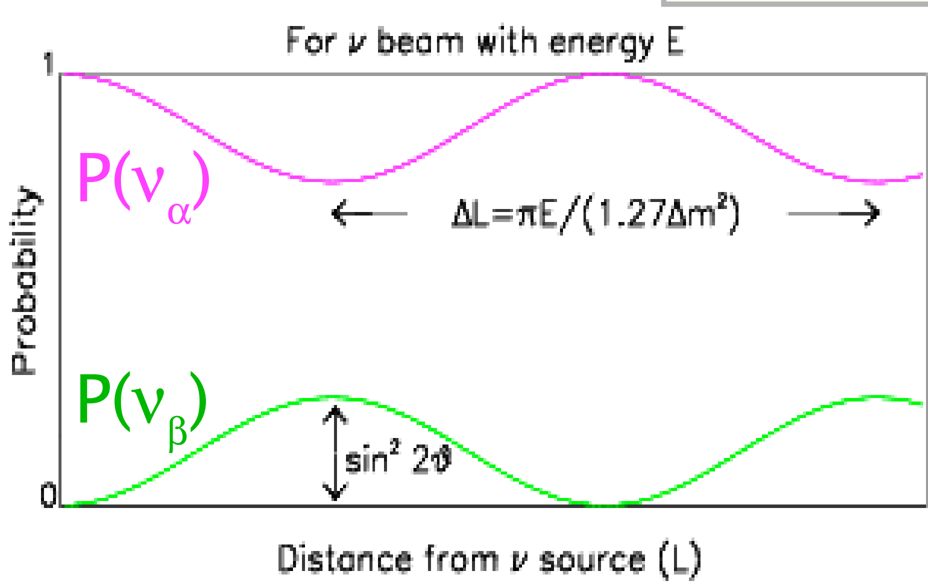
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



$$U_{\mathrm{PMNS}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}}_{\text{Atmospheric}} \underbrace{\begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}}_{\text{Reactor}} \underbrace{\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar}}$$

Two-neutrino case:

$$P(\nu_\alpha \to \nu_\beta) \;=\; |\langle \nu_\beta | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2[\mathrm{eV}^2]L[\mathrm{km}]}{E[\mathrm{GeV}]}\right)$$



$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$$

$$P(\nu_\alpha \Rightarrow \nu_\beta) = \sin^2(2\theta)\sin^2\left(\Delta m^2_{12} L / 4E\right)$$

disappearance of ν_α

appearance of ν_β

θ	: mixing angle
Δm^2	: mass squared difference
L [km]	: the distance traveled
E (GeV)	: the energy of neutrino

中微子振盪

如果中微子有質量，則不同類中微子間會產生振盪現象。

舉ν_μ及ν_τ為例：

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

ν_μ, ν_τ : 弱作用本徵態

ν_1, ν_2 : 質量本徵態

在t=0時，ν_μ在大氣中產生，則

$$|\nu_\mu(0)\rangle \equiv |\nu_\mu\rangle = \cos\theta|\nu_1(0)\rangle + \sin\theta|\nu_2(0)\rangle$$

$$|\nu_\tau\rangle = -\sin\theta|\nu_1(0)\rangle + \cos\theta|\nu_2(0)\rangle$$

到了時間t，上述狀態演變為

$$|\nu_\mu(t)\rangle = \exp(-iE_1 t/\hbar)\cos\theta|\nu_1(0)\rangle + \exp(-iE_2 t/\hbar)\sin\theta|\nu_2(0)\rangle,$$

Using $t \approx L$ and $E - P_i = E - \sqrt{E^2 - m_i^2} \approx m_i^2/2E$,

如果 $m_1 \neq m_2$，則$E_1 \neq E_2$，因此$|\nu_\mu(t)\rangle$不再垂直於 $|\nu_\tau\rangle$!

$$\Delta m_{21}^2 = m_2^2 - m_1^2$$

$$P(\nu_\alpha \Rightarrow \nu_\beta) = \sin^2(2\theta)\sin^2\left(\Delta m_{12}^2 L / 4E\right)$$

$$m_1 \neq m_2$$

Neutrino Oscillations

☞ Neutrinos have masses and mix with each other.

$$P(\nu_l \to \nu_{l'}) = \sum_j |U_{lj}|^2 |U_{l'j}|^2 + 2\sum_{j>k} |U_{l'j} U^*_{lj} U_{lk} U^*_{l'k}|^2 \cos\left(\frac{\Delta m^2_{jk}}{2p} L - \varphi_{l'l;jk}\right)$$

$$\Delta m^2_{jk} = m^2_j - m^2_k$$

$$\varphi_{l'l;jk} = \arg(U_{l'j} U^*_{lj} U_{lk} U^*_{l'k})$$

ν_{atm} SK UP-DOWN ASYMMETRY

θZ–, L/E– dependences of μ–like events

Dominant $\nu_\mu \to \nu_\tau$ **K2K, MINOS, T2K; CNGS (OPERA)** → $|\Delta m^2_{32}|$, $\sin^2\theta_{23}$

$\nu_\odot$ **Homestake, Kamiokande, SAGE, GALLEX/GNO**
Super-Kamiokande, SNO, BOREXINO; KamLAND → Δm^2_{21}, $\sin^2\theta_{12}$

$\bar{\nu}_e$ (from reactors): **Daya Bay, RENO, Double Chooz**

Dominant $\nu_e \to \nu_{\mu,\tau}$ → $\sin^2\theta_{13}$

T2K, MINOS (ν_μ from accelerators): $\nu_\mu \to \nu_e$

Experiments on solar neutrinos

$$\Delta m_{21}^2 = \left(7.54^{+0.26}_{-0.22}\right) \times 10^{-5}\,\mathrm{eV}^2$$

$$\Delta m_{sun}^2 = |m_2|^2 - |m_1|^2 > 0$$

Neutrinos born in Cosmic ray collisions and on earth

$$|\Delta m_{31}^2| = \begin{cases} (2.43 \pm 0.06) \times 10^{-3}\,\mathrm{eV}^2 & \text{normal hierarchy,} \\ (2.38 \pm 0.06) \times 10^{-3}\,\mathrm{eV}^2 & \text{inverted hierarchy,} \end{cases}$$

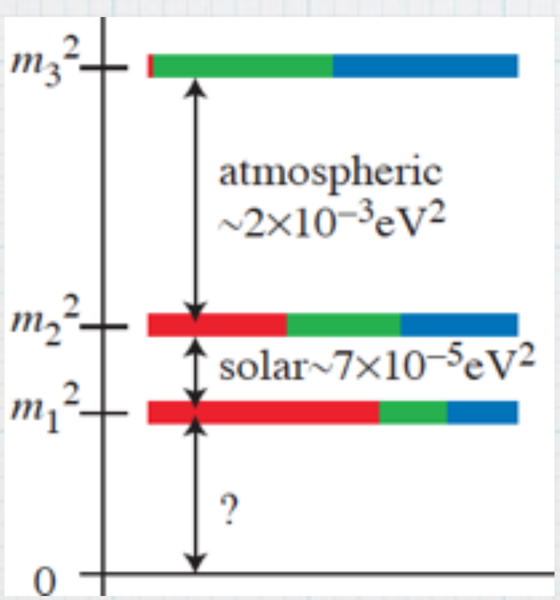
$$\Delta m_{atm}^2 = |\Delta m_{31}^2|$$

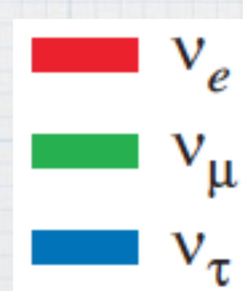
$$\Delta m_{31}^2 = |m_3|^2 - |m_1|^2$$

Normal hierarchy

$$|m_{\nu_1}| < |m_{\nu_2}| \ll |m_{\nu_3}|$$

$$m_1 \simeq 0,\ m_2^2 \simeq \Delta m_\odot^2, \text{ and } m_3^2 \simeq \Delta m_{atm}^2.$$



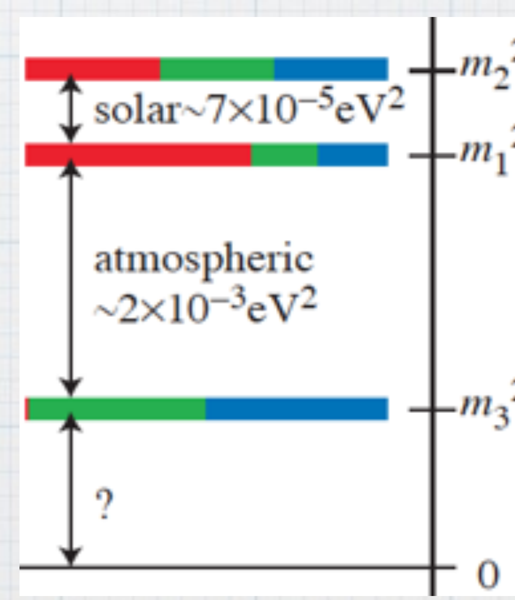


Δm^2

Inverted hierarchy

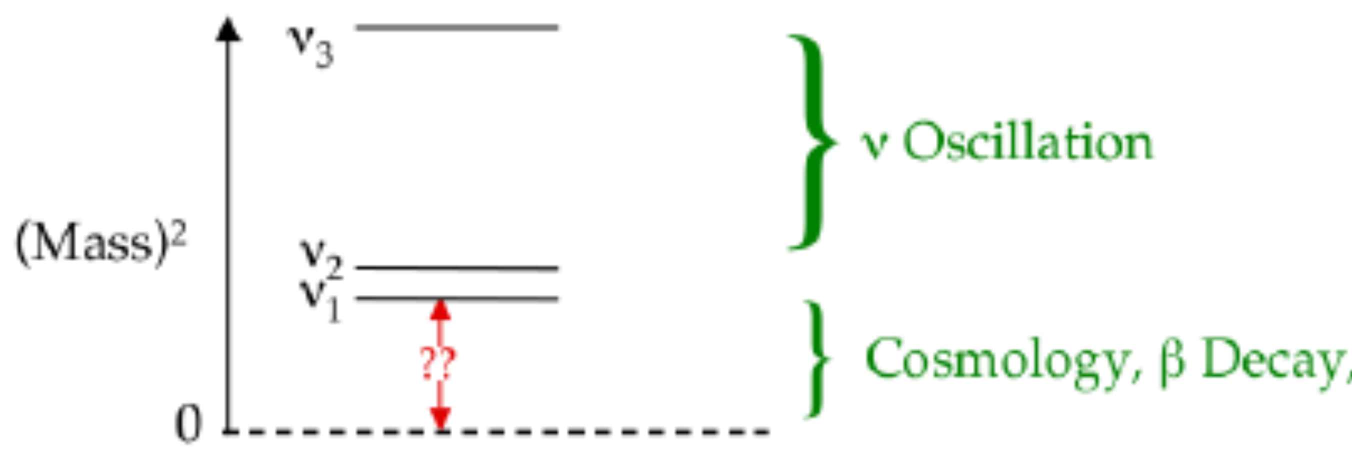
$$|m_{\nu_1}| \simeq |m_{\nu_2}| \gg |m_{\nu_3}|$$

$$m_1^2 \simeq m_2^2 \simeq \Delta m_{atm}^2 \gg m_3^2$$



Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

The Troitzk and Mainz ^{3}H β-decay experiments $m_{\bar{\nu}_e} < 2.05$ eV (95% C.L.) → KATRIN → 0.2 eV

$$^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

Improved β energy resolution requires a ***BIG*** β spectrometer.

KATRIN

Improved β energy resolution requires a ***BIG*** β spectrometer.

KATRIN

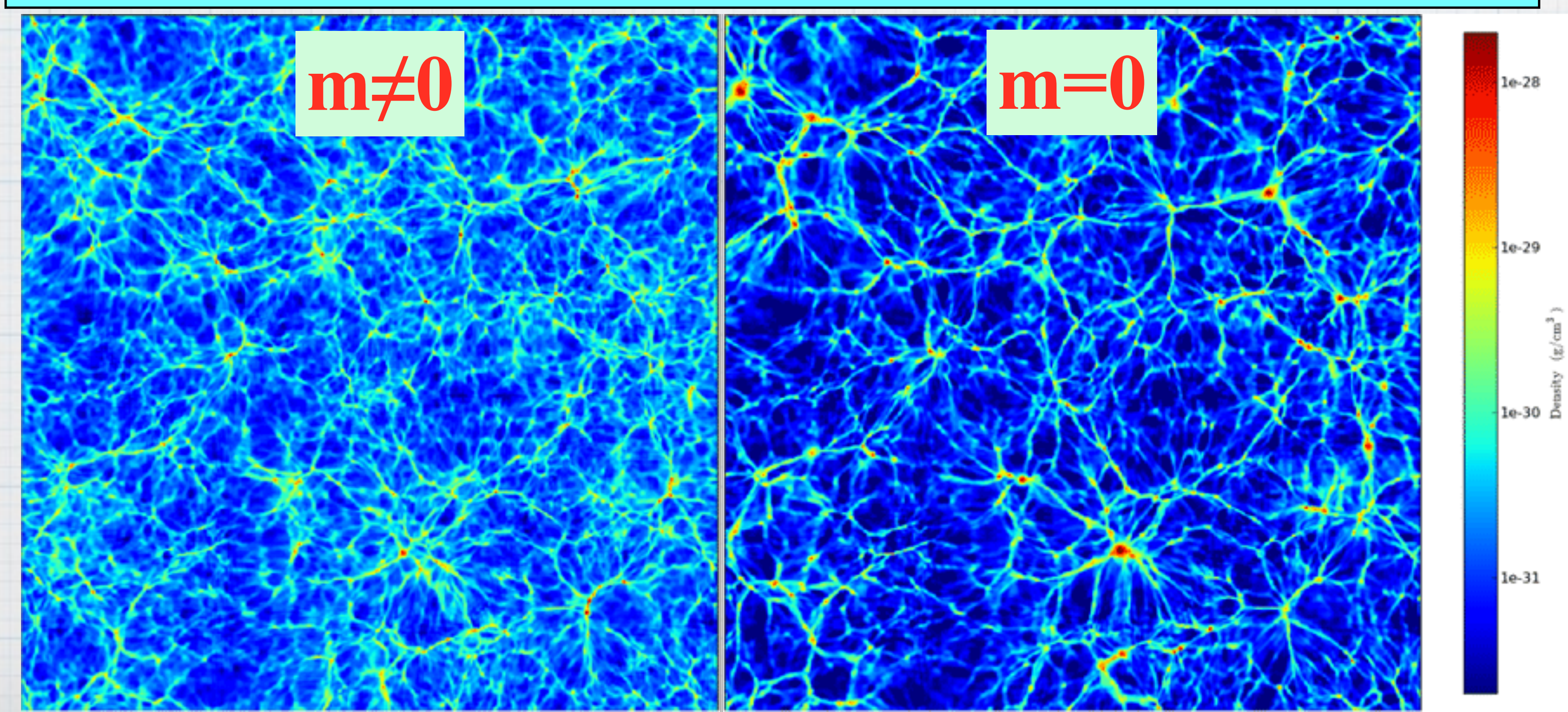
The cosmological bound on m_ν

- Number density

$$n_\nu = \int \frac{d^3p}{(2\pi)^3} f_\nu(p,T_\nu) = \frac{3}{11} n_\gamma = \frac{6\zeta(3)}{11\pi^2} T^3_{CMB}$$

- Energy density

$$\rho_{\nu_i} = \int \sqrt{p^2 + m^2_{\nu_i}} \frac{d^3p}{(2\pi)^3} f_\nu(p,T_\nu) \rightarrow \begin{cases} \frac{7\pi^2}{120}\left(\frac{4}{11}\right)^{4/3} T^4_{CMB} & \text{Massless} \\ m_{\nu_i} n_\nu & \text{Massive } m_\nu \gg T \end{cases}$$



The cosmological bound on m_ν

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Contribution to the energy density of the Universe

$$\Omega_\nu h^2 = 1.7 \times 10^{-5}$$ Massless

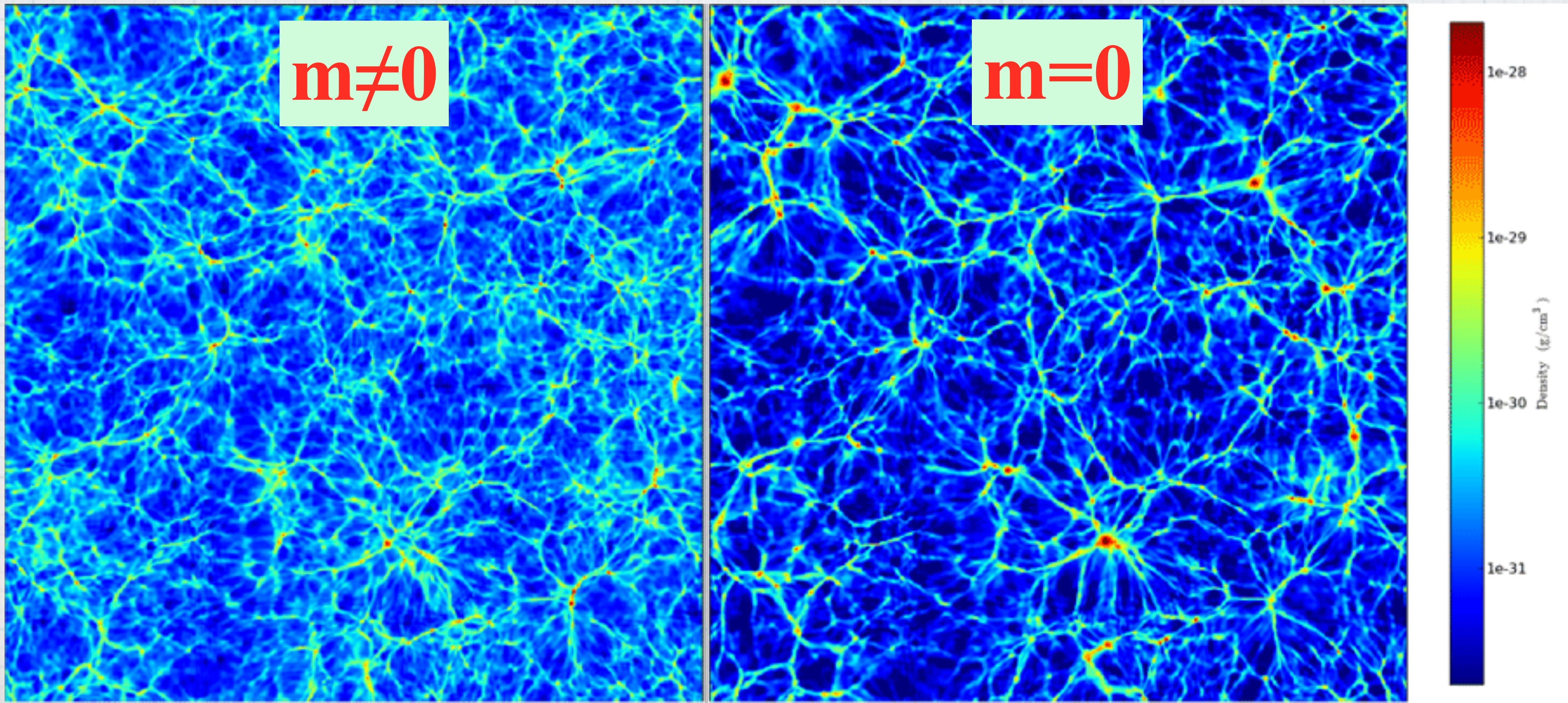
$$\Omega_\nu h^2 = \frac{\sum_i m_i}{94.1\ \mathrm{eV}}$$ Massive $m_\nu \gg T$

The best bound to absolute values of neutrino masses from Cosmology

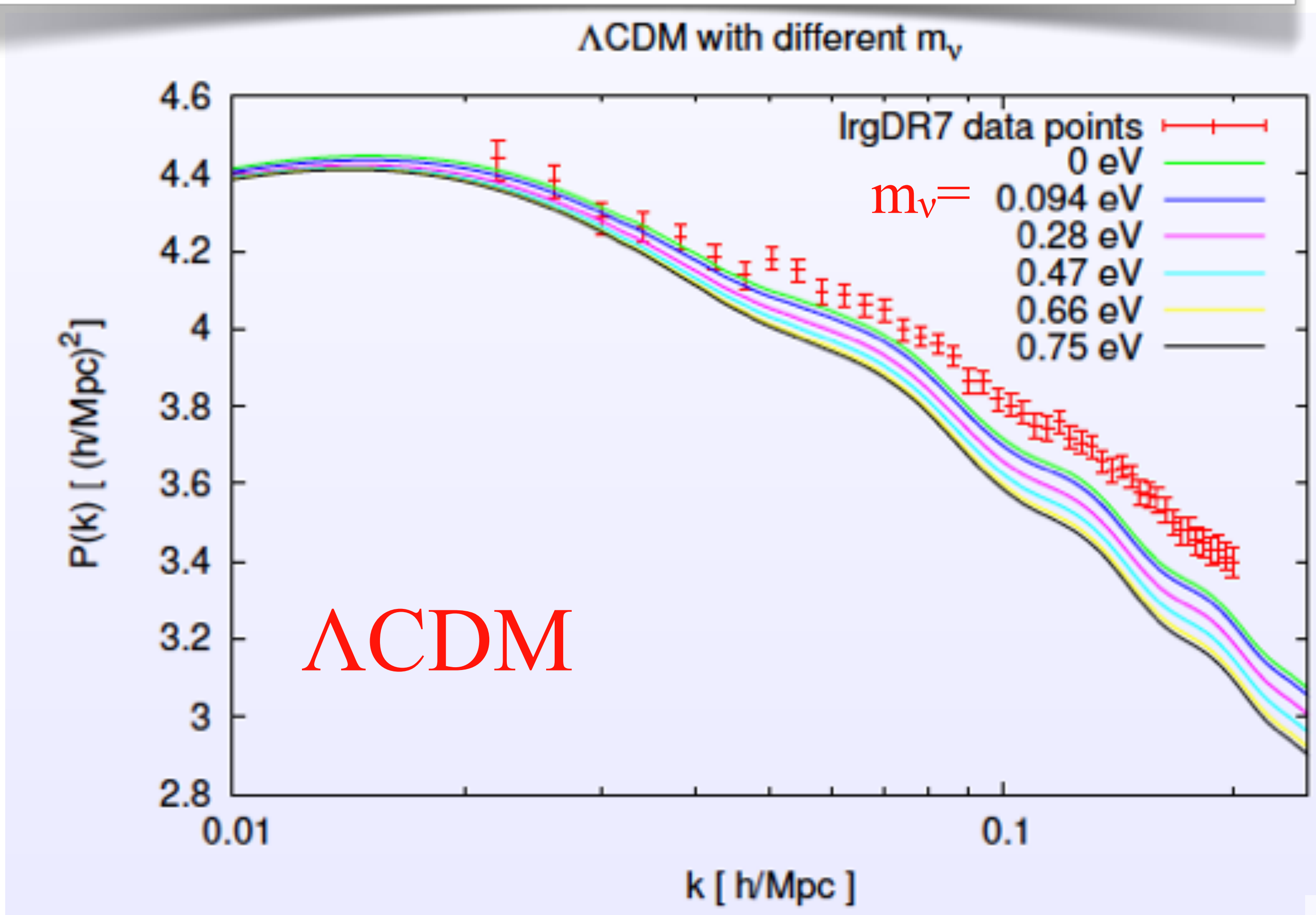
$$\sum_i m_{\nu_i} < 0.23\ \mathrm{eV}$$

95%CL (Planck+other data)

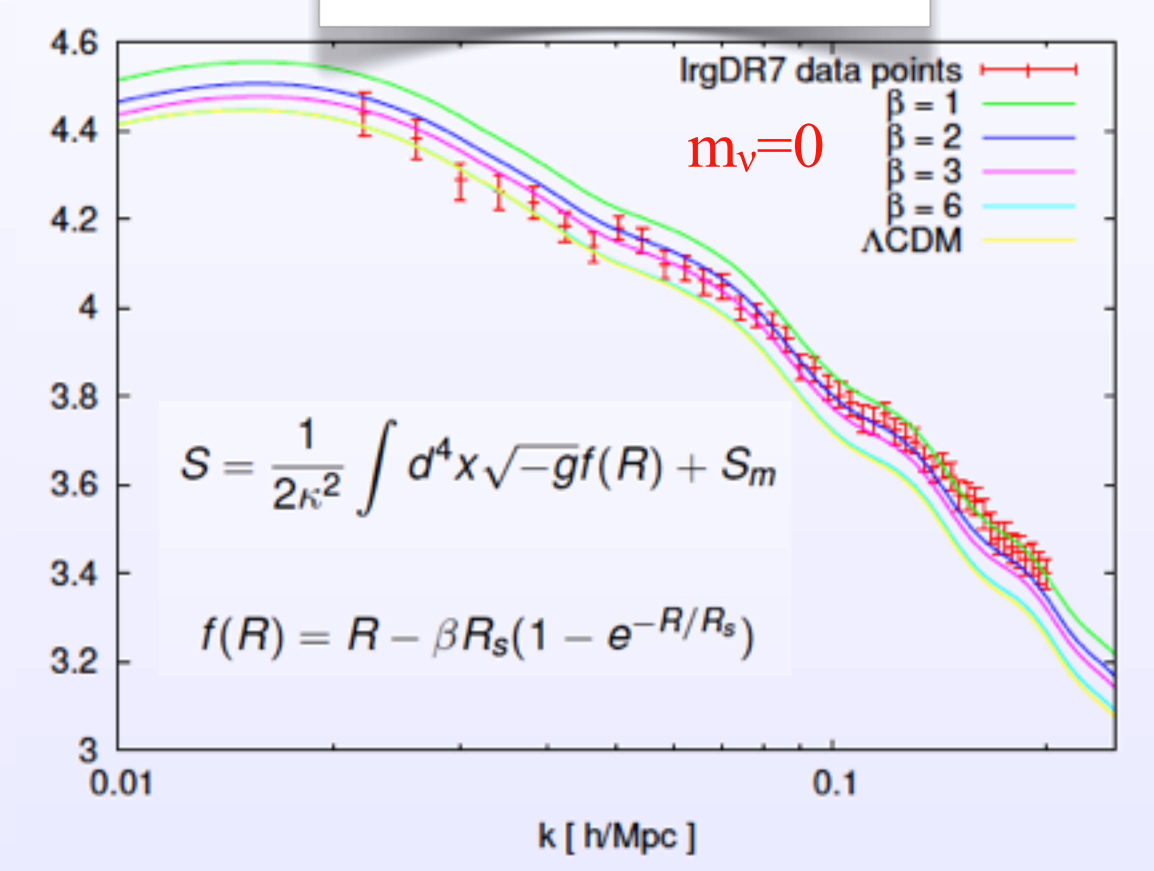
ΛCDM

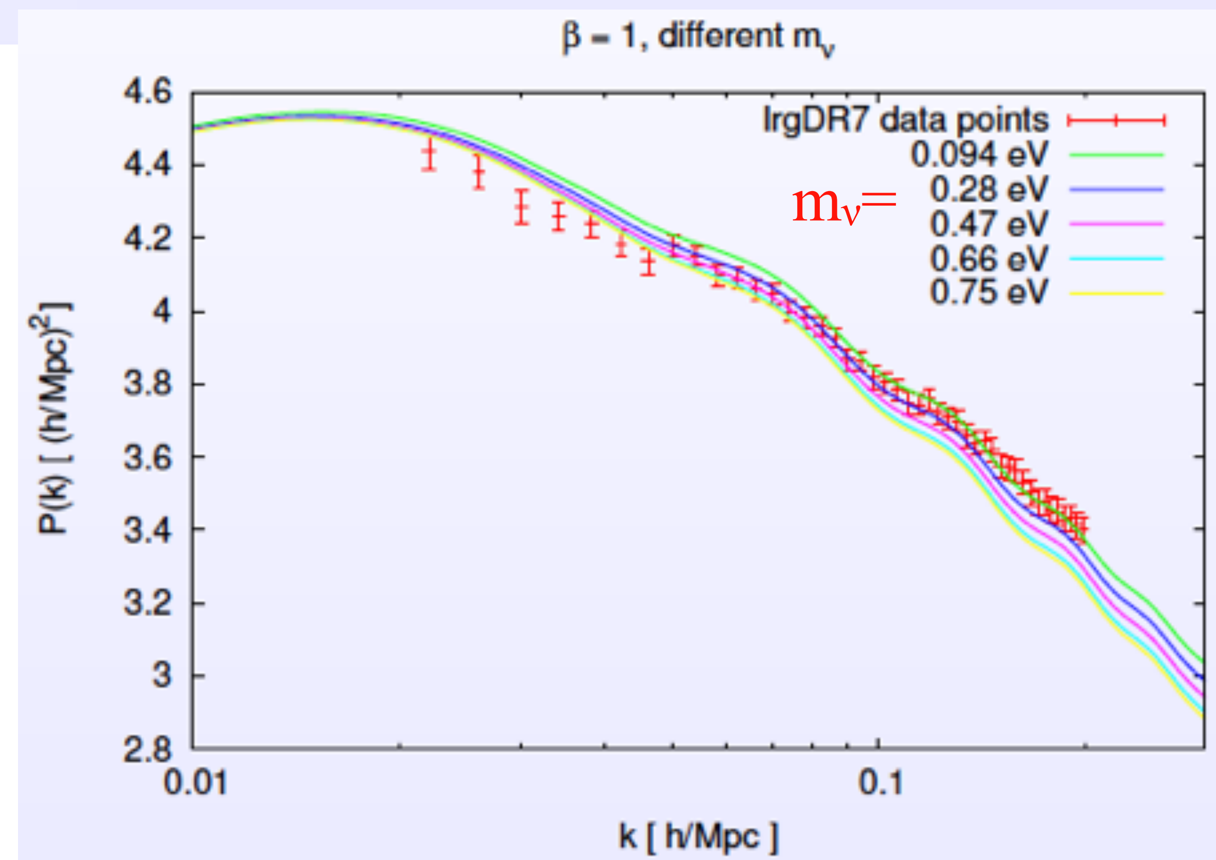


Matter power spectrum in ΛCDM and f(R)



Exponential f(R) model





$f(R)$ model	Σm_ν
ΛCDM	< 0.200 eV
Starobinsky	$0.248^{+0.203}_{-0.232}$ eV
Exponential	< 0.214 eV

CQG+CC.Lee, J.L.Shen, PLB740,285(2015)

In terms of the PMNS mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \rightarrow$$

$$V_{PMNS} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

PDG2016

$\sin^2\theta_{12} = 0.308 \pm 0.017$, $\sin^2\theta_{23} = 0.437^{+0.033}_{-0.023}$, $\sin^2\theta_{13} = 0.0234^{+0.020}_{-0.019}$, $\delta = (1.39^{+0.38}_{-0.27})\pi$ ← **NH**

$\sin^2\theta_{12} = 0.308 \pm 0.017$, $\sin^2\theta_{23} = 0.455^{+0.039}_{-0.031}$, $\sin^2\theta_{13} = 0.0240^{+0.019}_{-0.020}$, $\delta = (1.31^{+0.29}_{-0.33})\pi$ ← **IH**

center values

$\theta_{12} \approx 34^o$

$\theta_{23} \approx 42^o$

$\theta_{13} \approx 9^o$

$\delta \approx 1.4\pi$

Bimaximal Matrix

$$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

$\theta_{12} = 45^o$, $\theta_{23} = 45^o$, $\theta_{13} = 0$

Tribimaximal Matrix

$$\begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

0 ← **Daya-Bay**

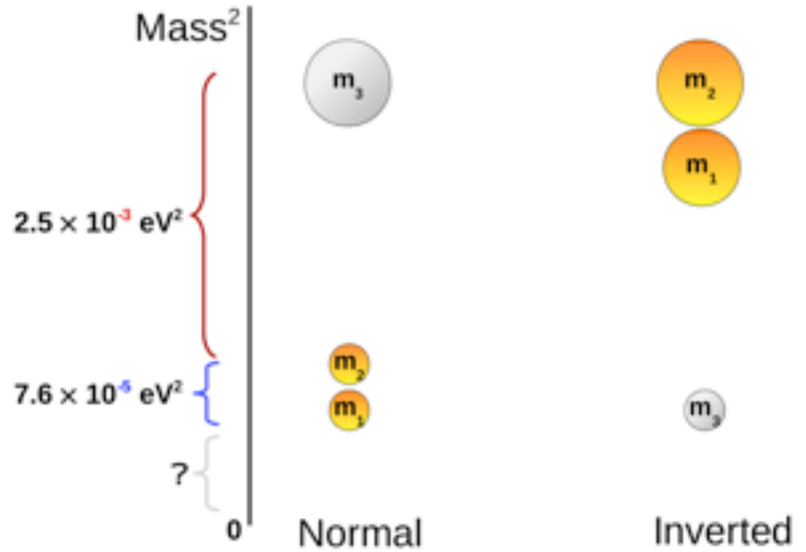
$\theta_{12} = 35.3^o$, $\theta_{23} = 45^o$, $\theta_{13} = 0$

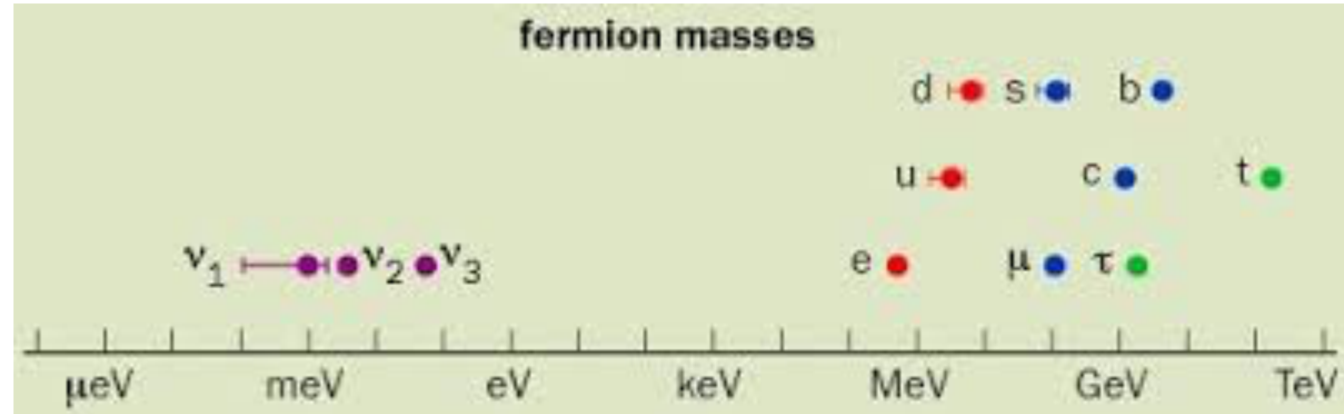
$|U_{PMNS}| \sim$ 

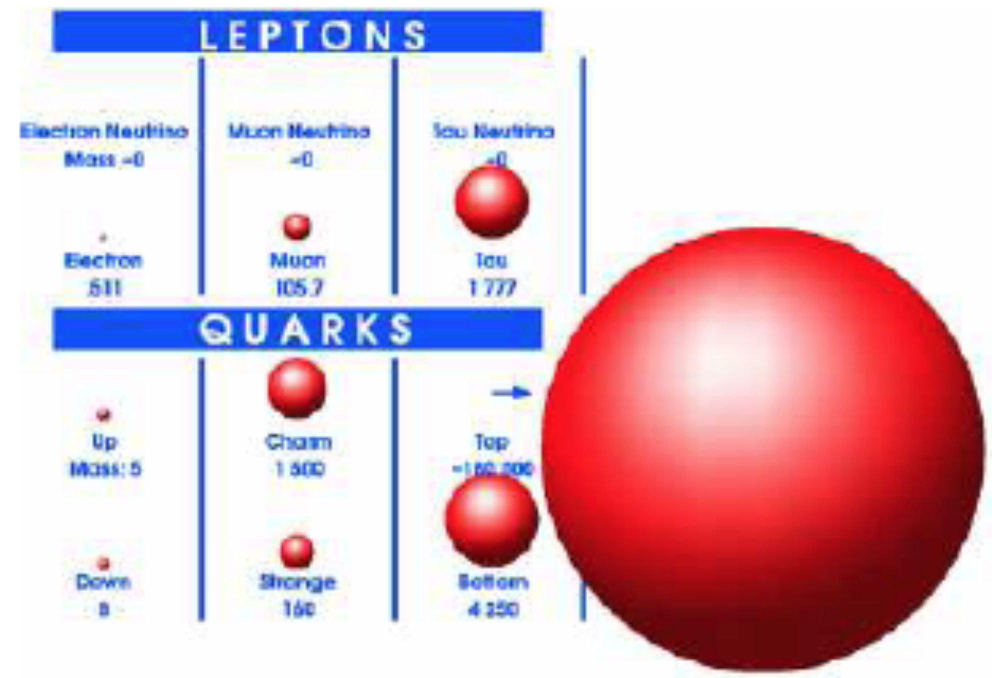
$|V_{CKM}| \sim$ 

A brief overview of neutrino mass generation

Fermion Mass Problem







$$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q,\ q = u,c,t,d,s,b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

Questions: Where do the

- quark mass hierachy
- small neutrino masses
- small quark mixings and
- large lepton mixings

originate from?

如果中微子有質量，它們的質量為什麼遠小於相對的帶電輕子及夸克的質量？

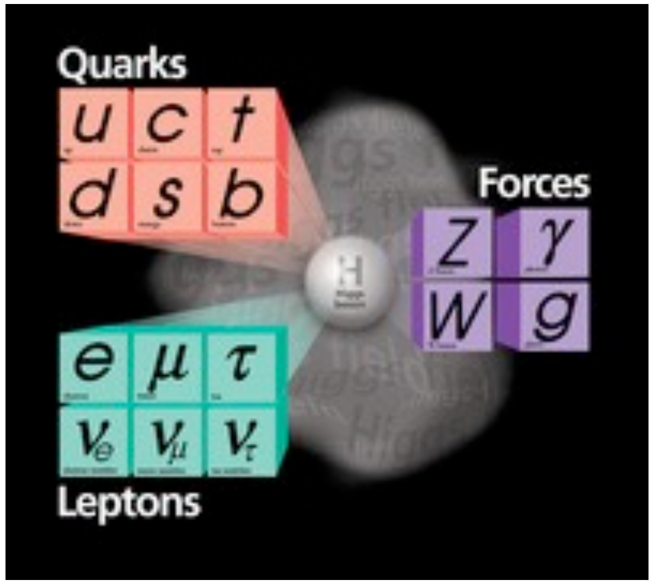
- **The standard model:** $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$Q_L : \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad L_L : \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$U_R : u_R \quad c_R \quad t_R$$

$$D_R : d_R \quad s_R \quad b_R \qquad E_R : e_R \quad \mu_R \quad \tau_R$$

Higgs : H^0 Gauge Bosons : $W^\pm$, Z, γ, g



Yukawa interactions: $Y = \sum_{i,j} h^d_{ij} \bar{Q}_L \phi D_R + h^u_{ij} \bar{Q}_L \tilde{\phi} U_R + h^e_{ij} \bar{L}_L \phi E_R + h.c.$

$\Phi=\Phi_0=(-\mu^2/2\lambda)^{1/2}$
SSB

$$V_L^{d+} M_d V_R^d = M_d^{diag.} \;,\; D_{L(R)_j} = (V^d_{L(R)})_{ji} D'_{L(R)_i}$$

$$V_L^{u+} M_u V_R^u = M_u^{diag.} \;,\; U_{L(R)_j} = (V^u_{L(R)})_{ji} U'_{L(R)_i}$$

- **What about neutrinos?**

- **Do neutrinos get their masses like charged fermions?**

Neutrino masses: Dirac or Majorana

Dirac neutrino mass:

$$\mathcal{L}_D = -m_D \overline{\nu_L} \nu_R + \text{h.c.}$$

☞ Introduce ν_R (not in the SM)

☺ the lepton number L is conserved

Majorana neutrino mass:

$$\mathcal{L}_M = -m_M \overline{\nu^c} \nu \quad + \text{h.c.} \qquad \nu \leftrightarrow \bar{\nu}$$

☞ FORBIDDEN IN THE SM.

• the lepton number L is violated

Neutrino oscillations measure Δm^2, but they do not provide information about the absolute neutrino spectrum and cannot distinguish pure Dirac and Majorana neutrinos.

In the SM:

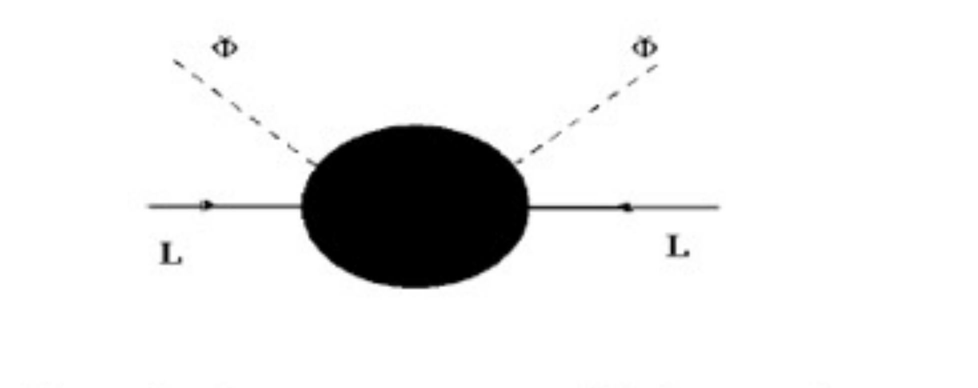
- No Dirac mass term (no right-handed neutrino).
- No Majorana mass term either (ν_L is an SU(2) doublet).

	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
$L_a = (\nu_a, l_a)_L^T$	$(1, 2, -1)$
e^c_{aL}	$(1, 1, 2)$
$Q_a = (u_a, d_a)_L^T$	$(3, 2, 1/3)$
u^c_{aL}	$(\bar{3}, 1, -4/3)$
d^c_{aL}	$(\bar{3}, 1, 2/3)$
Φ	$(1, 2, 1)$

Table 1: Matter and scalar multiplets of the Standard Model (SM)

■ **Effective Dim-5 operator:**

S. Weinberg, Phys. Rev. D22, 1694 (1980).



Dimension five operator responsible for neutrino mass

$$O=(\lambda_0/M_X)L\Phi L\Phi$$

↓ *SSB*

$$m_\nu = \lambda_0 \frac{\langle\Phi\rangle^2}{M_X},$$ (Majorana)

For $\lambda_0 \sim 1$, $<\Phi>\sim 100$ GeV, $M_X \sim M_P$ → $m_\nu \sim 10^{-6}$ eV (too small)

$\Delta m^2_{21} \sim 7\times10^{-5}\ eV^2$ $|\Delta m^2_{31}| \sim 2\times10^{-3}\ eV^2$

Neutrino masses beyond the SM:

■ **If there are right handed neutrinos ν_R :** $\nu_R=(1,1,0)$

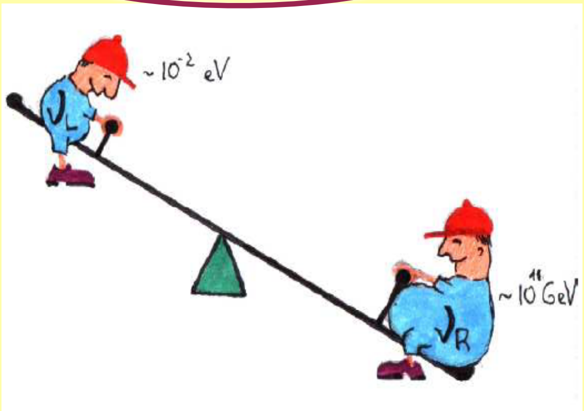
$$\mathcal{L}_Y = Y_\nu \bar{L}\Phi\nu_R + h.c. \quad \Rightarrow \quad m_\nu^D = Y_\nu < \Phi >$$

The observed neutrino masses would require $Y_\nu \leq 10^{-13} - 10^{-12}$ (unnatural)?

■ **Majorana mass for ν_R:** $M_R \nu_R^T C^{-1} \nu_R + h.c.$

Type-I see-saw mechanism:

$$\mathcal{M}_\nu = -m_D^T M_R^{-1} m_D.$$



(naturally small?+Majorana)

The Seesaw Mechanism

The Seesaw mechanism refers to the neutrino mass matrix of the form:

$$L_m = -\frac{1}{2}(\nu_L^c, \nu_R)\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

For one generation, if $M_R >> m_D$, the diagonal masses are

$$m_\nu \approx -m_D M_R^{-1} m_D^T, \quad m_N \approx M_R$$

$$m_{\nu_1} = m_e^2/M_R$$
$$m_{\nu_2} = m_\mu^2/M_R$$
$$m_{\nu_3} = m_\tau^2/M_R$$

How large M_R needs to be?

For $m_{\nu_1} = 0.1\text{eV}, M_R = 2.5\text{TeV}$

For $m_{\nu_2} = 0.1\text{eV}, M_R = 10^8\text{GeV}$

For $m_{\nu_3} = 0.1\text{eV}, M_R = 3 \times 10^{10}\text{GeV}.$

A very nice way to explain why light neutrino masses are so much lighter than their charged lepton partners

(Minkowski (1977); Gell-Mann, Ramond, and Slansky (1979); Yanagida (1979); Glashow (1980); Mohapatra and Senjanovic(1980))

Inverse Seesaw

Basis ***(ν, ν^c, S)*****:**

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix},$$

Mohapatra & Valle, 1986

After EWSB, the effective light neutrino mass matrix is given by

$$M_\nu = m_D {M^T}^{-1} \mu M^{-1} m_D^T.$$

"Inverse" seesaw, because:

$$M_\nu \Rightarrow 0 \qquad \text{IF} \qquad \mu \Rightarrow 0$$

Effective Operators

d = 5:

$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_i H)(L_j H)$$

Weinberg, 1980

d = 7:

$$\mathcal{O}_2 \propto LLLe^cH$$

$$\mathcal{O}_3 \propto LLQd^cH$$

$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^cH$$

$$\mathcal{O}_8 \propto L\bar{e}^c\bar{u}^cd^cH$$

$$\mathcal{O} \propto (LH)(LH)(H_uH_d)$$

Babu & Leung, 2001

de Gouvea & Jenkins, 2007

d = 9:

$$\mathcal{O}_5 \propto LLQd^cHHH^\dagger$$

$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^cHH^\dagger H$$

$$\mathcal{O}_7 \propto LQ\bar{e}^c\bar{Q}HHH^\dagger$$

$$\mathcal{O}_9 \propto LLLe^cLe^c \quad .$$

$$\mathcal{O}_{10} \propto LLLe^cQd^c$$

$$\mathcal{O}_{11} \propto LLQd^cQd^c$$

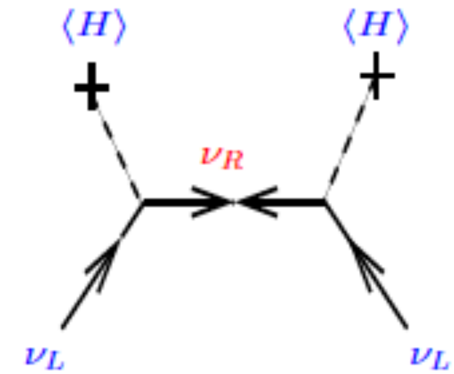
$\cdots\cdots$

Example realization:

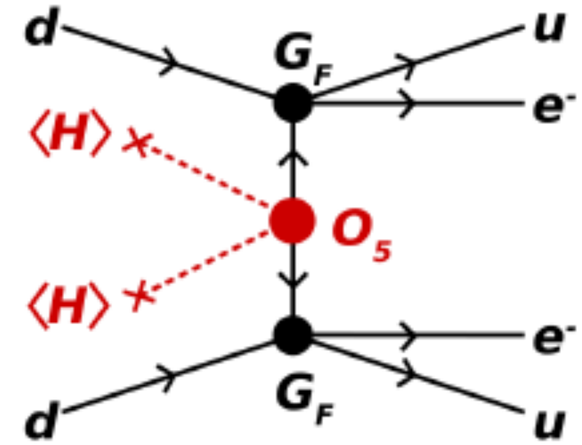
d = 5: $\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_iH)(L_jH)$

$\Lambda \simeq M_{\nu_{R_k}}$

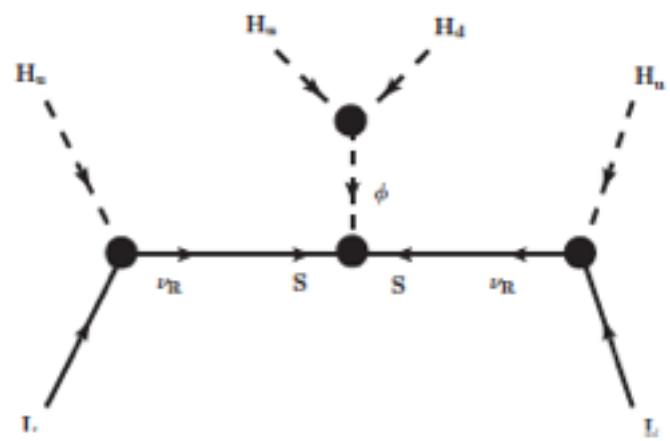
$c_{ij} \propto Y^{\nu}_{ik} Y^{\nu}_{jk}$



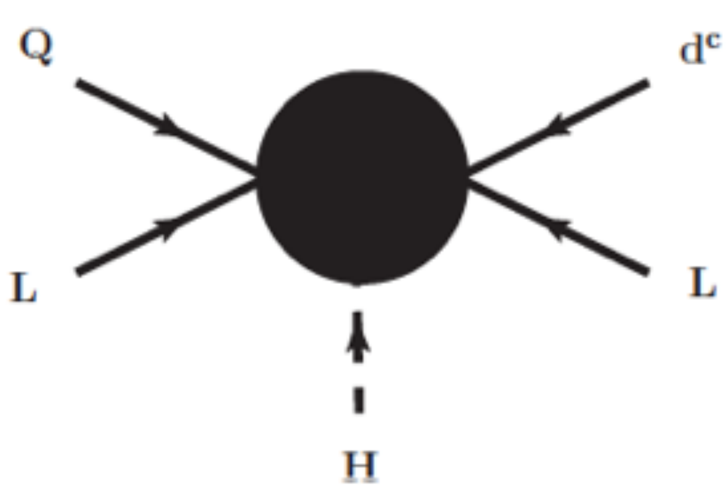
0νββ decay:

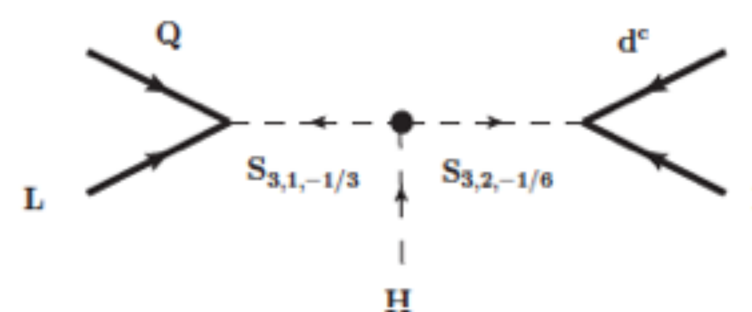


d = 7: $\mathcal{O} \propto (LH)(LH)(H_uH_d)$



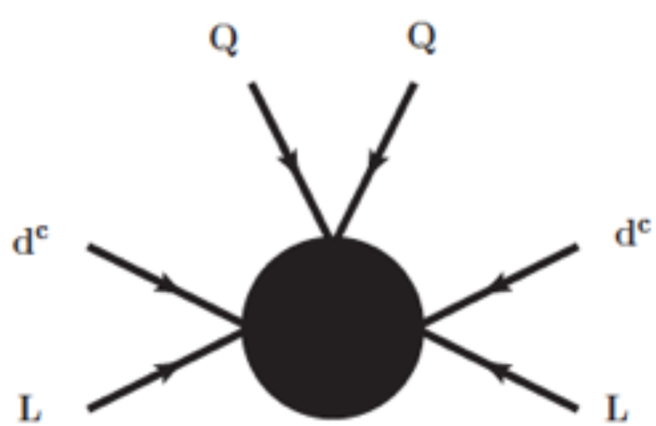
$\mathcal{O}_3 \propto LLQd^cH$

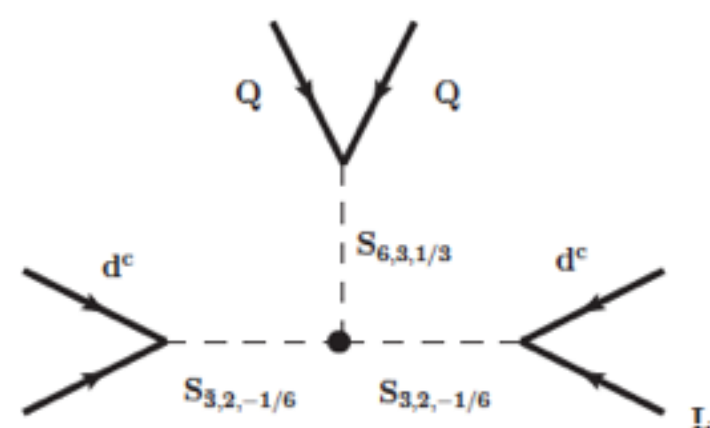




$S_{3,1,-1/3}$ - singlet leptoquark
$S_{3,2,1/6}$ - doublet leptoquark

d = 9: $\mathcal{O}_{11} \propto LLQd^cQd^c$





- 產生Majorana中微子質量簡介

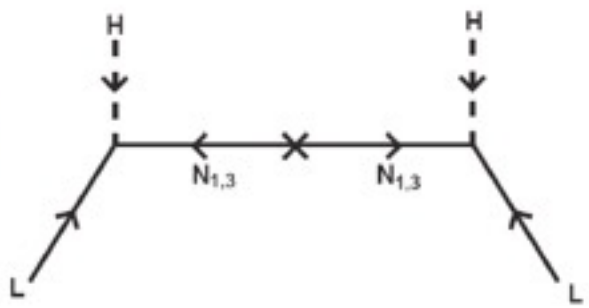
Without ν_R

Tree level

Minkowski 1977; ... **Xiao-Gang He** et al 1989

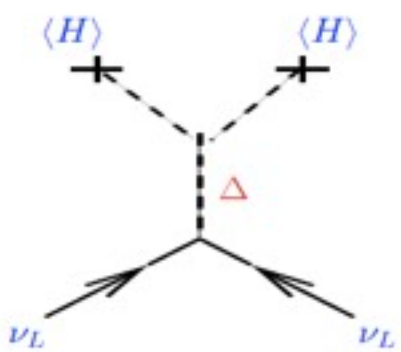
Type-III See-Saw Xiao-Gang He

Type (I,III) seesaw



$N_1 : (1,1,0)$

$N_3 : (1,3,0)$



Type II seesaw

Schechter & Valle, 1980, 1982
Cheng & Li, 1980
Mohapatra, Senjanovic, 1981
...

$$\Delta \equiv \begin{pmatrix} H^- & -\sqrt{2}\, H^0 \\ \sqrt{2}\, H^{--} & -H^- \end{pmatrix} = (1,3,2)$$ **scalar triplet**

$$\mathcal{L}_{\rm TypeII} = {\rm Tr}(D_\mu \Delta)^\dagger (D^\mu \Delta) - (Y_\nu\, l_L^T\, C\, i\sigma_2\, \Delta\, l_L + {\rm h.c.}) - V(H,\Delta),$$

$$V(H,\Delta) = M_\Delta^2\, Tr\Delta^\dagger\Delta + (\mu\, H^T\, i\sigma_2\, \Delta^\dagger H + h.c.) + \lambda_1\, (H^\dagger H) Tr\Delta^\dagger\Delta + \lambda_2\, (Tr\Delta^\dagger\Delta)^2 + \lambda_3\, Tr\,(\Delta^\dagger\Delta)^2 + \lambda_4\, H^\dagger\Delta\Delta^\dagger H.$$

$$M_\nu = \sqrt{2}\, Y_\nu\, v_\Delta$$

$$v_\Delta = \frac{\mu\, v^2}{\sqrt{2}\, M_\Delta^2}$$

$M_\Delta \sim 250$ GeV, $\mu \sim 0.1$ eV, $Y_\nu \sim 1$

$\longrightarrow$ $M_\nu \sim 0.1$ eV

Type III seesaw

Foot, Lew, X.G.He and Joshi, 1989

$$\Sigma = \begin{pmatrix} \Sigma_L^0/\sqrt{2} & \Sigma_L^+ \\ \Sigma_L^- & -\Sigma_L^0/\sqrt{2} \end{pmatrix}$$

the triplet Σ=(1, 3, 0)

$$\mathcal{L} = Tr[\overline{\Sigma} i \not{D} \Sigma] - \frac{1}{2} Tr[\overline{\Sigma} M_\Sigma \Sigma^c + \overline{\Sigma^c} M_\Sigma^* \Sigma] - \tilde{H}^\dagger \overline{\Sigma^c} \sqrt{2} Y_\Sigma L_L - \overline{L_L} \sqrt{2} {Y_\Sigma}^\dagger \Sigma^c \tilde{H}$$

The mass terms are given by:

$$\mathcal{L}_{\Downarrow} = -(\overline{l_R}\ \overline{\Psi}_R) \begin{pmatrix} m_l & 0 \\ Y_\Sigma v & M_\Sigma \end{pmatrix} \begin{pmatrix} l_L \\ \Psi_L \end{pmatrix} - (\overline{\nu_L^c}\ \overline{\Sigma_L^{0c}}) \begin{pmatrix} 0 & {Y_\Sigma}^T v/2\sqrt{2} \\ Y_\Sigma v/2\sqrt{2} & M_\Sigma/2 \end{pmatrix} \begin{pmatrix} \nu_L \\ \Sigma_L^0 \end{pmatrix}$$

$$v \equiv \sqrt{2}\langle\phi^0\rangle = 246 \text{ GeV}. \ \Psi \equiv \Sigma_L^{+c} + \Sigma_L^-$$

$$M_\Sigma \sim 100 \text{ GeV} \Rightarrow Y_\Sigma \sim 10^{-7}$$

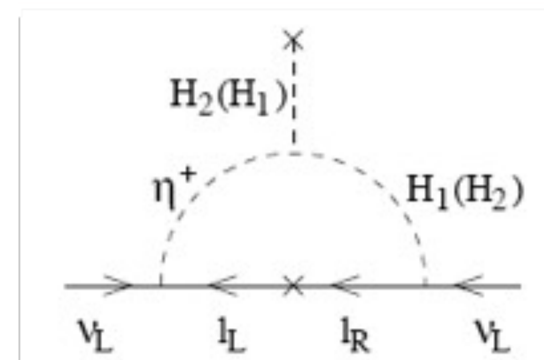
Loop level

1-loop:

1980

a. Zee model (with charged scalar singlet and additional scalar doublets).

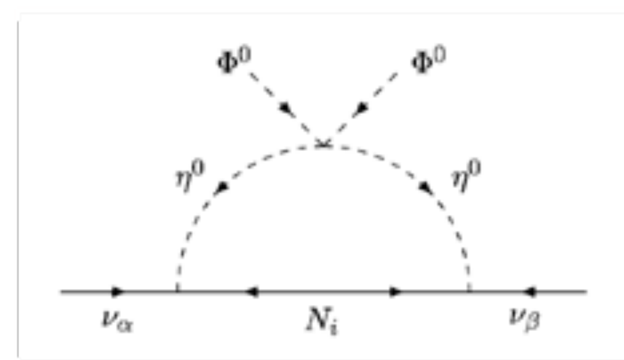
$$l^T \hat{f} i\sigma_2 l \eta^+ + \sum_{i=1,2} \bar{l} \hat{f}_i e H_i,$$



2006

b. Ma model (with fermion singlet N_i and additional scalar doublet η).

$$h_{\alpha i}(\nu_\alpha \eta^0 - l_\alpha \eta^+) N_i$$

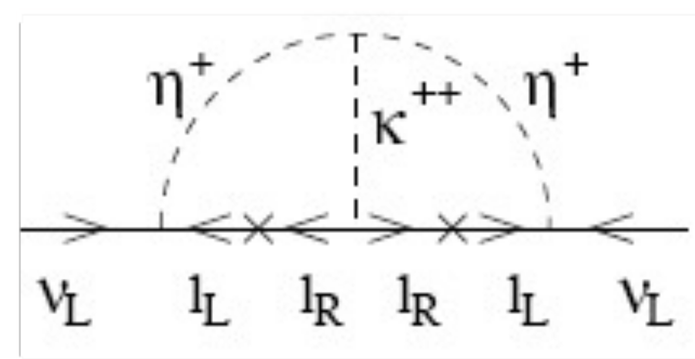


2-loop:

1986 1988

Zee-Babu model (with doubly and singly charged scalars).

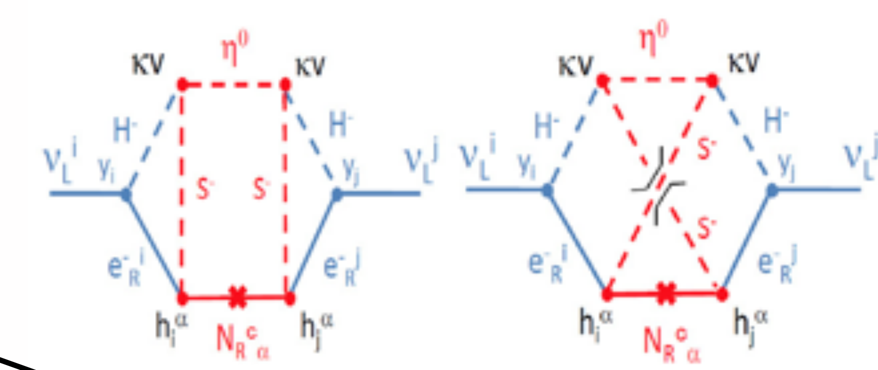
$$l^T \hat{f} l \eta^+ + l_R^T \hat{h} l_R k^{++}$$



3-loop:

	Q^i	u_R^i	d_R^i	L^i	e_R^i	Φ_1	Φ_2	$S^\pm$	η	N_R^α
Z_2 (exact)	+	+	+	+	+	+	+	−	−	−
$\tilde{Z}_2$ (softly broken)	+	−	−	+	+	+	−	+	−	+

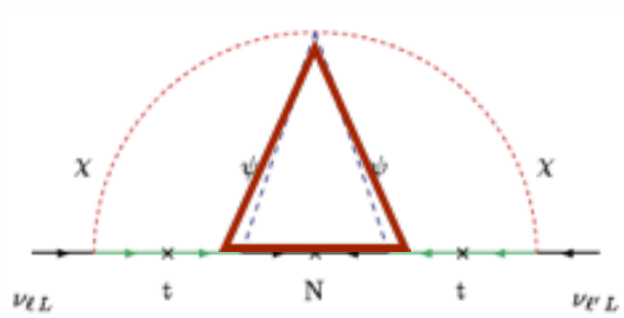
M.Aoki,S.Kanemura,O.Seto, PRL102,051805(2009)



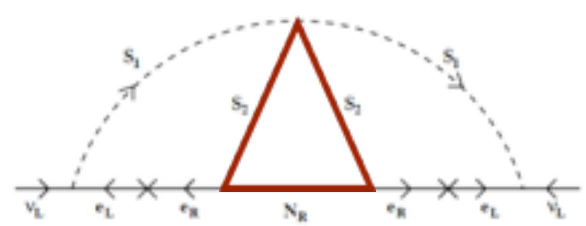
Dark Matter

Top quark as a dark portal

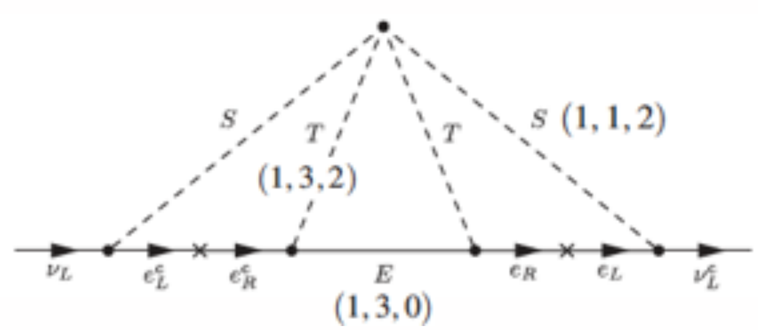
John N. Ng, Alejandro de la Puente 2013

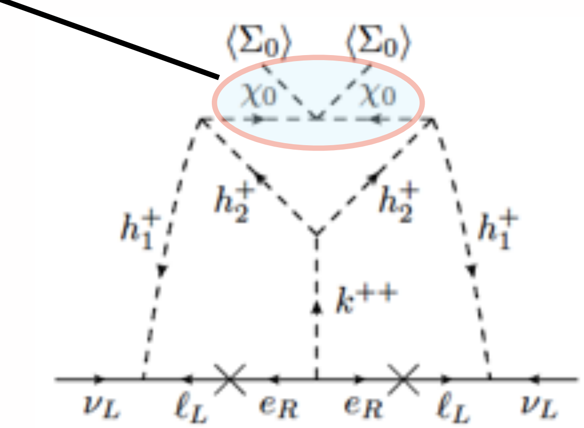


L.Krauss,S.Nasri,and M.Trodden, 2002



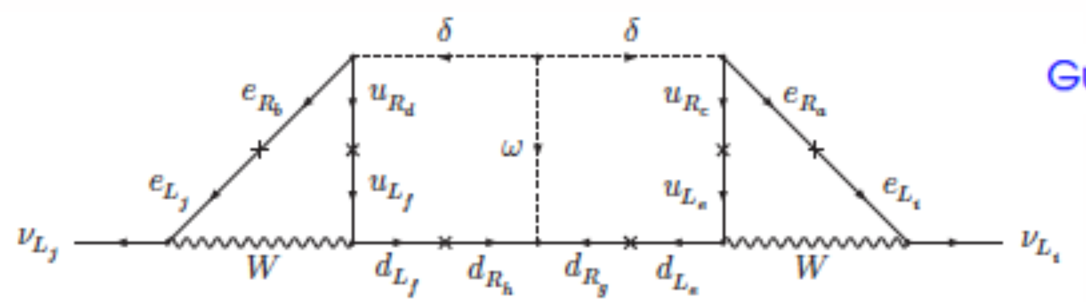
C.S.Chen,K.L.McDonald,S.Nasri, 2014





Hatanaka,Nishiwaki,Okada,Orikasa, arXiv:1412.8664

4-loop:



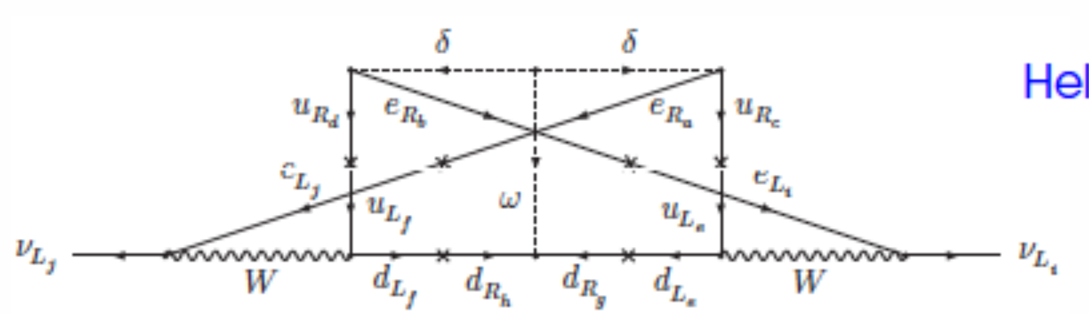
Gu, 2011

$m_\nu \simeq 10^{-8}$ eV

... because $d = 9$ 4-loop

Needs (Quasi)-Dirac ν's

to explain oscillation data

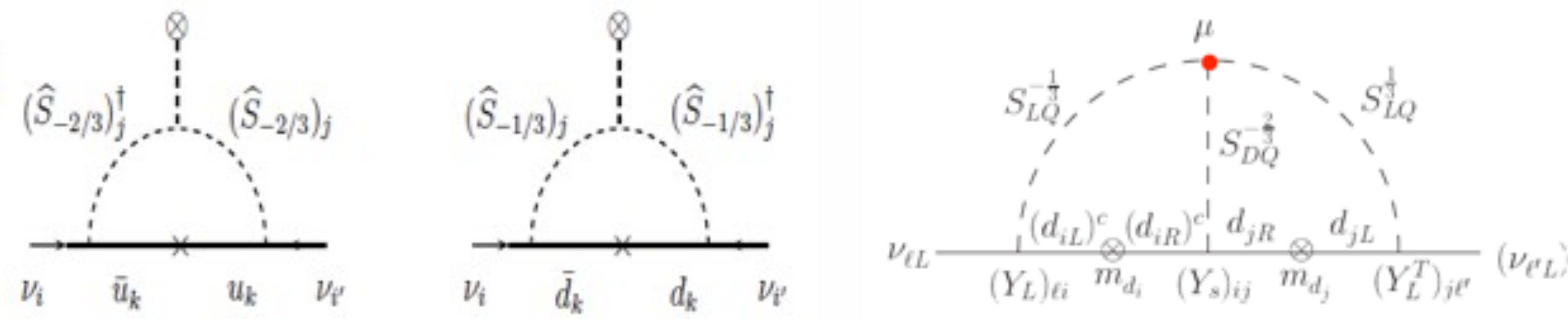


Helo et al., 2015

Other models with loops:

Hirsch et al. 1996, Aristizabal et al. 2008

Leptoquarks

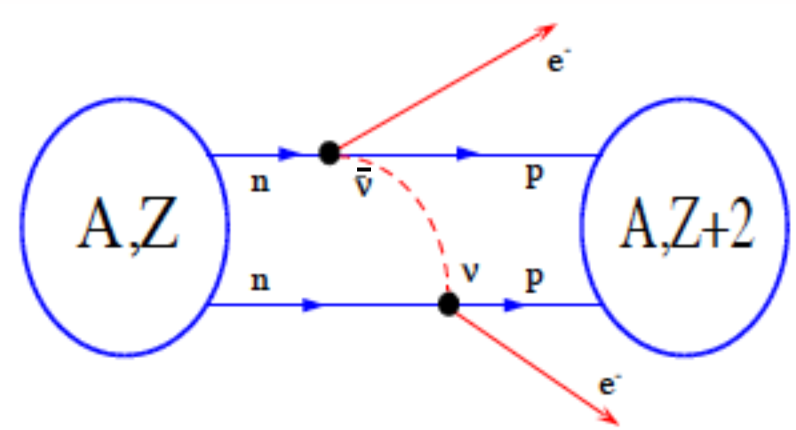


Suppressed 0νββ in all these loop models!

Double beta decay: $(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \overline{\nu}_e + \overline{\nu}_e$

Neutrinoless double beta decay:

0νββ $(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

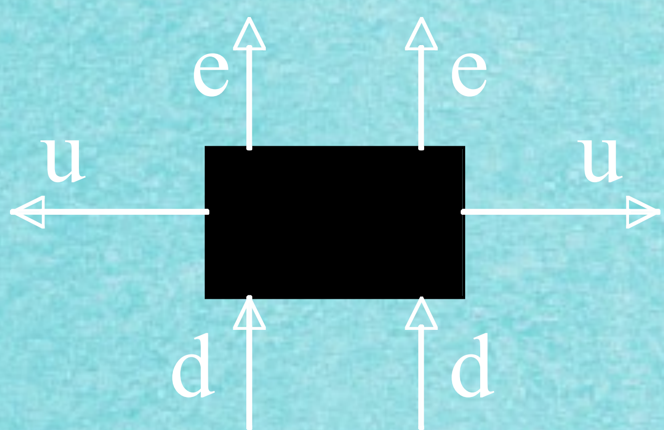


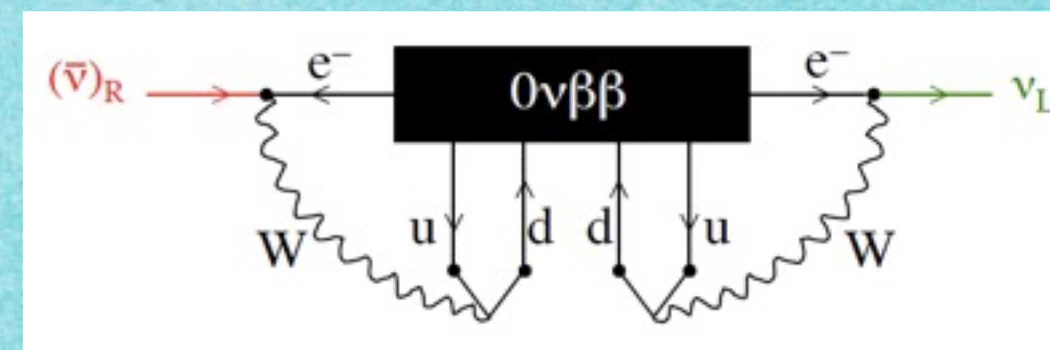
Majorana nature of ν $\nu \leftrightarrow \bar{\nu}$

“Black Box” theorem

J. Schechter and J.W.F. Valle, Phys.Rev. D 25, 2951 (1982)

``Any mechanism inducing the 0νββ decay produces an effective Majorana neutrino mass term, which must therefore contribute to this decay.”





0νββ decay ? ⟷ **Majorana neutrino mass**

☞ **The multi-loop with 0νββ to m_ν is too small, $\sim O(10^{-25})$ eV.** *M.Duerr, M.Lindner, A.Merle, JHEP1106, 091 (2011) .....*

The theorem does not state if the mechanism for 0νββ from M_ν is the dominant one.

In some models, the dominant contributions to 0νββ are generated without directly involving ν_M or M_ν.

• A special class of models to generate M_ν

C.S.Chen+CQG+J.N.Ng, PRD75,053004(2007)

	$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
$L_a = (\nu_a, l_a)_L^T$	$(1, 2, -1)$
e^c_{aL}	$(1, 1, 2)$
$Q_a = (u_a, d_a)_L^T$	$(3, 2, 1/3)$
u^c_{aL}	$(\bar{3}, 1, -4/3)$
d^c_{aL}	$(\bar{3}, 1, 2/3)$
Φ	$(1, 2, 1)$

Table 1: Matter and scalar multiplets of the Standard Model (SM)

No ν_R added

New scalars: a triplet T (1,3,2) + a singlet Ψ (1,1,4)

$$V(\phi, T, \psi) = -\mu^2\phi^\dagger\phi + \lambda_\phi(\phi^\dagger\phi)^2 - \mu_T^2\,\mathrm{Tr}(T^\dagger T) + \lambda_T[\mathrm{Tr}(T^\dagger T)]^2 + \lambda_T'\,\mathrm{Tr}(T^\dagger T T^\dagger T) + m^2\Psi^\dagger\Psi + \lambda_\Psi(\Psi^\dagger\Psi)^2$$
$$+ \kappa_1\,\mathrm{Tr}(\phi^\dagger\phi T^\dagger T) + \kappa_2\phi^\dagger T T^\dagger\phi + \kappa_\Psi\phi^\dagger\phi\Psi^\dagger\Psi + \rho\,\mathrm{Tr}(T^\dagger T\Psi^\dagger\Psi)$$
$$+ [\lambda(\phi^T T\phi\Psi^\dagger) - M(\phi^T T^\dagger\phi) + \mathrm{H.c.}]$$

New Yukawa term: $Y_{ab}\bar{l}^c_{aR} l_{bR}\Psi$ ← lepton # for Ψ is 2

No Yukawa coupling for the triplet: ~~*LLT*~~ ← Forbidden by a symmetry

Φ_1 and Φ_2 *Z_2 or T-parity: $\Phi_1 \to \Phi_1$; $\Phi_2 \to -\Phi_2$; $T \to -T$; $L \to L$*

C.S.Chen+CQG, PRD82,105004(2010)

***Symmetry:** two Higgs doublets (Φ_1 and Φ_2) with Z_2-symmetry or T-parity

CS.Chen,CQG,PRD82,105004(2010)

T-parity: $\Phi_1 \to \Phi_1$; $\Phi_2 \to -\Phi_2$; $T \to -T$; $L \to L$

~~LLT~~

$$
\begin{aligned}
V = & -\mu_1^2\phi_1^\dagger\phi_1 + \lambda_1(\phi_1^\dagger\phi_1)^2 - \mu_2^2\phi_2^\dagger\phi_2 + \lambda_2(\phi_2^\dagger\phi_2)^2 \\
& - \mu_T^2 \mathrm{Tr}(T^\dagger T) + \lambda_T[\mathrm{Tr}(T^\dagger T)]^2 + \lambda_T' \mathrm{Tr}(T^\dagger T T^\dagger T) \\
& + m^2\Psi^\dagger\Psi + \lambda_\Psi(\Psi^\dagger\Psi)^2 + \kappa_{\phi_1}\mathrm{Tr}(\phi_1^\dagger\phi_1 T^\dagger T) \\
& + \kappa_{\phi_1}'\phi_1^\dagger T T^\dagger\phi_1 + \kappa_{\Psi_1}\phi_1^\dagger\phi_1\Psi^\dagger\Psi \\
& + \kappa_{\phi_2}\mathrm{Tr}(\phi_2^\dagger\phi_2 T^\dagger T) + \kappa_{\phi_2}'\phi_2^\dagger T T^\dagger\phi_2 \\
& + \kappa_{\Psi_2}\phi_2^\dagger\phi_2\Psi^\dagger\Psi + \lambda_3\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2 \\
& + \lambda_4\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1 + \rho\,\mathrm{Tr}(T^\dagger T\Psi^\dagger\Psi) + (M\phi_1^T T^\dagger\phi_2 \\
& + \lambda_5\phi_1^\dagger\phi_2\phi_1^\dagger\phi_2 + \lambda\tilde\phi_1^\dagger T\tilde\phi_2^*\Psi + \mathrm{H.c.}),
\end{aligned}
$$

No effect for other couplings

Chen,CQG,Huang,Tsai, PRD87,077702 (2013)

Without Symmetry: $\xi(1,N,2) + \Psi(1,1,4)$ ➡ ~~$LL\xi$~~ *if $N>3$*

There are only three possible new renormalizable Yukawa interactions involving SM fermions with non-Higgs scalars:

~~$f_{ab}\bar{L}^c_a L_b s$~~ , $y_{ab}\bar{\ell}^c_{R_a}\ell_{R_b}\Psi$, ~~$g_{ab}\bar{L}^c_a L_b T$~~

(Colorless scalars)

Zee model — Zee-Babu model — Type-II seesaw

We will consider higher dimensional multiplets so that NO LL-like term is allowed in the Yukawa interactions.

- We replace s=(1,1,2) and T=(1,3,2) by ξ=(1,N,2)

Chen, CQG, Huang, Tsai, PRD87,077702 (2013)

N>3 (=4, 5, 6, 7,…) is the quantum # under $SU(2)_L$ and Y=2 is the hypercharge with $Q_{em}=I_3+Y/2$

→ **Multi High Charged Scalars** e.g. for N=5 ☞ $\xi = (\xi^{+++}, \xi^{++}, \xi^{+}, \xi^{0}, \xi^{-})^T$

enhance

Higgs → 2 photon ~~1.6 times of excess at the LHC~~

- The scalar potential reads

$$V(\Phi,\xi,\Psi) = -\mu_\Phi^2|\Phi|^2 + \lambda_\Phi|\Phi|^4 + \mu_\xi^2|\xi|^2 + \lambda_\xi^\alpha|\xi|_\alpha^4 + \mu_\Psi^2|\Psi|^2 + \lambda_\Psi|\Psi|^4$$
$$+\lambda_{\Phi\xi}^\beta(|\Phi|^2|\xi|^2)_\beta + \lambda_{\Phi\Psi}|\Phi|^2|\Psi|^2 + \lambda_{\xi\Psi}|\xi|^2|\Psi|^2$$
$$+[\mu\xi\xi\Psi + \text{h.c.}]$$

No N=4, 6, 8, 10,…, even dimensions due to their antisymmetric products → N=5, 7, …, odd dimensions

Some detail calculations: doublet Φ(1,2,-1) + triplet T (1,3,2) + singlet Ψ (1,1,4)

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}_{-1} \qquad T = \begin{pmatrix} T^0 & \frac{T^-}{\sqrt{2}} \\ \frac{T^-}{\sqrt{2}} & T^{--} \end{pmatrix}_{-2} \qquad \psi_4^{++}$$

The must general potential is

$$\begin{aligned}
V(\phi,T,\psi) &= -\mu^2\phi^\dagger\phi + \lambda_\phi(\phi^\dagger\phi)^2 - \mu_T^2 Tr(T^\dagger T) + \lambda_T[Tr(T^\dagger T)]^2 + \lambda_T' Tr(T^\dagger T T^\dagger T) \\
&+ m^2\psi^\dagger\psi + \lambda_\psi(\psi^\dagger\psi)^2 + \kappa_{T1} Tr(\phi^\dagger\phi T^\dagger T) + \kappa_{T2}\phi^\dagger T T^\dagger\phi + \kappa_\psi\phi^\dagger\phi\psi^\dagger\psi \\
&+ \rho Tr(T^\dagger T\psi^\dagger\psi) + \lambda(\phi^\dagger T\phi\psi + h.c.) \\
&= -\mu^2(\phi^+\phi^- + |\phi^0|^2) + \lambda_\phi(|\phi^0|^4 + 2|\phi^0|^2\phi^+\phi^- + (\phi^+\phi^-)^2) \\
&- \mu_T^2(|T^0|^2 + T^{++}T^{--} + T^+T^-) \\
&+ (\lambda_T + \lambda_T')[|T^0|^4 + 2|T^0|^2T^+T^- + (T^{++}T^{--})^2 + 2T^{++}T^{--}T^+T^-] \\
&+ (\lambda_T + \frac{\lambda_T'}{2})(T^+T^-)^2 + \lambda_T'(T^0T^{++}T^-T^- + T^{0*}T^{--}T^+T^+) \\
&+ m^2(\psi^{++}\psi^{--}) + \lambda_\psi(\psi^{++}\psi^{--})^2 \\
&+ (\kappa_{T1} + \kappa_{T2})(|\phi^0|^2|T^0|^2 + \phi^+\phi^-T^{++}T^{--}) \\
&+ (\kappa_{T1} + \frac{\kappa_{T2}}{2})(|\phi^0|^2T^+T^- + \phi^+\phi^-T^+T^-) + \kappa_{T1}(|T^0|^2\phi^+\phi^- + |\phi^0|^2T^{++}T^{--}) \\
&- \frac{\kappa_{T2}}{\sqrt{2}}(\phi^0T^0\phi^+T^- + \phi^0T^+\phi^+T^{--} + h.c.) + \kappa_\psi(|\phi^0|^2 + \phi^+\phi^-)\psi^{++}\psi^{--} \\
&+ \rho(|T^0|^2 + T^{++}T^{--} + T^+T^-)\psi^{++}\psi^{--} \\
&+ \lambda(T^{0*}\phi^-\phi^-\psi^{++} + \sqrt{2}\phi^{0*}T^-\phi^-\psi^{++} + \phi^{0*}\phi^{0*}T^{--}\psi^{++} + h.c.) \qquad (4)
\end{aligned}$$

The vacuum energy

$$-\frac{\mu^2 v^2}{2} + \frac{\lambda_\phi v^4}{4} - \frac{\mu_T^2 v_T^2}{2} + (\lambda_T + \lambda_T')\frac{v_T^4}{4}$$

Tadpole terms

$$+[-\mu^2 v + \lambda_T v^3 + (\kappa_{T1} + \kappa_{T2})\frac{v v_T^2}{2}]\phi_1 + [-\mu_T^2 v_T + (\lambda_T + \lambda_T')v_T^3 + (\kappa_{T1} + \kappa_{T2})\frac{v^2 v_T}{2}]T_1$$

The mass terms

$$\begin{aligned}
&-\mu^2(\frac{\phi_1^2 + \phi_2^2}{2} + \phi^+\phi^-) + \frac{\lambda_T}{2}(3v^2\phi_1^2 + v^2\phi_2^2 + 2v^2\phi^+\phi^-) - \mu_T^2(\frac{T_1^2 + T_2^2}{2} + T^+T^- + T^{++}T^{--}) \\
&+\frac{\lambda_T + \lambda_T'}{4}(6v_T^2 T_1^2 + 2v_T^2 T_2^2 + 4v_T^2 T^+T^-) + m^2(\psi^{++}\psi^{--}) \\
&+\frac{\kappa_{T1} + \kappa_{T2}}{4}[v^2(T_1^2 + T_2^2) + v_T^2(\phi_1^2 + \phi_2^2) + 4 v v_T \phi_1 T_1] + (\kappa_{T1} + \frac{\kappa_{T2}}{2})\frac{v^2}{2}T^+T^- \\
&+\kappa_{T1}(\frac{v_T^2}{2}\phi^+\phi^- + \frac{v^2}{2}T^{++}T^{--}) - \frac{\kappa_{T2}}{\sqrt{2}}\frac{v v_T}{2}(\phi^+T^- + \phi^-T^+) + \frac{\lambda v^2}{2}(T^{--}\psi^{++}) \\
&+(\frac{\kappa_\psi v^2}{2} + \frac{\rho v_T^2}{2})\psi^{++}\psi^{--} \qquad (7)
\end{aligned}$$

Trilinear couplings

$$\begin{aligned}
&\frac{\lambda_\phi}{4}(4v\phi_1^3 + 4v\phi_1\phi_2^2 + 8v\phi_1\phi^+\phi^-) + \frac{\lambda_{T1} + \lambda_T'}{4}(4v_T T_1^3 + 4v_T T_1 T_2^2 + 8 v_T T_1 T^+T^-) \\
&+\frac{\lambda_T'}{\sqrt{2}}(v_T T^{++}T^-T^- + v_T T^{--}T^+T^+) + \frac{\kappa_{T1} + \kappa_{T2}}{2}[v\phi_1(T_1^2 + T_2^2) + v_T T_1(\phi_1^2 + \phi_2^2)] \\
&+(\kappa_{T1} + \frac{\kappa_{T2}}{2})v\phi_1 T^+T^- + \kappa_{T1}(v_T T_1 \phi^+\phi^- + v\phi_1 T^{++}T^{--}) \\
&-\frac{\kappa_{T2}}{\sqrt{2}}[\frac{v}{2}(T_1 + iT_2)\phi^+T^- + \frac{v_T}{2}(\phi_1 + i\phi_2)\phi^+T^- + \frac{v}{\sqrt{2}}T^+\phi^+T^{--} + h.c.] \\
&+(\kappa_\psi v\phi_1 + \rho v_T T_1)\psi^{++}\psi^{--} \\
&+\lambda[\frac{v_T}{\sqrt{2}}\phi^-\phi^-\psi^{++} + vT^-\phi^-\psi^{++} + v(\phi_1 - i\phi_2)T^{--}\psi^{++} + h.c.]
\end{aligned}$$

The quartic couplings

$$\frac{\lambda_\phi}{4}[\phi_1^4+\phi_2^4+\phi_1^2\phi_2^2+4\phi_1^2\phi^+\phi^-+4\phi_2^2\phi^+\phi^-+4(\phi^+\phi^-)^2]$$
$$+(\lambda_T+\lambda_T')[\frac{1}{4}(T_1^4+T_2^4+T_1^2T_2^2)+(T_1^2+T_2^2)T^+T^-+(T^{++}T^{--})^2+2T^{++}T^{--}T^+T^-]$$
$$+(\lambda_T+\frac{\lambda_T'}{2})(T^+T^-)^2+\frac{\lambda_T'}{\sqrt{2}}[(T_1+iT_2)T^{++}T^-T^-+(T_1-iT_2)TT^{--}T^+T^+]$$
$$+(\kappa_{T1}+\kappa_{T2})[\frac{1}{4}(\phi_1^2+\phi_2^2)(T_1^2+T_2^2)+\phi^+\phi^-T^{++}T^{--}]$$
$$+(\kappa_{T1}+\frac{\kappa_{T2}}{2})(\frac{\phi_1^2+\phi_2^2}{2}T^+T^-+\phi^+\phi^-T^+T^-)+\frac{\kappa_{T1}}{2}[(T_1^2+T_2^2)\phi^+\phi^-+(\phi_1^2+\phi_2^2)T^{++}T^{--}]$$
$$-\frac{\kappa_{T2}}{\sqrt{2}}[(\frac{\phi_1T_1-\phi_2T_2}{2}+i\frac{\phi_2T_1+\phi_1T_2}{2})\phi^+T^-+\frac{\phi_1+i\phi_2}{\sqrt{2}}T^+\phi^+T^{--}+h.c.]$$
$$+[\kappa_\psi\phi^+\phi^-+\frac{\kappa_\psi}{2}(\phi_1^2+\phi_2^2)+\frac{\rho}{2}(T_1^2+T_2^2)+\rho T^{++}T^{--}+\rho T^+T^-]\psi^{++}\psi^{--}$$
$$+\lambda[\frac{T_1-iT_2}{\sqrt{2}}\phi^-\phi^-\psi^{++}+(\phi_1-i\phi_2)T^-\phi^-\psi^{++}+\frac{\phi_1^2-\phi_2^2-2i\phi_1\phi_2}{2}T^{--}\psi^{++}+h.c.] \quad (9)$$

Let

$$\phi^0=\frac{v+\phi_1+i\phi_2}{\sqrt{2}} \quad \text{and} \quad T^0=\frac{v_T+T_1+iT_2}{\sqrt{2}}$$

Mass mixing matrices

$$\begin{pmatrix}\phi_1 & T_1\end{pmatrix}\begin{pmatrix}-\frac{\mu^2}{2}+\frac{3}{2}\lambda_\phi v^2+\frac{(\kappa_{T1}+\kappa_{T2})}{4}v_T^2 & \frac{(\kappa_{T1}+\kappa_{T2})}{2}vv_T \\ \frac{(\kappa_{T1}+\kappa_{T2})}{2}vv_T & -\mu_T^2+\frac{3}{2}(\lambda_T+\lambda_T')v_T^2+\frac{(\kappa_{T1}+\kappa_{T2})}{4}v^2\end{pmatrix}\begin{pmatrix}\phi_1 \\ T_1\end{pmatrix}$$

$$\begin{pmatrix}\phi_2 & T_2\end{pmatrix}\begin{pmatrix}-\frac{\mu^2}{2}+\frac{\lambda_\phi}{2}v^2+\frac{(\kappa_{T1}+\kappa_{T2})}{4}v_T^2 & 0 \\ 0 & -\frac{\mu_T^2}{2}+\frac{(\lambda_T+\lambda_T')}{2}v_T^2+\frac{(\kappa_{T1}+\kappa_{T2})}{4}v^2\end{pmatrix}\begin{pmatrix}\phi_2 \\ T_2\end{pmatrix}$$

$$\begin{pmatrix}\phi^+ & T^+\end{pmatrix}\begin{pmatrix}-\mu^2+\lambda_\phi v^2+\frac{\kappa_{T1}}{2}v_T^2 & -\frac{\kappa_{T2}}{2\sqrt{2}}vv_T \\ -\frac{\kappa_{T2}}{2\sqrt{2}}vv_T & -\mu_T^2+(\lambda_T+\lambda_T')v_T^2+(\frac{\kappa_{T1}}{2}+\frac{\kappa_{T2}}{4})v^2\end{pmatrix}\begin{pmatrix}\phi^- \\ T^-\end{pmatrix}$$

$$\begin{pmatrix}T^{++} & \psi^{++}\end{pmatrix}\begin{pmatrix}-\mu_T^2+\lambda_T v_T^2 v_T^2+\frac{\kappa_{T1}}{2}v^2 & \frac{\lambda}{2}v^2 \\ \frac{\lambda}{2}v^2 & m^2+\frac{\kappa_\psi}{2}v^2+\frac{\rho}{2}v_T^2\end{pmatrix}\begin{pmatrix}T^{--} \\ \psi^{--}\end{pmatrix}$$

The covariant derivative

$$D_\mu T = \partial_\mu T - i\tfrac{g}{2}([\vec{\sigma}\cdot\vec{W}_\mu T] + [\vec{\sigma}\cdot\vec{W}_\mu]^t) - i\tfrac{g'}{2} Y B_\mu T$$

$$\begin{aligned}
Tr[(D_\mu T)^\dagger (D^\mu T)] &= (\partial_\mu T^{0*})(\partial^\mu T^0) + (\partial_\mu T^-)(\partial^\mu T^+) + (\partial_\mu T^{--})(\partial^\mu T^{++}) \\
&+ (\partial^\mu T^0)[ig(T^{0*}W^3_\mu + T^+ W^-_\mu) - ig' B_\mu T^{0*}] + h.c. \\
&+ (\partial^\mu T^+)[-ig(T^{--}W^+_\mu + T^0 W^-_\mu) + ig' B_\mu T^-] + h.c. \\
&+ (\partial^\mu T^{++})[-ig(W^-_\mu T^- - T^{--}W^3_\mu) + ig' B_\mu T^{--}] + h.c. \\
&+ [g(T^{0*}W^3_\mu + T^+ W^-_\mu) - g' B_\mu T^{0*}]^2 \\
&+ [g(W^+_\mu T^{--} + T^0 W^-_\mu) - ig' B_\mu T^-]^2 \\
&+ [g(W^-_\mu T^- - T^{--}W^3_\mu) - g' B_\mu T^{--}]^2
\end{aligned}$$

$$\begin{aligned}
(D_\mu\phi)^\dagger(D^\mu\phi) &= (\partial_\mu\phi^0)(\partial^\mu\phi^{0*}) + (\partial_\mu\phi^+)(\partial^\mu\phi^-) \\
&+ \frac{1}{2}(\partial_\mu\phi^+)[ig(W^3_\mu\phi^- - \sqrt{2}W^-_\mu\phi^0) + ig' B_\mu\phi^-] + h.c. \\
&+ \frac{1}{2}(\partial_\mu\phi^0)[ig(\sqrt{2}W^{-\mu}\phi^+ + W^{3\mu}\phi^{0*}) - ig' B^\mu\phi^{0*}] + h.c. \\
&+ \frac{1}{4}[g(\sqrt{2}W^-_\mu\phi^0 - W^3_\mu\phi^-) - g' B^\mu\phi^-]^2 \\
&+ \frac{1}{4}[g(\sqrt{2}W^-_\mu\phi^+ - W^3_\mu\phi^{0*}) - g' B_\mu\phi^{0*}]^2
\end{aligned}$$

$$\begin{aligned}
(D_\mu\psi)^\dagger(D^\mu\psi) &= (\partial_\mu\psi^{--})(\partial^\mu\psi^{++}) + 2ig' B_\mu\psi^{--}(\partial^\mu\psi^{++}) + h.c. \\
&+ 4g'^2 B_\mu B^\mu \psi^{--}\psi^{++}
\end{aligned}$$

The masses of gauge bosons are $M_W^2 W_\mu^+ W^{-\mu} = \frac{g^2}{4}(v^2 + 2v_T^2) W_\mu^+ W^{-\mu}$

$$M_W^2 = \frac{g^2}{4}(v^2 + 2v_T^2)$$

The mixing matrix

$$\frac{1}{2}\begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix}\begin{pmatrix} \frac{g^2}{8}(v^2 + 4v_T^2) & -\frac{gg'}{8}(v^2 - 4v_T^2) \\ -\frac{gg'}{8}(v^2 - 4v_T^2) & \frac{g'^2}{8}(v^2 + 4v_T^2) \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\tan 2\theta_W = \frac{2\frac{g'}{g}\frac{v^2 - 4v_T^2}{v^2 + 4v_T^2}}{1 - \frac{g'^2}{g^2}}$$

$$M_Z^2 = \frac{g^2}{4\cos^2\theta_W}(v^2 + 4v_T^2)$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{v^2 + 2v_T^2}{v^2 + 4v_T^2}$$

$$\begin{pmatrix} G^- \\ P^- \end{pmatrix} = \frac{1}{\sqrt{v^2 + 2v_T^2}}\begin{pmatrix} v & \sqrt{2}v_T \\ -\sqrt{2}v_T & v \end{pmatrix}\begin{pmatrix} \phi^- \\ T^- \end{pmatrix}$$

Gauge-scalar trilinear interactions :

$W_\mu^+ - T^+ - T^0$:

$$\frac{ig}{\sqrt{2}} W^{-\mu}[(\partial_\mu T_1)T^+ - (\partial_\mu T^+)T_1] + \frac{g}{\sqrt{2}} W^{-\mu}[(\partial_\mu T^+)T_2 - (\partial_\mu)T^+] + h.c.$$

$W_\mu^+ - T^+ - T^{--}$:

$$igW^{+\mu}[(\partial_\mu T^{--})T^+ - (\partial_\mu T^+)T^{--}] + h.c.$$

$Z_\mu(A_\mu) - T^+ - T^-$

$$ig' \sin\theta_W (\partial_\mu T^-)T^+ Z^\mu - ig \sin\theta_W (\partial_\mu T^-)T^+ A^\mu + h.c$$

$A_\mu - T^{++} - T^{--}$

$$-i\frac{g}{\cos\theta_W}(\partial_\mu T^{--})T^{++}A^\mu$$

$Z_\mu - T^0 - T^0$

$$\frac{g}{\cos\theta_W} Z^\mu[(\partial_\mu T_1)T_2 - (\partial_\mu T_2)T_1]$$

$A_\mu - \phi^+ - \phi^-$

$$-\frac{ig}{2\cos\theta}(\partial_\mu \phi^-)\phi^+ A^\mu + h.c.$$

$W_\mu^+ - \phi^- - \phi^0$

$$\frac{ig}{2} W_\mu^+[(\partial^\mu \phi^-)\phi_1 - (\partial^\mu \phi_1)\phi^-] + \frac{g}{2} W_\mu^+[(\partial^\mu \phi^-)\phi_2 - (\partial^\mu \phi_2)\phi^-] + h.c.$$

Gauge-scalar trilinear interactions :

$W^+_\mu - T^+ - T^0$:	$\frac{ig}{\sqrt{2}}W^{-\mu}[(\partial_\mu T_1)T^+ - (\partial_\mu T^+)T_1] + \frac{g}{\sqrt{2}}W^{-\mu}[(\partial_\mu T^+)T_2 - (\partial_\mu)T^+] + h.c.$
$W^+_\mu - T^+ - T^{--}$:	$igW^{+\mu}[(\partial_\mu T^{--})T^+ - (\partial_\mu T^+)T^{--}] + h.c.$
$Z_\mu(A_\mu) - T^+ - T^-$	$ig' \sin\theta_W(\partial_\mu T^-)T^+ Z^\mu - ig\sin\theta_W(\partial_\mu T^-)T^+ A^\mu + h.c$
$A_\mu - T^{++} - T^{--}$	$-i\frac{g}{\cos\theta_W}(\partial_\mu T^{--})T^{++}A^\mu$
$Z_\mu - T^0 - T^0$	$\frac{g}{\cos\theta_W}Z^\mu[(\partial_\mu T_1)T_2 - (\partial_\mu T_2)T_1]$
$A_\mu - \phi^+ - \phi^-$	$-\frac{ig}{2\cos\theta}(\partial_\mu\phi^-)\phi^+ A^\mu + h.c.$
$W^+_\mu - \phi^- - \phi^0$	$\frac{ig}{2}W^+_\mu[(\partial^\mu\phi^-)\phi_1 - (\partial^\mu\phi_1)\phi^-] + \frac{g}{2}W^+_\mu[(\partial^\mu\phi^-)\phi_2 - (\partial^\mu\phi_2)\phi^-] + h.c.$
$Z_\mu - \phi^0 - \phi^0$	$\frac{g}{2\cos\theta_W}Z^\mu[(\partial_\mu\phi_1)\phi_2 - (\partial_\mu\phi_2)\phi_1]$
$Z_\mu - Z_\mu - T^0$	$\frac{g^2}{\cos^2\theta_W}v_T Z_\mu Z^\mu T_1$
$T^+ - W^-_\mu - Z_\mu$	$\frac{g^2 + g^2\sin^2\theta_W}{\cos\theta_W}(\frac{v_T}{\sqrt{2}})Z^\mu W^-_\mu T^+ + h.c.$
$T^+ - W^-_\mu - A_\mu$	$-g^2\sin\theta_W\frac{v_T}{\sqrt{2}}T^+W^-_\mu A^\mu + h.c.$
$T^0 - W^{+\mu} - W^-_\mu$	$g^2 v_T T_1 W^{+\mu}W^-_\mu$
$T^{--} - W^{+\mu} - W^+_\mu$	$\frac{g^2}{\sqrt{2}}v_T T^{--}W^{+\mu}W^+_\mu + h.c.$
$\phi^0 - W^{+\mu} - W^-_\mu$	$\frac{g^2 v}{2}W^{+\mu}W^-_\mu\phi_1$
$W^{-\mu} - A_\mu - \phi^+$	$-\frac{g^2 v}{4\cos\theta_W}W^{-\mu}A_\mu\phi^+ + h.c$
$\phi^0 - Z^\mu - Z_\mu$	$\frac{g^2 v}{4\cos^2\theta_W}Z^\mu Z_\mu\phi_1$
$W^{-\mu} - Z_\mu - \phi^+$	$\frac{g^2 v}{4\cos\theta_W}W^{-\mu}Z_\mu\phi^+ + h.c.$

$T^+ - T^- - Z_\mu - Z_\mu$

$$-g'^2 \sin^2\theta_W T^+T^- Z^\mu Z_\mu$$

$T^+ - T^- - A^\mu - A_\mu$

$$g'^2 \cos^2\theta_W T^+T^- A^\mu A_\mu$$

$T^+ - T^- - Z^\mu - A_\mu$

$$-2g'^2 \sin\theta_W \cos\theta_W T^+T^- Z^\mu A_\mu$$

$T^{--} - T^+ - W^{+\mu} - Z_\mu$

$$gg' \sin\theta_W W^{+\mu} Z_\mu T^+ T^{--} + h.c.$$

$T^{--} - T^+ - W^{+\mu} - A_\mu$

$$-(gg'\cos\theta_W + \frac{g^2}{\cos\theta_W}) W^{+\mu} A_\mu T^+ T^{--} + h.c.$$

$T^0 - T^0 - W^{+\mu} - W^-_\mu$

$$\frac{g^2}{2}(T_1^2 + T_2^2) W^{+\mu} W^-_\mu$$

$T^0 - T^+ - W^{-\mu} - Z_\mu$

$$\frac{g^2}{\sqrt{2}}(\frac{1+\sin^2\theta_W}{\cos\theta_W}) T^+ Z_\mu W^{-\mu}(T_1 + iT_2) + h.c.$$

$T^0 - T^+ - W^{-\mu} - A_\mu$

$$-\frac{g^2 \sin\theta_W}{\sqrt{2}} T^+ A_\mu W^{-\mu}(T_1 + iT_2) + h.c$$

$T^0 - T^{--} - W^{+\mu} - W^+_\mu$

$$\frac{g^2}{\sqrt{2}}(T_1 - iT_1) T^{--} W^{+\mu} W^+_\mu + h.c.$$

$T^0 - T^0 - Z_\mu - Z^\mu$

$$\frac{g^2}{2\cos^2\theta_W}(T_1^2 + T_2^2) Z^\mu Z_\mu$$

$T^+ - T^- - W^{+\mu} - W^-_\mu$

$$2g^2 W^{+\mu} W^-_\mu T^+ T^-$$

$T^{++} - T^{--} - W^{+\mu} - W^-_\mu$

$$g^2 W^{+\mu} W^-_\mu T^{++} T^{--}$$

$T^{++} - T^{--} - A^\mu - A_\mu$

$$\frac{g^2}{\cos^2\theta_W} A^\mu A_\mu T^{++} T^{--}$$

$\phi^+ - \phi^- - A^\mu - A_\mu$

$$\frac{g^2}{4\cos^2\theta_W} A^\mu A_\mu \phi^+ \phi^-$$

$\phi^0 - \phi^0 - W^{+\mu} - W^-_\mu$

$$\frac{g^2}{4} W^{+\mu} W^-_\mu (\phi_1^2 + \phi_2^2)$$

$\phi^0 - \phi^+ - W^{-\mu} - A_\mu$

$$-\frac{g^2}{4\cos\theta_W} W^{-\mu} A_\mu \phi^+ (\phi_1 + i\phi_2) + h.c.$$

$\phi^0 - \phi^0 - Z^\mu - Z_\mu$

$$\frac{g^2}{8\cos^2\theta_W} Z^\mu Z_\mu (\phi_1^2 + \phi_2^2)$$

$\phi^+ - \phi^- - W^{-\mu} - W^+_\mu$

$$\frac{g^2}{2} W^{+\mu} W^-_\mu \phi^+ \phi^-$$

$\phi^0 - \phi^+ - W^{-\mu} - Z_\mu$

$$\frac{g^2}{4\cos\theta_W} W^{-\mu} Z_\mu \phi^+ (\phi_1 + i\phi_2) + h.c.$$

$Z^\mu(A^\mu) - Z_\mu(A_\mu) - \psi^{++} - \psi^{--}$

$$4g'^2 \sin^2\theta_W Z^\mu Z_\mu \psi^{++}\psi^{--} + 4g'^2 \cos^2\theta_W A^\mu A_\mu \psi^{++}\psi^{--} - 8g'^2 \sin\theta_W \cos\theta_W A^\mu Z_\mu$$

Constraints on the models:

VEVs: $\langle\phi^0\rangle \equiv \frac{v}{\sqrt{2}}$ and $\langle T^0\rangle \equiv \frac{v_T}{\sqrt{2}}$.

$$M_W^2 = \frac{g^2}{4}(v^2 + 2v_T^2)\,, \qquad M_Z^2 = \frac{g^2}{4\cos^2\theta_W}(v^2 + 4v_T^2)\,,$$

$$\rho = 1.0002^{+0.0007}_{-0.0004} \quad \rightarrow \quad v_T < 4.41\ \text{GeV}$$

Two doubly charged scalars:

$$T = \begin{pmatrix} T^0 & \frac{T^-}{\sqrt{2}} \\ \frac{T^-}{\sqrt{2}} & T^{--} \end{pmatrix}$$ and Ψ^{++}

or for N=5 $\xi = (\xi^{+++}, \xi^{++}, \xi^{+}, \xi^{0}, \xi^{-})^T$

Mass eigenstates:

$$\begin{pmatrix} P_1^{\pm\pm} \\ P_2^{\pm\pm} \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} T^{\pm\pm} \\ \Psi^{\pm\pm} \end{pmatrix}$$

$$\sin 2\delta = \left[1 + \left(\frac{2m^2 + (2\lambda_T' + \rho)v_T^2}{2\lambda v^2} + \frac{\kappa_2 + \kappa_\Psi}{2\lambda} - \frac{\omega}{\lambda}\right)^2\right]^{-\frac{1}{2}}$$

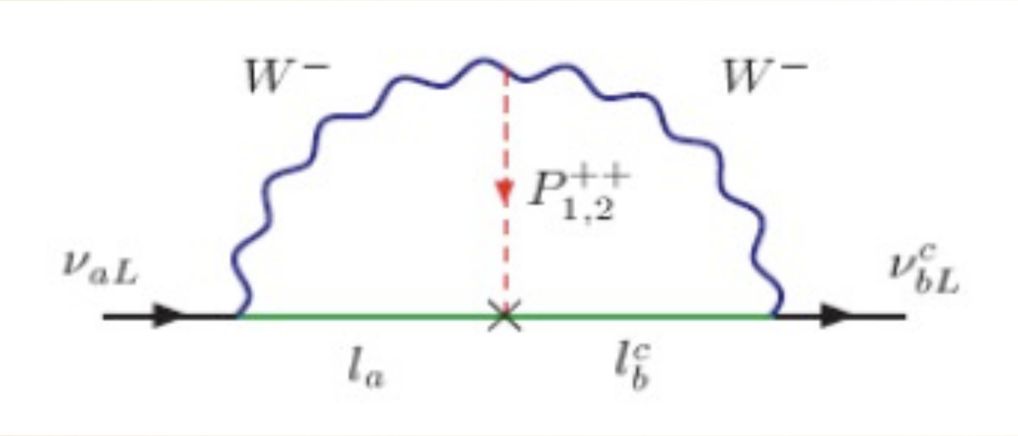
$$M_{P_{1,2}}^2 = \frac{1}{2}\left[a + c \mp \sqrt{4b^2 + (c-a)^2}\right]$$

$$\omega \equiv \frac{M}{\sqrt{2}v_T}$$

$$a = \frac{1}{2}(2\omega - \kappa_2)v^2 - \lambda_T' v_T^2\,, \qquad b = \frac{1}{2}\lambda v^2\,, \qquad c = m^2 + \frac{1}{2}(\kappa_\Psi v^2 + \rho v_T^2)\,.$$

- **Neutrino mass generation:**

The neutrino masses are generated radiatively at two-loop level



$$a, b = e, \mu, \tau.$$

$$(m_\nu)_{ab} = \frac{1}{\sqrt{2}} g^4 m_a m_b v_T Y_{ab} \sin(2\delta) \left[I(M_W^2, M_{P_1}^2, m_a, m_b) - I(M_W^2, M_{P_2}^2, m_a, m_b) \right]$$

$$I(M_W^2, M_{P_i}^2, m_a^2, m_b^2) = \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_a^2} \frac{1}{k^2 - M_W^2} \frac{1}{q^2 - M_W^2} \frac{1}{q^2 - m_b^2} \frac{1}{(k-q)^2 - M_{P_i}^2}.$$

$$M_{P_{1,2}} > M_W$$

$$I(M_W^2, M_{P_i}^2, 0, 0) \sim \frac{1}{(4\pi)^4} \frac{1}{M_{P_i}^2} \log^2\left(\frac{M_W^2}{M_{P_i}^2}\right)$$

$$m_\nu = \tilde{f}(M_{P_1}, M_{P_2}) \times \begin{pmatrix} m_e^2 Y_{ee} & m_e m_\mu Y_{e\mu} & m_e m_\tau Y_{e\tau} \\ m_e m_\mu Y_{e\mu} & m_\mu^2 Y_{\mu\mu} & m_\tau m_\mu Y_{\mu\tau} \\ m_e m_\tau Y_{e\tau} & m_\tau m_\mu Y_{\mu\tau} & m_\tau^2 Y_{\tau\tau} \end{pmatrix}$$

$$= f(M_{P_1}, M_{P_2}) \times \begin{pmatrix} 2.6 \times 10^{-7} Y_{ee} & 5.4 \times 10^{-5} Y_{e\mu} & 9.1 \times 10^{-4} Y_{e\tau} \\ 5.4 \times 10^{-5} Y_{e\mu} & 1.1 \times 10^{-2} Y_{\mu\mu} & 0.19 Y_{\mu\tau} \\ 9.1 \times 10^{-4} Y_{e\tau} & 0.19 Y_{\mu\tau} & 3.17 Y_{\tau\tau} \end{pmatrix}$$

normal hierarchy:

$$\rightarrow \begin{pmatrix} \varepsilon' & \varepsilon & \varepsilon \\ \varepsilon & 1+\eta & 1+\eta \\ \varepsilon & 1+\eta & 1+\eta \end{pmatrix}$$

$$\tilde{f}(M_{P_1}, M_{P_2}) = \frac{\sqrt{2} g^4 v_T \sin(2\delta)}{128\pi^4} \left[\frac{1}{M_{P_1}^2} \log^2\left(\frac{M_W}{M_{P_1}}\right) - \frac{1}{M_{P_2}^2} \log^2\left(\frac{M_W}{M_{P_2}}\right) \right]$$

$$f = \tilde{f} \times (1\text{GeV}^2)$$

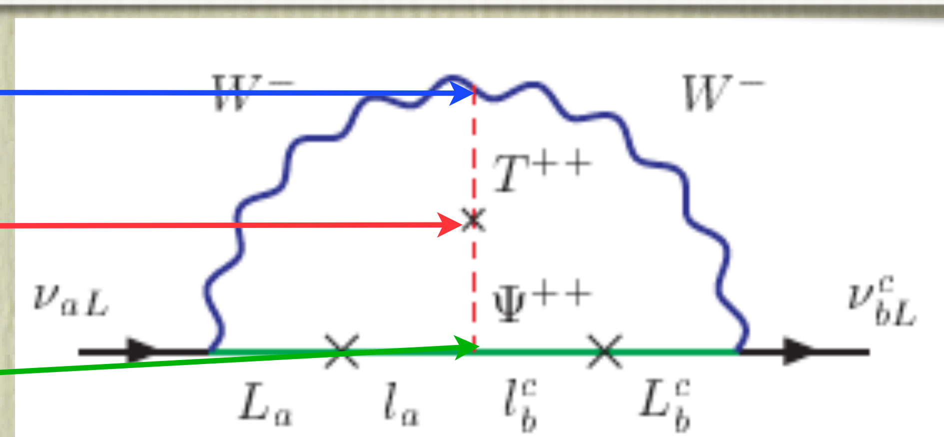
$$Y_{ee} < 0.17, \quad Y_{e\mu} < 0.2, \quad Y_{e\tau} < 0.2$$
$$Y_{\mu\mu} < 3.5, \quad Y_{\mu\tau} < 0.2, \quad Y_{\tau\tau} < 0.02$$

The neutrino masses are generated radiatively at two-loop level

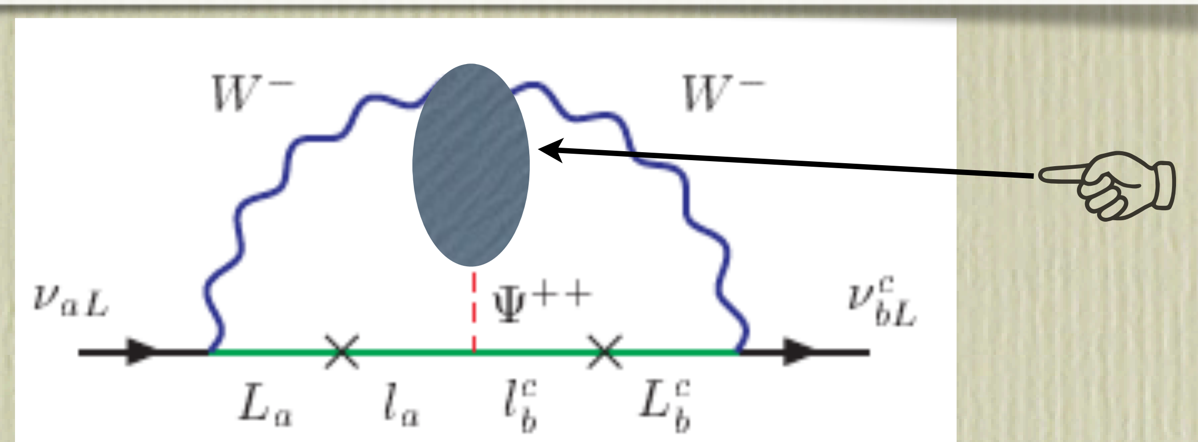
$Tr[(D_\mu T)^\dagger (D^\mu T)]$

$\lambda \Phi^T T \Phi \Psi$

$Y_{ab} \overline{l^c_{aR}} l_{bR} \Psi$

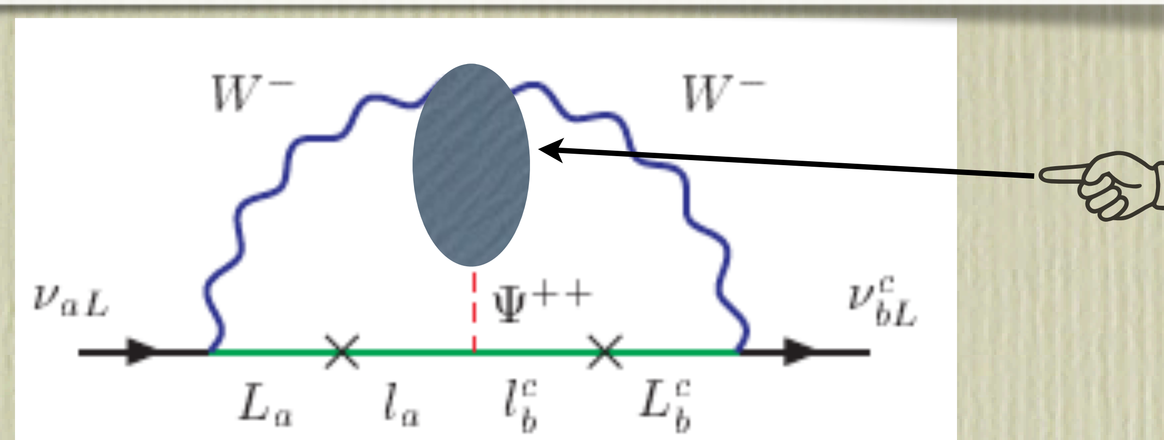


The neutrino masses are generated radiatively at two-loop level



$W^{\pm}W^{\pm}\,\Psi^{\mp\mp}$

The neutrino masses are generated radiatively at two-loop level

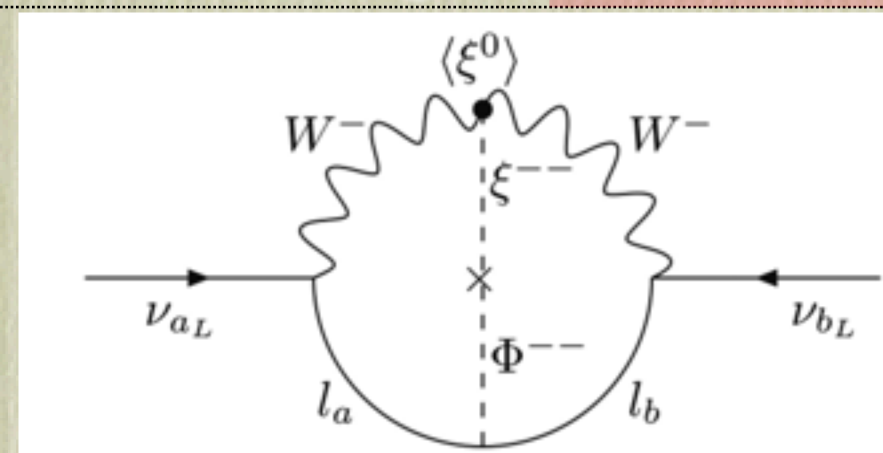


$W^\pm W^\pm\ \Psi^{\mp\mp}$

New model: a multiplet ξ **(1,5,2)** + a singlet Φ (1,1,4)

Chen,CQG,Huang,Tsai, PRD87, 077702 (2014)

Without Symmetry: ~~$LL\xi$~~

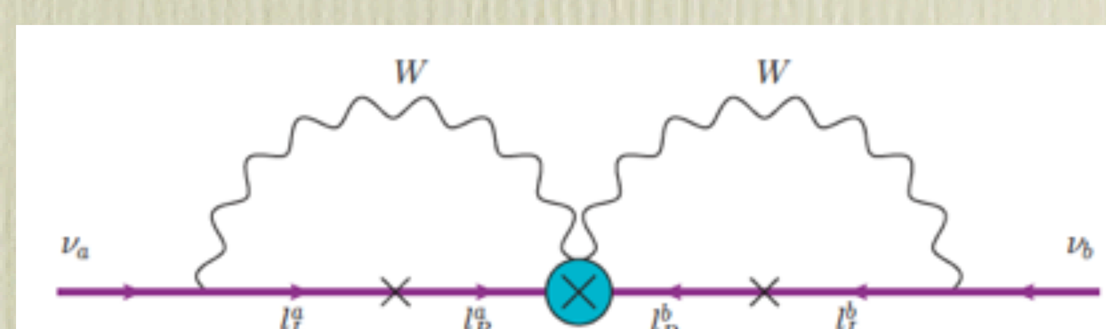


M.Gustafsson,J.M.No,M.A.Rivera, PRD90, 013012 (2014)

dimension-9 L violating O

$$\mathcal{O}^9 \equiv C^{(9)}_{ab}\, \overline{\ell^c}_{R_a} \ell_{R_b} \left[(D_\mu H)^T\, i\sigma_2 H\right]^2$$

$$\frac{C^{(9)}_{ab}}{\tilde{\Lambda}}\, \overline{\ell^c}_{R_a} \ell_{R_b} W^+_\mu W^{+\mu}$$



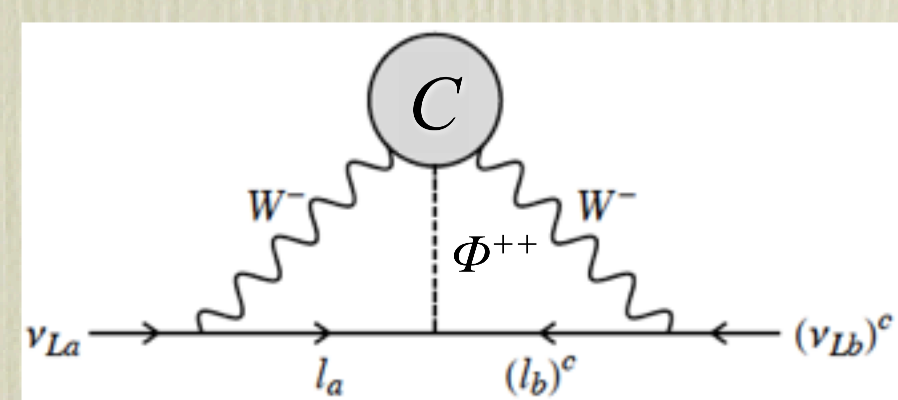
S.F.King,A.Merle,L.Panizzi, JHEP1411, 124 (2014)

dimension-7 O

$$\mathcal{O}^{(a)}_7 = \Phi\, (H \otimes H)_{\underline{3}}\ [(D_\mu H) \otimes (D^\mu H)]_{\underline{3}}$$

$$\mathcal{O}^{(b)}_7 = \Phi\, [(D_\mu H) \otimes H]_{\underline{1}}\ [(D^\mu H) \otimes H]_{\underline{1}}$$

$$\mathcal{O}^{(c)}_7 = \Phi\, [(D_\mu H) \otimes H]_{\underline{3}}\ [(D^\mu H) \otimes H]_{\underline{3}}$$



$$(M_\nu)_{ab} \propto m_a m_b Y_{ab}$$

For $Y_{ab} \sim O(1)$ $\quad (M_\nu)_{ee} \ll (M_\nu)_{e\mu} \ll (M_\nu)_{e\tau} \ll (M_\nu)_{\mu\mu} \ll (M_\nu)_{\mu\tau} \ll (M_\nu)_{\tau\tau}$

→ ***Normal hierarchy***

Z.-z.Xing,PLB530,159(2002);PLB539,85(2002);
Frampton,Glashow,Marfatia,PLB536,79(2002);
W.L.Guo,Z.-z.Xing,PRD67,053002(2003);
……

With $(M_\nu)_{ee} \simeq (M_\nu)_{e\mu} \simeq 0$ and the center values of PDG2014 :

$$\sin^2\theta_{12} = 0.308 \pm 0.017\,,\ \sin^2\theta_{23} = 0.437^{+0.033}_{-0.023}\,,\ \sin^2\theta_{13} = 0.0234^{+0.0020}_{-0.0019}\,,$$

$$\Delta m^2_{21} = \left(7.54^{+0.26}_{-0.22}\right) \times 10^{-5}\,\mathrm{eV}\,,\ \Delta m^2_{32} = (2.43 \pm 0.06) \times 10^{-3}\,\mathrm{eV},$$

CQG+L.H.Tsai, Annals Phys. 365,210 (2016)
CQG, MPLA30,1530018(2015)

$$\rightarrow\quad M_\nu \simeq \begin{pmatrix} 0 & 0 & 1.0e^{-i\eta} \\ 0 & 2.4\,e^{i(\frac{\pi}{2}+\eta)} & 2.3\,e^{i\frac{\pi}{2}} \\ 1.0\,e^{-i\eta} & 2.3\,e^{i\frac{\pi}{2}} & 2.8\,e^{i(\frac{\pi}{2}+\frac{2\eta}{3})} \end{pmatrix} \times 10^{-2}\ \mathrm{eV}$$

Dirac and Majorana phases:

$$\delta = \frac{3}{2}\pi - \frac{3}{2}\eta\,,\ \alpha_{21} = \pi + \frac{3}{2}\eta\,,\ \alpha_{31} = \frac{3}{2}\pi - \frac{1}{2}\eta\,,\quad (\eta \simeq 0.07\pi)$$

Agree well with Global Best-Fit → $\delta/\pi = 1.39^{+0.38}_{-0.27}$

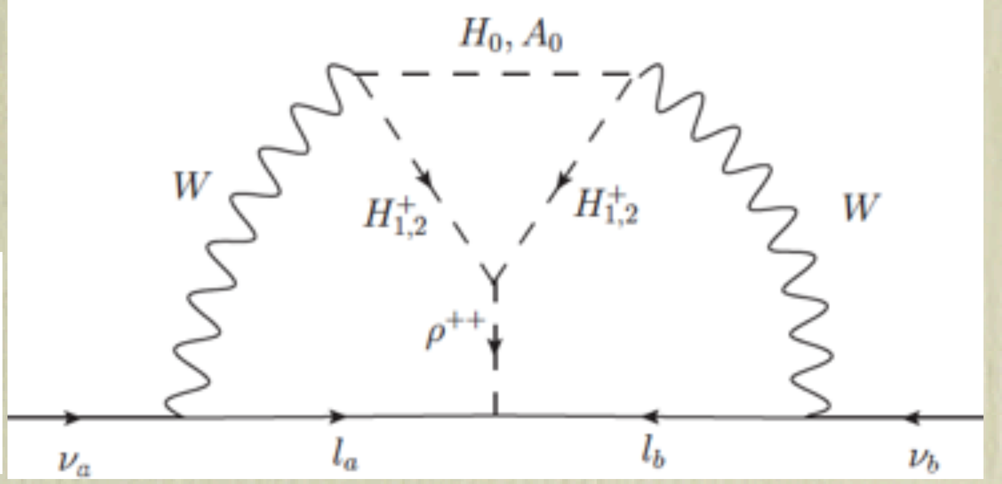
At three-loop level

THE COCKTAIL MODEL FOR NEUTRINO MASSES

M.Gustafsson, J.M.No, M.A.Rivera, PRL110, 211802 (2013); Erratum, PRL112, 259902 (2014).

	$SU(2)_L$	$U(1)_Y$	Z_2
Φ_2	**2**	1	$-$
S^+	**1**	2	$-$
ρ^{++}	**1**	4	$+$

$$-\mathcal{L}_{\rm dark} = \frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \kappa_1\Phi_2^T i\sigma_2\Phi_1 S^- + \kappa_2\rho^{++}S^-S^- + \xi\Phi_2^T i\sigma_2\Phi_1 S^+\rho^{--} + C_{ab}\overline{\ell^c}_{aR}\ell_{bR}\rho^{++} + \text{h.c.},$$



CQG, D.Huang, L.H.Tsai, PRD90, 113005 (2014)

$$(m_\nu)_{ab} = (x_a C_{ab} x_b)\frac{s_{2\beta}}{(16\pi^2)^3}(\mathcal{A}_1\mathcal{I}_1 + \mathcal{A}_2\mathcal{I}_2),$$

$$\mathcal{A}_1 = \frac{[\kappa_2 s_{2\beta} + (\xi v)c_{2\beta}]}{m_\rho^2}\frac{(\Delta m_+^2)^2\Delta m_0^2}{m_\rho^2 v^2},$$

$$\mathcal{A}_2 = \frac{\xi v}{m_\rho^2}\frac{\Delta m_+^2\Delta m_0^2}{v^2},$$

$$\Delta m_+^2 = m_{H_2}^2 - m_{H_1}^2, \quad \Delta m_0^2 = m_{A^0}^2 - m_{H^0}^2$$

$$x_a = m_a/v, \quad I_1 \sim I_2 \sim O(1)$$

For $C_{ab} \sim O(1)$

$$m_\nu = \begin{pmatrix} \approx 0 & \approx 0 & 10.1 \\ \approx 0 & -5.01 & 0.0980 \\ 10.1 & 0.0980 & -4.77 \end{pmatrix} \times 10^{-3} + i \begin{pmatrix} \approx 0 & \approx 0 & 0.23 \\ \approx 0 & -2.37 & -2.33 \\ 0.23 & -2.33 & -2.74 \end{pmatrix} \times 10^{-2}\ \text{eV}$$

$$C_{ab} = \begin{pmatrix} \leq \mathcal{O}(10^{-2}) & \leq \mathcal{O}(10^{-2}) & e^{0.224i} \\ \leq \mathcal{O}(10^{-2}) & 1.90\times 10^{-1}e^{-1.78i} & 1.08\times 10^{-2}e^{-1.56i} \\ e^{0.224i} & 1.08\times 10^{-2}e^{-1.56i} & 7.73\times 10^{-4}e^{-1.74i} \end{pmatrix} \times |C_{e\tau}|$$

- **0νββ decays:**

Y_{ee}

C.S.Chen+CQG+J.N.Ng, PRD75,053004(2007)

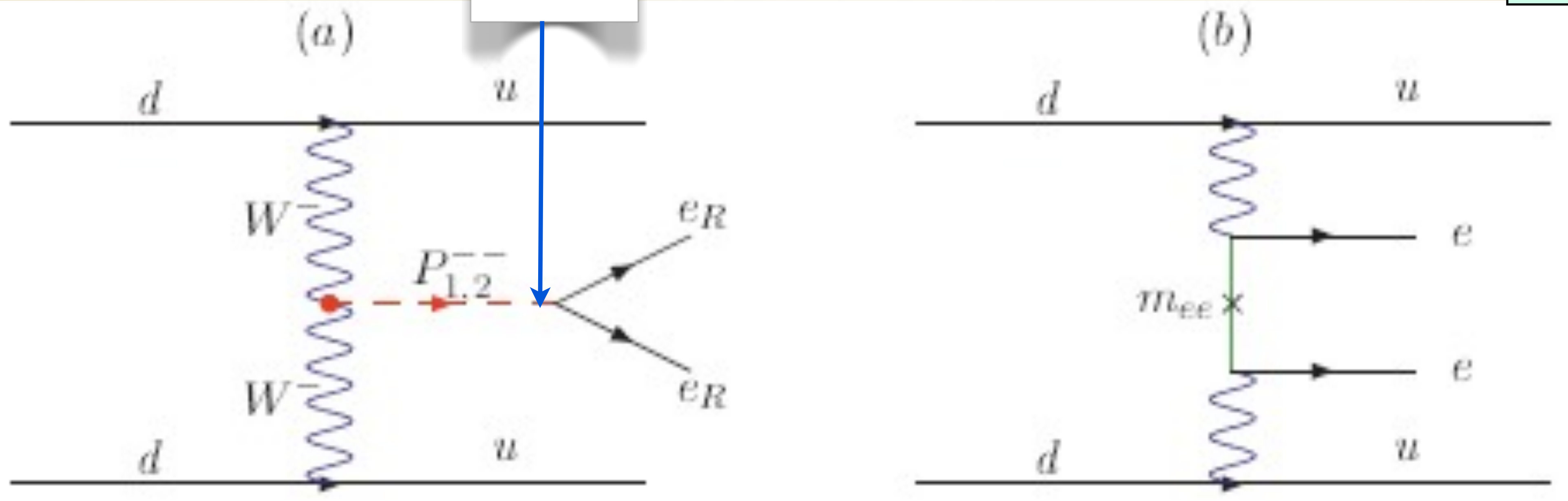


Figure 9: $0\nu\beta\beta$ decays via exchange of: (a) doubly charged Higgs and (b) light Majorana neutrinos.

$$A_{P^{--}_{1,2}} \sim \frac{g^4 Y_{ee} v_T \sin 2\delta}{16\sqrt{2} M_W^4}\left(\frac{1}{M^2_{P_1}} - \frac{1}{M^2_{P_2}}\right)$$

»

$$A_\nu \sim \frac{g^4}{M_W^4}\frac{m_{ee}}{<p>^2}$$

$<p> \sim 0.1\,\mathrm{GeV}$

$$A_\nu / A_{P^{--}_{1,2}} \lesssim 10^{-7}$$

☞ **Black box** theorem is irrelevant as 0νββ dominantly arises from the SD contribution

No other strong constraint on Y_{ee} except the rate of 0νββ itself. So the rate of 0νββ can be very large, which would correlate with the LHC searches.

- **0νββ decays:** Y_{ee}

C.S.Chen+CQG+J.N.Ng, PRD75,053004(2007)

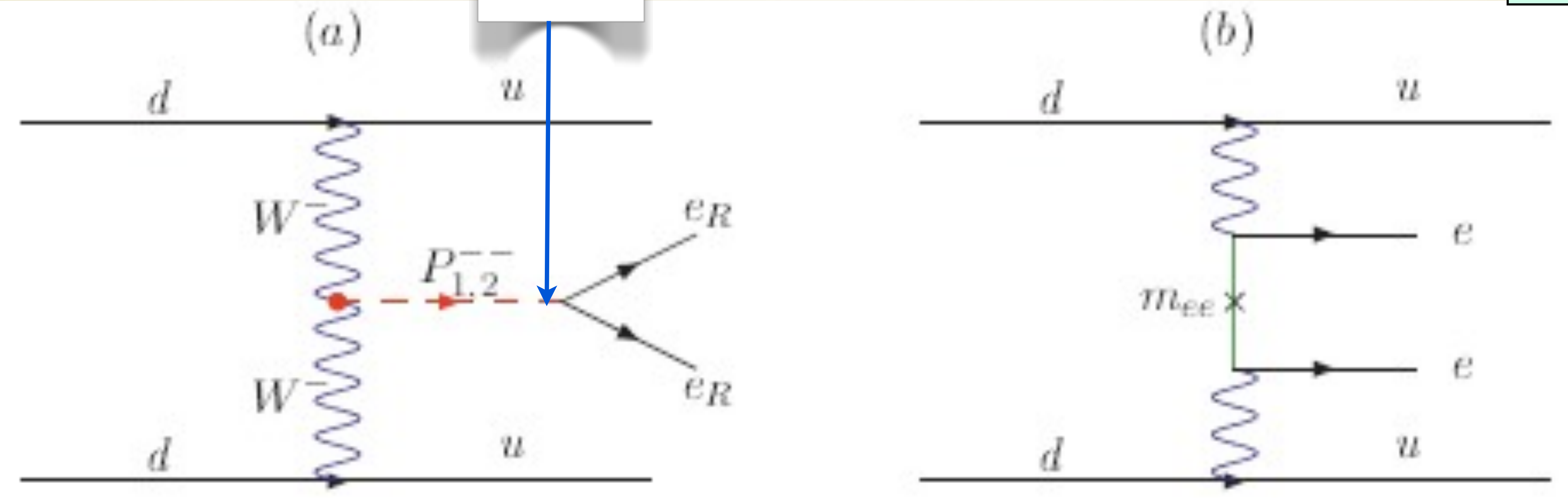


Figure 9: $0\nu\beta\beta$ decays via exchange of: (a) doubly charged Higgs and (b) light Majorana neutrinos.

$$A_{P_{1,2}^{--}} \sim \frac{g^4 Y_{ee} v_T \sin 2\delta}{16\sqrt{2} M_W^4}\left(\frac{1}{M_{P_1}^2} - \frac{1}{M_{P_2}^2}\right)$$

»

$$A_\nu \sim \frac{g^4}{M_W^4}\frac{m_{ee}}{<p>^2}$$

$<p> \sim 0.1\,\text{GeV}$

$$A_\nu / A_{P_{1,2}^{--}} \lesssim 10^{-7}$$

The smallness of this ratio is due to the fact that in our model, m_{ee} is suppressed not only by a two-loop factor, it is also suppressed by the electron mass factor $(m_e/M_W)^2$ coming from the doubly charged scalar coupling.

☞ **Black box** theorem is irrelevant as 0νββ dominantly arises from the SD contribution

No other strong constraint on Y_{ee} except the rate of 0νββ itself. So the rate of 0νββ can be very large, which would correlate with the LHC searches.

Currently, $Y_{ee} < O(10^{-2})$ for $M_P \sim O(1)$ TeV

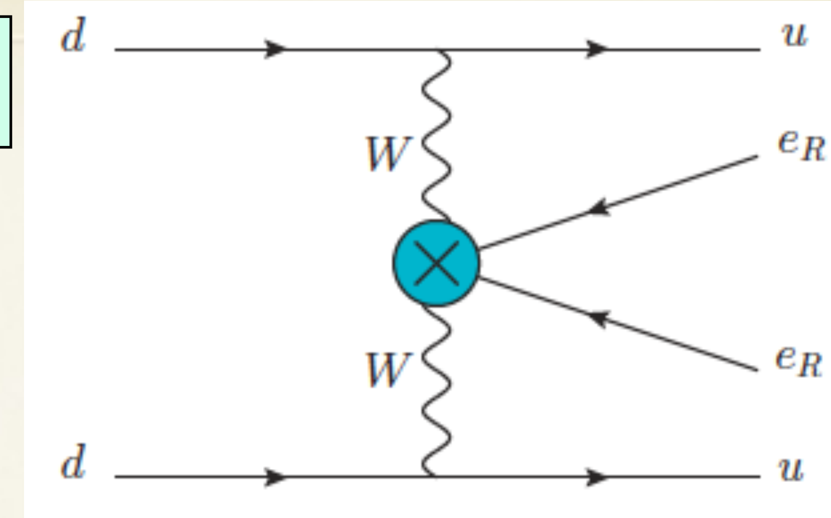
Dimension-9 L violating O

$$\frac{C^{(9)}_{ab}}{\tilde{\Lambda}}\,\overline{\ell^c}_{R_a}\ell_{R_b}W^+_\mu W^{+\mu}$$

M.Gustafsson,J.M.No,M.A.Rivera, PRD90, 013012 (2014)

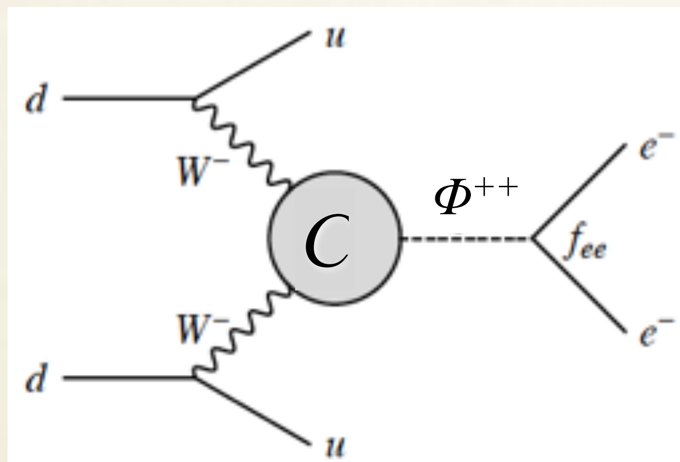
$$\mathcal{L}_{0\nu\beta\beta} = \frac{G_F^2}{2\,m_p}\,\epsilon_3\,J^\mu\,J_\mu\,\bar{e}(1-\gamma_5)e^c$$

$$J^\mu = \bar{u}\gamma^\mu(1-\gamma_5)d \qquad \epsilon_3 = -2\,m_p\,\mathcal{A}^{\rm SD}_{0\nu\beta\beta}$$



Dimension-7 O

S.F.King,A.Merle,L.Panizzi, JHEP1411, 124 (2014)



$$\mathcal{L}^{eff}_{0\nu\beta\beta} = \frac{C\,f_{ee}}{4M_\phi^2\Lambda^3}J_{L\mu}J_L^\mu\bar{e}(1-\gamma_5)e^c$$

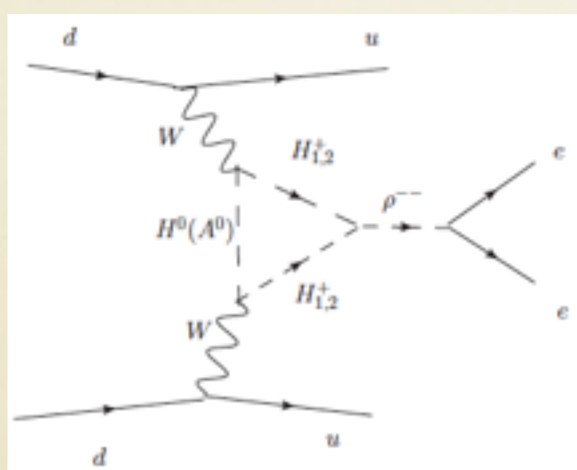
$$\Downarrow$$

$$\frac{C\,f_{ee}}{M_\phi^2\Lambda^3} < 4.0\times 10^{-3}\,TeV^{-5}$$

Cocktail model

Gustafsson,No,Rivera, PRD90, 013012 (2014)

CQG,D.Huang,L.H.Tsai, PRD90, 113005 (2014)



	$> T_{\rm exp}(10^{25}{\rm yr})$	$\lvert C_{ee}\rvert_{\rm max}$
GERDA-1(^{76}Ge) [22]	2.1	0.0015
KamLAND-Zen(^{136}Xe) [23]	1.9	0.0011
NEMO-3(^{150}Nd) [24]	0.0018	0.0060
CUORICINO(^{130}Te) [25]	0.3	0.0016
NEMO-3(^{82}Se) [26, 27]	0.036	0.0059
NEMO-3(^{100}Mo) [27]	0.11	0.0021

$$\mathcal{A}^{\rm loop}_{0\nu\beta\beta} = \frac{\Delta m_+^2 s_{2\theta^+}}{8\pi^2 m_\rho^2}C_{ee}\{[\Delta m_+^2 s_{2\theta^+} - \xi v(c^2_{\theta^+}m^2_{H_2^+} + s^2_{\theta^+}m^2_{H_1^+})][F_{H_1^+,H_2^+,H_0} - F_{H_1^+,H_2^+,A_0}] - \xi v[m^2_{H_0}F_{H_1^+,H_2^+,H_0} - m^2_{A_0}F_{H_1^+,H_2^+,A_0}]\}$$

• Other physics:

Multi Charged Scalars

a. Lepton flavor physics:

1. *Muonium anti-muonium conversion* $\mu^+e^- - \mu^-e^+$ $H_{M\bar{M}} = \frac{Y_{ee}Y_{\mu\mu}}{2M_{--}^2}\bar{\mu}\gamma^\mu e_R\bar{\mu}\gamma_\mu e_R + h.c.,$

2. *Effective* $e^+e^- \to l^+l^-$, $l = e, \mu, \tau$, *contact interactions* $\frac{Y_{ee}^2}{M_{--}^2}\bar{e}_R\gamma^\mu e_R\bar{e}_R\gamma_\mu e_R$

3. *Rare* $\mu \to 3e$ *decays and its* τ *counterparts*

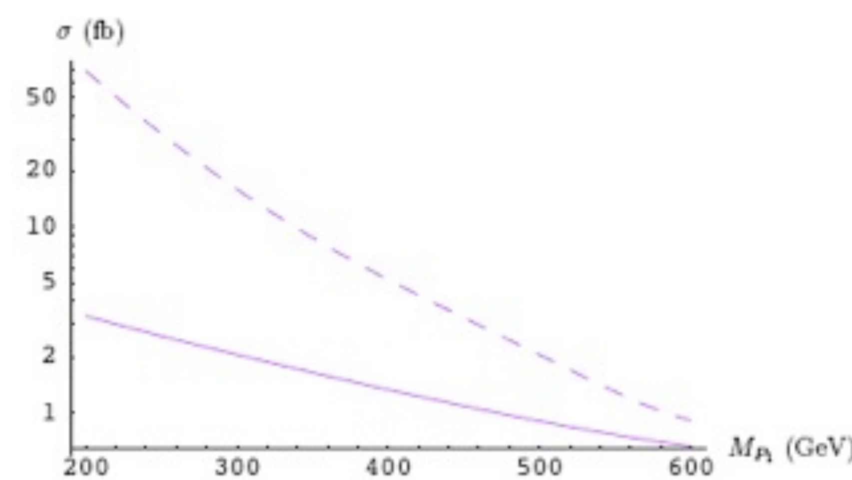
4. *Radiative flavor violating charged leptonic decays* $Br(\mu \to e\gamma) = \frac{\alpha}{3\pi G_F^2}\sum_{l=e,\mu,\tau}\left(\frac{Y_{l\mu}Y_{le}}{M_{--}^2}\right)^2$

b. Doubly charged scalars at the LHC:

1 Production of the doubly charged Higgs

The WW fusion processes similar to 0νββ decays + the Drell-Yan annihilation processes:

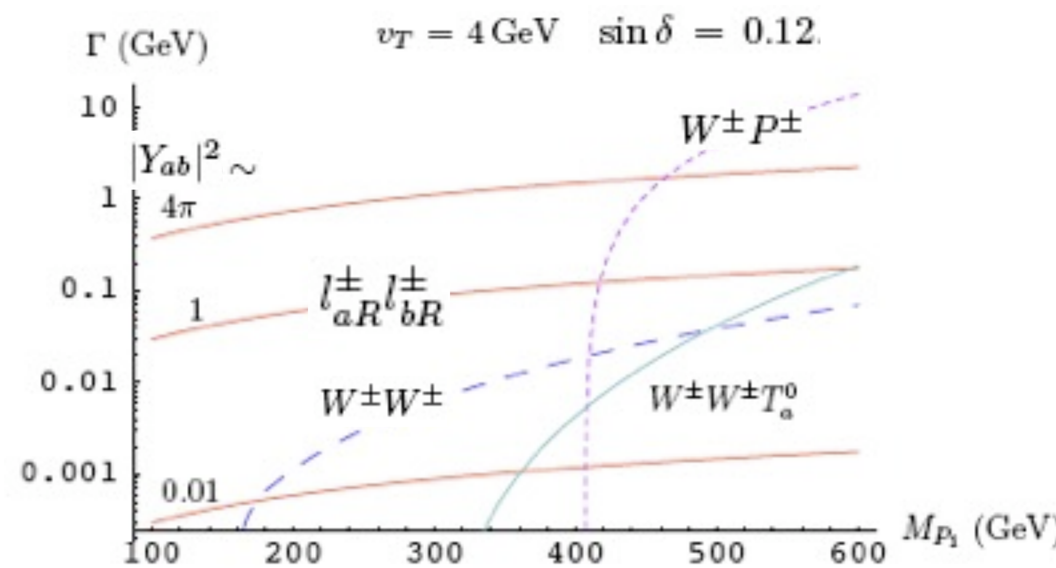
$$q\bar{q} \to \gamma^*, Z^* \to P_1^{++}P_1^{--} \quad (q = u, d)$$



2 The decay of $P_1^{\pm\pm}$

(1) $P_1^{\pm\pm} \to l_{aR}^\pm l_{bR}^\pm \quad (a, b = e, \mu, \tau)$,
(2) $P_1^{\pm\pm} \to W^\pm W^\pm$,
(3) $P_1^{\pm\pm} \to P^\pm W^\pm$,
(4) $P_1^{\pm\pm} \to P^\pm P^\pm$,
(5) $P_1^{\pm\pm} \to W^\pm W^\pm X^0$, $\quad X^0 = T_a^0, h^0, P^0$
(6) $P_1^{\pm\pm} \to P^\pm P^\pm X^0$.

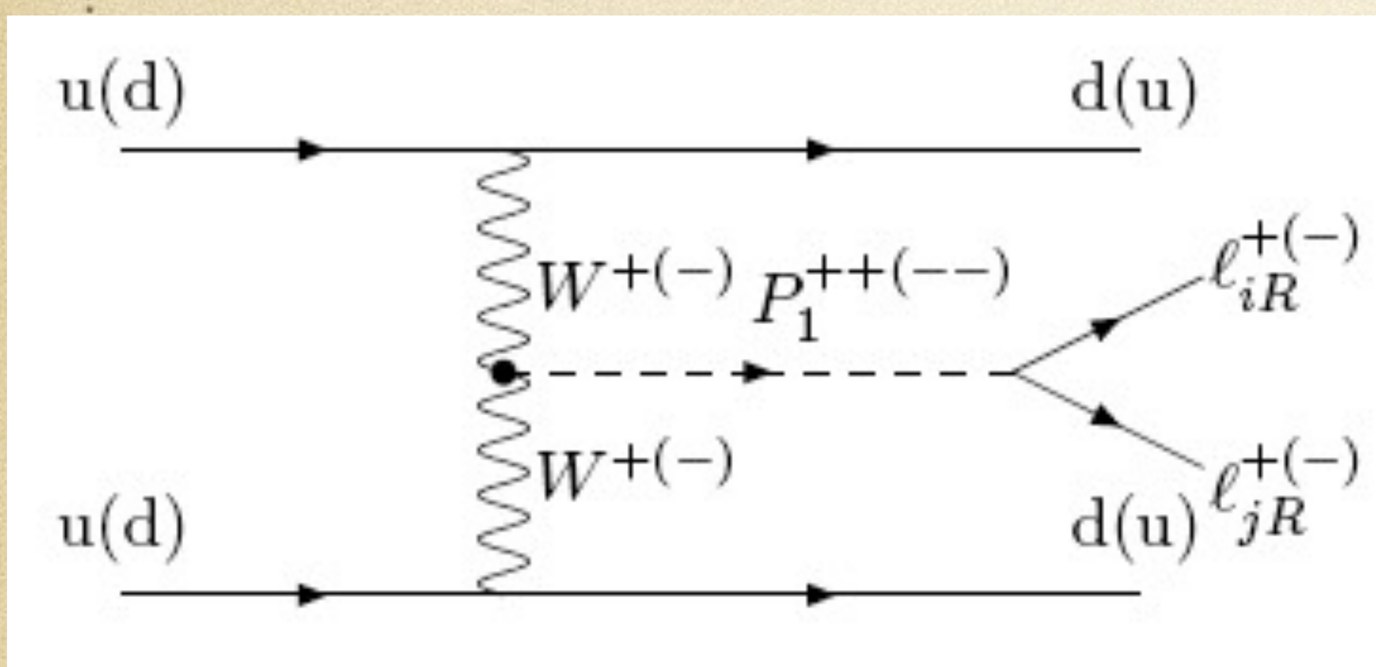
(4) and (6) are not allowed in our model.



C.S.Chen+CQG+J.Ng+ J.Wu, JHEP0708, 22 (07)

c. Same-sign single dilepton signatures:

$$pp \to \ell_i^\pm \ell_j^\pm \underline{X} \quad \leftarrow JJ$$



Chen, CQG, Zhuridov, Eur.Phys.J.C60,119(2009)

$$\frac{d\sigma_\pm^{pp}}{d\cos\theta} = A\left(\lambda_1^{ij}\right)^2 H_\pm^{pp}$$

$$A = \frac{G_F^4 M_W^6}{2^7\pi^5} = 50\ \text{ab},\ \ \lambda_1^{ij} = \sqrt{2-\delta_{ij}}\,|Y_{ij}|\,c_\delta s_\delta,$$

$$H_\pm^{pp} = \left(\frac{v_T}{M_W}\right)^2 \int_{z_0}^1 \frac{dz}{z}\int_z^1 \frac{dy}{y}\int_y^1 \frac{dx}{x} p_\pm(x, xs)\, p_\pm\left(\frac{y}{x}, \frac{y}{x}s\right) l\left(\frac{z}{y}\right) h\left(\frac{s}{M_{P_1}^2}z\right)$$

Remarks:

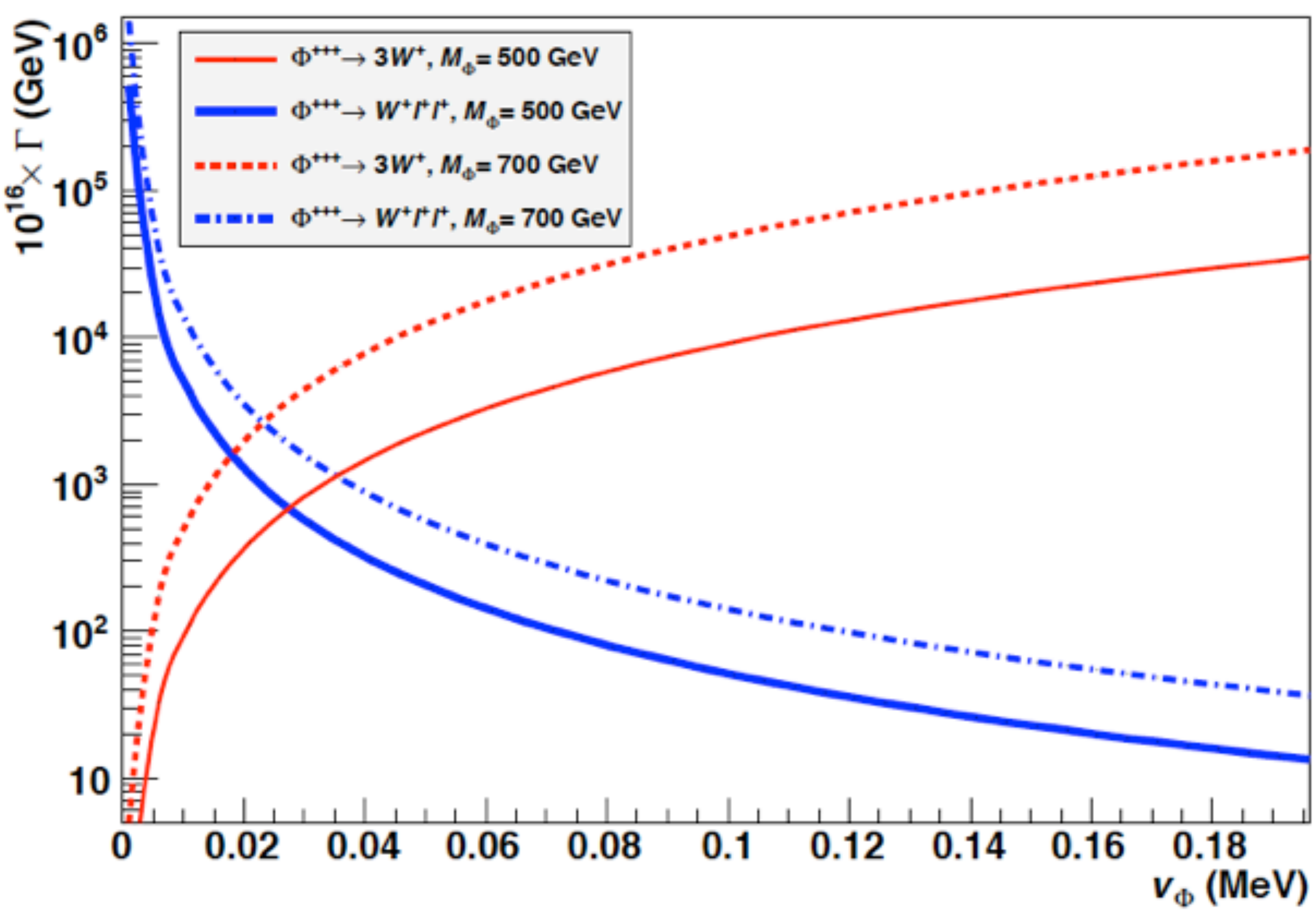
(i) In our model, the final state charged leptons are right-handed. Hence, in principle, helicity measurements can be used to distinguish between our model and those whose doubly charged Higgs coupling only to left-handed leptons (LLT).

(ii) $P_1^{\pm\pm}$ will directly produce spectacular lepton # violating signals from like-sign dileptons such as eμ, eτ and μτ.

d. Triply charged scalar decays:

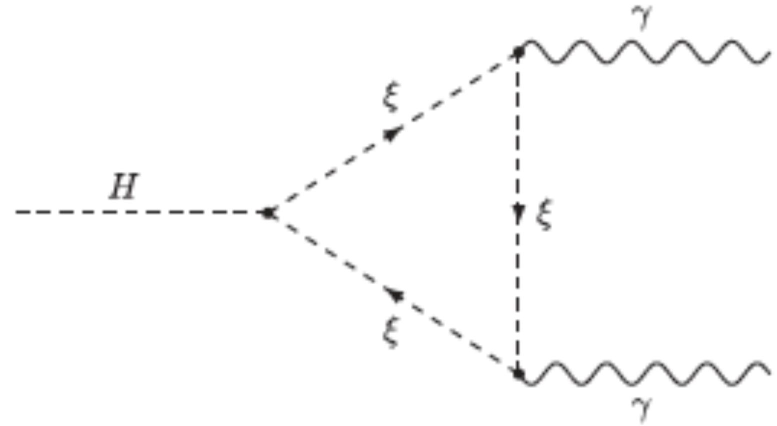
$$\Gamma(\Phi^{+++} \to 3W) = \frac{3g^6}{2048\pi^3}\frac{v_\Phi^2 M_\Phi^5}{m_W^6}$$

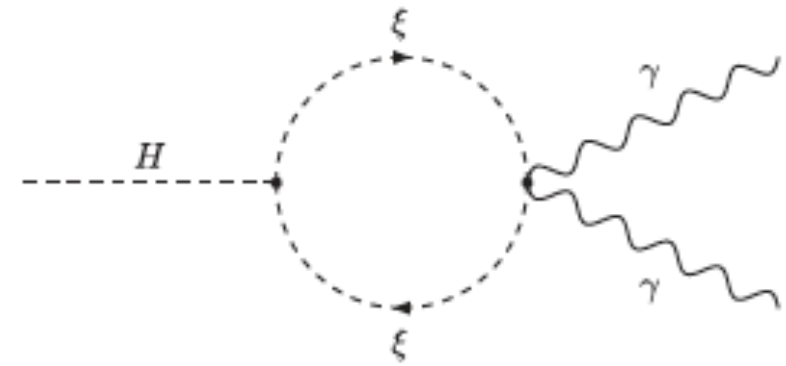
$$\Gamma(\Phi^{+++} \to W^+\ell^+\ell^+) = \frac{g^2}{6144\pi^3}\frac{M_\Phi \sum_i m_i^2}{v_\Phi^2}$$



e. Multi charged scalar contributions to H→ γγ and H→ Zγ :

$$\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 m_H^3}{128\sqrt{2}\pi^3} \left| \sum_f N_f^c Q_f^2 A_{\frac{1}{2}}(\tau_f) + A_1(\tau_W) + \sum_{I_3} (I_3+1)^2 \frac{v}{2} \frac{\mu_s}{m_s^2} A_0(\tau_s) \right|^2,$$





ξ=(1, N, 2) with N=3, 5, ...

e.g. $\xi = (\xi^{+++}, \xi^{++}, \xi^{+}, \xi^{0}, \xi^{-})^T$ **for N=5**

I_3=(-N+3)/2 to (N+1)/2

Chen, CQG, Huang, Tsai, PRD87, 077702 (2013)

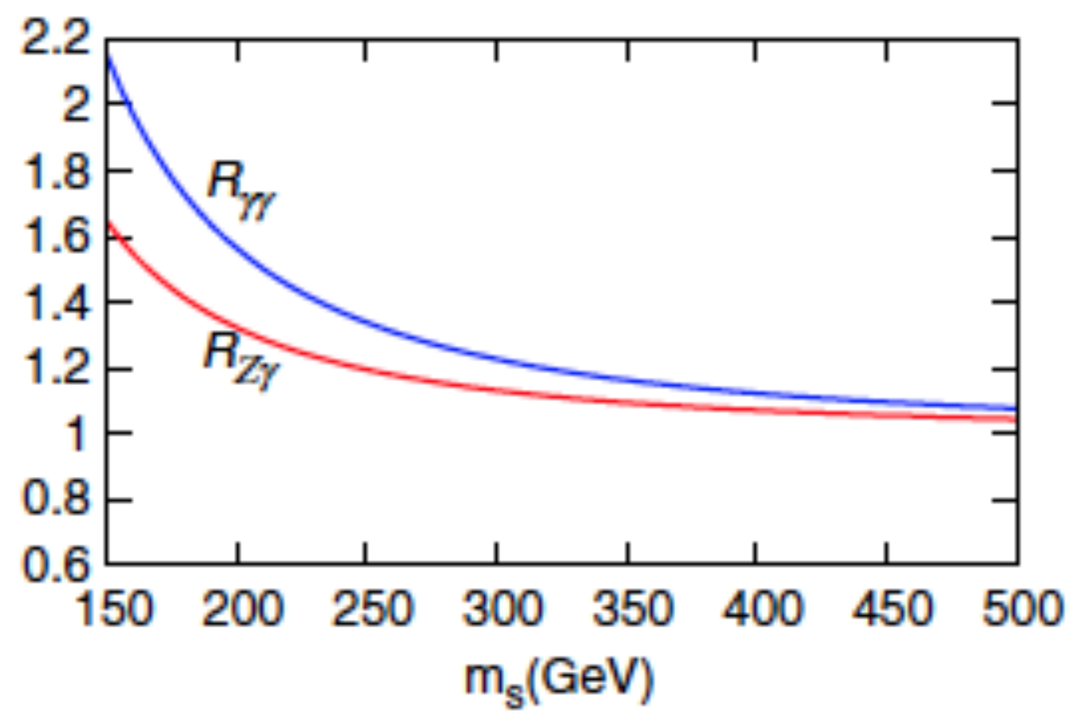


FIG. 4 (color online). $R_{\gamma\gamma} \equiv \Gamma(H \to \gamma\gamma)/\Gamma(H \to \gamma\gamma)_{SM}$ and $R_{Z\gamma} \equiv \Gamma(H \to Z\gamma)/\Gamma(H \to Z\gamma)_{SM}$ as functions of the degenerate mass factor m_s of the multicharged scalar states with $\mathbf{n} = 5$ and the universal trilinear coupling to Higgs, $\mu_s = -100$ GeV.

☺ Open questions in neutrino physics:

1. What are the masses of the neutrino mass eigenstates (ν_i)?

2. Are the neutrino mass eigenstates Dirac or Majorana particles?

3. If $0\nu\beta\beta$ is observed, is ν a Majorana particle?

4. Can we understand the mixing angles in the neutrino sector? Is there a symmetry behind them?

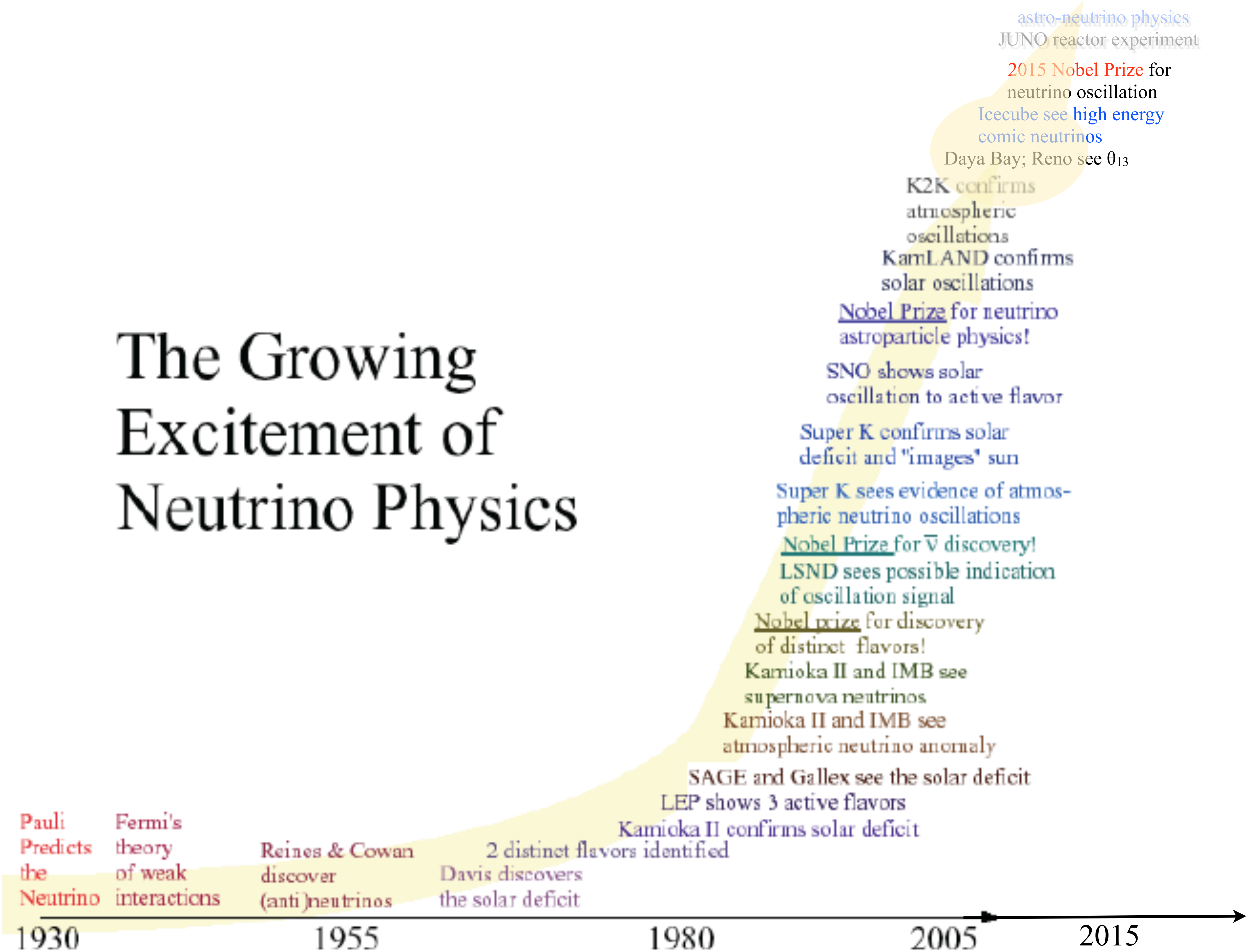
5. Is there CP violation in the neutrino sector?

6. Others: sterile neutrino, dark radiation?

New field: Astro-Neutrino Physics

A real window for new physics

The Growing Excitement of Neutrino Physics



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